PHYSICS

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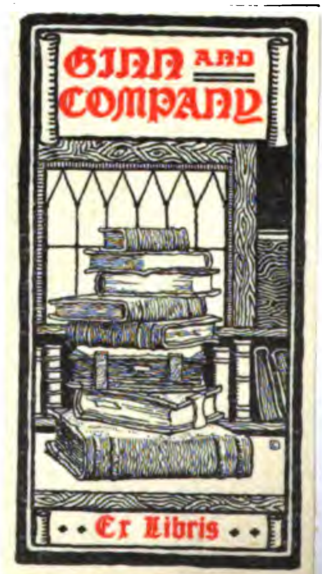
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PLATE I. THE 20TH CENTURY LIMITED

PHYSICS

BY

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SCOTT, FORESMAN AND COMPANY

1905


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be shown to consist in that body of organized knowledge which makes invention possible. Beginning arguments with inventions, or general observations of phenomena, may not be the logical order, but it is more nearly the order in which Nature herself teaches, and the result of the argument does not lose in definiteness, clearness, or accuracy, provided the laboratory is continually held up as the final court of appeal where all doubtful questions are settled.

Each chapter in this book is a continuous argument toward some principle or principles, and the entire book is an argument toward the conclusions stated in the last chapter. This treatment is intended to develop and foster the habit of scientific thinking. The attempt is made (1), to interest the student in observing carefully and accurately first the familiar things about him, and then the things in the laboratory; (2), to interest him in detecting analogies and similarities among the things observed; (3), to train him in keeping his mind free from bias and in drawing conclusions tentatively; (4), to make him see the value of verifying the conclusions and accepting the result, whether it confirms or denies his inferences. The arguments in the various parts of the book are not all alike; there are many forms in which the scientific method may be used.

We have tried deliberately to give the student the impression that science leads to no absolute results—that, at best, it is merely a question of close approximation; of doing the best we can, and accepting the result tentatively, until we can do better. This attitude places the teacher also in the position of a learner and prohibits him from making use of didactic or dogmatic statements; for these are the bane of science as well as of other things. Science instruction that does not develop mental integrity, freedom of the personal judgment, and tolerance, fails in a very vital spot.

III. History. References are given to books in which the biographies of the great men of science may be read, and the student is urged to read them and report. The arguments used by some of the great thinkers have been briefly sketched, and the methods devised by them for reaching conclusions have been given. The attempt has been made to present them as they live

in the ideas which they have handed down to us; to picture their mental processes and attitude, and to show how one thing leads to another as the subject develops in the discoverer's mind.

We wish to call the attention of our colleagues to several practical points. In the first place, although each chapter is a continuous argument, the paragraphs are headed in black type, so that the important steps are well marked; and a summary and set of questions are added at the end of each chapter, to assist the student in fixing the subject-matter in mind. The teacher will, we think, find these latter very helpful to his pupils in both advance and review work.

In the second place, the continuity of the treatment is not interrupted by the insertion of descriptions of laboratory and lecture experiments in fine type. Judged from our own experience, such experiments, thus inserted, confuse rather than assist the student. It goes without saying, that we expect both laboratory and lecture experiments to be given in connection with this book; but every laboratory experiment made by the student, and every experimental demonstration by the teacher should have a definite relation in time, place, and subject matter to the general argument as presented in the text. An experiment is simply an incumbrance and a source of distraction to the student unless its relation to the general scheme of the lessons in the classroom is perfectly obvious. A detailed description of a lecture experiment which he has not seen is of relatively small value to the student, and ordinarily there is no interest or profit to him in obtruding on his attention the distracting details of setting up and manipulating the apparatus. If such description of an experiment occurs in the text book, while the teacher chooses to make it with some other style of apparatus, different in its details, his confusion is all the worse, for his attention is distracted from the principle to be illustrated, and lost in the details of the apparatus.

On the other hand, when the student is to make an experiment himself in the laboratory, he must be given many details in order that he may manipulate, observe, and record successfully and without loss of time. It is the province of the laboratory manual to give these details, for they can not be included in a text

book without encumbering it to the exclusion of important theoretical matter, and destroying its unity. We have therefore preferred to leave the choice of illustrative experiments largely in the hands of the teacher, who may thus select them according to his individuality, his equipment, and the circumstances and limitations of his class and community.

We have based the argument wherever possible on the pupils' experience, expecting this to be supplemented by the teacher with lecture demonstrations and laboratory experiments, chosen in accordance with the conditions which he has to meet and with his own taste and judgment. But when a particular kind of experimental evidence is necessary to the argument, it has been used, without manipulatory details and in uniform type with the other subject matter.

In the third place, many of the old and familiar landmarks of the elementary physics text do not appear in these pages. Among these may be mentioned the division of levers into classes; the wedge; the classification of equilibrium as stable, unstable, and neutral; specific gravity as distinguished from density; the electrophorous and the electrostatic machine; the concave and convex mirrors; multiple reflection; and the formulas concerned with the radii of curvature of lenses. These have been omitted because they seem of less interest and importance than the following new subjects which we have been able to introduce in the space thus saved: The use of graphical methods and of vectors; the discussion of efficiencies of engines, both practical and theoretical; the relations among electrostatic charge, current, and magnetic field; the meaning of harmony; the nature of spectra; the reasons for the electromagnetic theory of light; and the electron theory of matter. We also believe that the presentation of the subjects of rotary motion and of optical instruments will be found much simpler and more satisfactory than those usually given.

The problems are also an innovation. They include no cases of forces *a*, *b*, and *c*, meeting at a point *q*, etc., but are, as far as possible, real, concrete cases, such as occur in actual practice, and which every boy or girl ought to know how to meet.

They also contain many of the subjects usually placed in the text and there explained; for example, the pulleys, distillation, and the Wheatstone bridge. We hope that this form of problem will interest the student, as most of them are problems in whose solution he can see some use.

Other devices for catching and holding the interest are the questions and the suggestions to students at the end of each chapter. We hope that these latter will be stimulating to the students and serve as hints which will lead them to suggest for themselves other home experiments. Are not such experiments, clumsy though they be, yet made with a genuine interest in finding out something—in getting the answer from Nature herself—far more useful than many that are made in some laboratories?

The illustrations are also a novelty. Great pains have been taken to have every picture a photograph of a real thing, for a photograph is always more interesting than a woodcut. It is believed that these will add much to the interest of the work.

We have been favored with the original photographs for many of these illustrations, by the firms and individuals mentioned on page x, whom we wish to thank for their courtesy.

We also desire to express our thanks to Professor R. D. Salisbury of the University of Chicago, Editor-in-Chief of the Lake Science Series for many valuable suggestions, and to Messrs. A. A. Knowlton of the Armour Institute of Technology, J. H. Kimmons of the Austin, Chicago High School, and C. Kirkpatrick of the High School, Seattle, Washington, for aid in the reading of the proof.

Many of the line diagrams are new and have been designed and executed with much thought and care, so as to present the essential ideas without complication by unnecessary details.

That great difficulties are involved in the working out of a method of instruction differing in principle from that in general use must be apparent to every one. We know better than any one else can that we have not produced a perfect book. This might be approximated by the concerted action of all teachers of physics. We therefore hope that members of the teaching fraternity will regard the result of our work as a first approximation, and will

join with us in making a united effort to lift our subject up to its proper place, and to inspire our young friends with an adequate appreciation of its interest, its majesty, and its grandeur. To this end we appeal to our colleagues to give us the benefit of their experience by sending us suggestions and criticisms, which will be gratefully received and carefully considered.

CHARLES RIBORG MANN,
GEORGE RANSOM TWISS.

ACKNOWLEDGMENT OF ILLUSTRATIONS

PLATE I. The Lake Shore and Michigan Southern Railway. Large copies of this picture in color may be obtained for 50 cents, by applying to Mr. A. J. Smith, General Passenger Agent, Cleveland, Ohio.

FIG. 11. The Electric Vehicle Co., Hartford, Conn.

FIGS. 16, 17, 18. The Eastman Kodak Co., Rochester, N. Y.

FIG. 19. Pawling, Harnischfeger & Co., Milwaukee, Wis.

PLATE II, and FIGS. 70, 156, 157, 158. The Niles-Bement-Pond Co., New York.

FIG. 31. The Manitou and Pike's Peak Railroad Co., Manitou, Col.

FIG. 51. Crowe Bros., House Movers, Chicago, Ill.

PLATES III, IV, VI. The Allis-Chalmers Co., Milwaukee, Wis.

FIG. 61. The Bausch and Lomb Optical Co., Rochester, N. Y.

FIGS. 65, 72. The Ingersoll-Sargeant Drill Co., New York.

FIG. 74. The Chicago Bridge and Iron Works Co., Chicago, Ill.

FIG. 77. The Century Co., New York.

FIGS. 91, 92. The Whitlock Coil Pipe Co., Hartford, Conn.

FIG. 100. The Otto Gas Engine Co., Philadelphia, Pa.

FIG. 101. Mr. Alfred Stieglitz, New York.

PLATE VII, and FIGS. 102, 131, 132, 133, 141, 142, 143, 144, 147, 148, 151. The Westinghouse Electric Co., Pittsburg, Pa.

"The Electric Spark in Nature," page 205. Mr. M. I'Anson, Newark, N. J.

FIG. 162. The Electric Controller and Supply Co., Cleveland, Ohio.

FIGS. 168, 169, 170. The Electric Storage Battery Co., Philadelphia, Pa.

PLATE VIII. The University of Chicago, Chicago, Ill.

FIGS. 236, 237. Wm. Scheidel & Co., Chicago, Ill.

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PHYSICS

INTRODUCTION

It has been said that man made his start on the long road toward enlightenment when he learned how to make a fire. For many centuries, our ancestors groped at a snail's pace along this road where we of the twentieth century are advancing by leaps and bounds.

By slow and painful steps, prehistoric man learned to use fire in order to keep himself warm, to cook his food, to get metals out of their ores, and to forge them into rude tools and weapons of defense. By means of signal fires on the hilltops, he sent his first wireless messages across the valleys. The magnetic force of the lodestone and the electric attraction of amber were known to the ancients, and the fact that steam pressure can be made to produce motion was known in the early centuries of our era. Why was it that so many centuries elapsed before man learned to subdue these forces of nature and make them do his will? Now we have the steam engine, the electric dynamo and motor, the power printing press, the power loom, the telephone, the wireless telegraph. By means of these and countless other inventions, one man can do the work of hundreds, the continents are linked together, darkness is turned into light, time and space are vanquished.

We can best realize how important are these inventions when we try to think how we should get on without them. And yet this great development of miracle working machinery has come within the space of three centuries, and the greater part of it within seventy-five years!

The stories of how these marvelous inventions came to be, of the struggles of the men who brought them into being, and of the pa-

tient researches and brilliant discoveries of the men of science who established the foundation principles upon which all these inventions rest are among the most important and most interesting chapters of history.

In the studies which follow, we shall endeavor to get an understanding of some of these principles, to gain at least a slight acquaintance with some of the great discoverers who formulated them, and to get some insight into the kind of thinking and the methods of experimentation by which their truth has been made plain. Such studies are of interest not only to those who expect to make practical use of them, but also to those who, in the pursuit of a liberal education, wish to learn how to think clearly, to express themselves precisely, and to test their conclusions accurately, as well as to get a properly balanced view of human life and activity.

The principles of physics are most easily understood by the beginner, and are also most interesting, when they are studied in connection with his own experiences. For no one can live long in this scientific age, surrounded as he is on all sides by the fruits of discovery and invention, without having a large amount of experience with the forces of Nature and without obtaining therefrom a large fund of general information.

A rapidly-moving railway train is certainly a familiar object to every one. Even a small child would not have to be told that Plate I is the picture of such a train. Moreover, we all know that the locomotive causes the train to move, and that it can not do so unless it has a fire in it. We are also familiar with the fact that the locomotive must be supplied with water, and that in the boiler this water is converted into steam, which somehow makes the big driving wheels turn. That such an engine warns us of its approach by means of a whistle and a bell, and that it lights its own path in front of it at night by means of a brilliant headlight, are well-known facts.

Now, although these and many other things about the locomotive are matters of everyday knowledge to most of us, how many of us can tell exactly how the steam makes the engine's driving wheels turn? And why is steam used at all? Why are some loco-

motives large, while others are small? How does the whistle work, and how does its sound get to us? How is the headlight made to send its light forward on the tracks?

A ride in a steam or trolley car is one of the most common of our experiences, and we all know that the car has many different kinds of motion. Wherein do these motions differ? How are speeds measured and compared with one another? What sort of velocity has the car while it is starting or stopping? Why are we thrown against the side of the car when it rounds a curve? How is it that some engines can go faster than others?

The picture of the Twentieth Century Limited (Plate I) was taken while the train was running at full speed. How do cameras and lenses work?

We can obtain the answers to these and to other similar questions without great difficulty, if we are willing to devote to the subject some careful study and thought. When we have done this, we shall find that the knowledge thus acquired gives us a greater control over the forces of Nature, and that the training thus obtained is of great service to us in everything we may wish to do.

CHAPTER I

MOTION, VELOCITY, ACCELERATION

1. The Motion of a Train. In order to find the answers to some of the questions just asked, let us suppose that a locomotive stands with steam up, ready to make the run to the next station. When it starts, we notice that at first it moves slowly, and that its velocity gradually increases until it has attained "full speed," when it runs for some time at a rate that is nearly constant. As the next station is approached, the speed gradually decreases; and the train comes to a full stop. What can we learn of its motion, of the way in which it rounds curves, of how it gets up speed, and of how it stops? How shall we describe and measure its velocity, and how take account of the energy that it must expend in order to move its load?

2. How Velocity is Measured. Since all motion implies both *distance* and *time*, and since distances and times must be measured in order to be compared, it is necessary to have units of length and of time in terms of which the measurements can be expressed.

The units adopted in all scientific work are purely arbitrary, and are chosen simply for convenience. The unit of length is the CENTIMETER, which is the one-hundredth part of the distance between two lines on a certain bar of platinum-iridium when the bar is at the temperature of zero degrees Centigrade. This bar is carefully preserved at Paris, and is called the International Prototype Meter. The symbol for centimeter is cm, and that for meter is m. The unit of time is the SECOND, which is the one-eighty-six-thousand-four-hundredth of the mean solar day. Its symbol is sec.

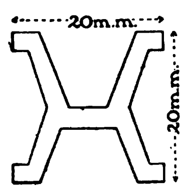


FIG. 1
CROSS-SECTION OF STANDARD METER

Now if a train, moving uniformly, attains in one minute a

distance of 150,000 cm from a given post in a certain direction, then in one second the change in its distance in that direction from the post will be $\frac{1}{60}$ of 150,000 or 2500 cm. Therefore it travels at such a rate that its distance from the post changes 2500 cm every second. This rate of change of distance is called LINEAR VELOCITY. Since linear velocity is rate of change of distance, and since distance is measured in a definite direction, linear velocity implies not only a rate but also a direction. *When both the rate and the direction are specified, the linear velocity is completely described.*

FIG. 2
STANDARD CLOCK

If, without changing the direction of its motion, a body traverses equal distances in equal times, no matter how small the time intervals are taken, its velocity is UNIFORM or constant. THE UNIT OF VELOCITY used in physics is the velocity of a body moving uniformly over one centimeter every second. It is read, one centimeter per second, and its symbol is $\frac{\text{cm}}{\text{sec}}$. Accordingly, the velocity of the train is written 2500 $\frac{\text{cm}}{\text{sec}}$. Note that the value of the linear velocity is obtained by dividing a number of units of length by a number of units of time.

3. There are Two Methods of Expressing Relations of this Kind. By one of these methods, the *graphical*, the relations are presented to the eye by means of a diagram, so that one can see at a glance, relations that would require many pages for their description. By the other method, the *analytical*, the same relations are presented by means of an algebraic equation. This method has the advantage of great conciseness, and also enables us to discover that relations belonging to one group of phenomena may also belong to other groups that at first sight appear to differ widely from one another; and thus we may express in a single equation a relation common to a large number of particular phenomena.

4. The Graphical Method. A graphical expression for uniform motion, like that of the train, may be obtained by drawing two straight lines OX and OY, Fig. 3, intersecting at right angles in a point O. The time intervals we represent by distances from OY in the direction OX, and the corresponding spaces traversed, by distances from OX in the direction OY. In each case we choose a *scale* that is convenient to the size of the page. In practice this plotting is done on squared paper, prepared expressly for this purpose.

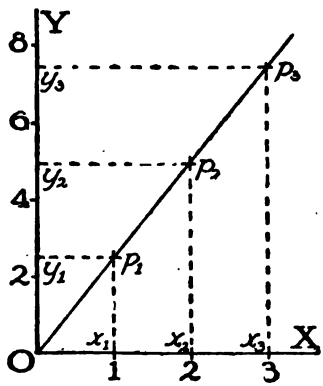


FIG. 3. UNIFORM VELOCITY

Let us begin to consider the motion at the instant when the front of the engine, going at the rate of 2500 $\frac{\text{cm}}{\text{sec}}$, passes a certain post. At this instant, since the time is zero and the distance is also zero, the condition of the train is represented by the point O. If we choose our scale so that 1 cm measured in the direction OX represents 1 sec, and 1 cm in the direction OY represents a distance of 2000 cm, then we shall find the point that represents the condition of our train at the end of 1 sec by laying off from O 1 cm along OX to a point x_1, and by laying off from O along OY 1.25 cm to a point y_1. Drawing dotted lines from x_1 and y_1 parallel to OY and OX, respectively, we find that they intersect in a point p_1. Since its distance from OY represents 1 sec, and its distance from OX 2500 cm, this point p_1 represents the condition of the train at the end of the first second with respect to both time and distance.

To find the point p_2, which represents the condition at the end of the second second, we must lay off from O 2 cm along OX to a point x_2, and also 2.5 cm along OY to a point y_2, and draw dotted lines from x_2 and y_2 parallel to OY and OX, respectively. The intersection of these lines is the point p_2, which is the point sought. In like manner the point p_3, distant 3 cm from OY and 3.75 cm from OX, represents the condition of the train at the end of the third second; and so on.

We next draw the straight lines Op_1, $p_1\,p_2$, $p_2\,p_3$, etc. Is the resulting line Op_3 straight? Do the points that represent the condition of the train's motion at 0.5, 1.7, 2.2, 2.5 sec also lie on this line? Is there on the line a point corresponding to every possible instant of time? Does every such point also represent a distance from the post? Does the line Op_3 completely represent the motion of the train with respect to both distance and time? How may the diagram be used to find the distance corresponding to any instant of time? How may it be used to find the time instant at which the train arrives at a given point?

Since we shall often use the graphical method, we shall need to know the names of the lines and points. The two lines OX and OY are called COÖRDINATE AXES. The distances Ox_1, Ox_2, Ox_3, etc., are called ABSCISSAS. They may be measured from any point on OY along a line parallel to OX; thus $y_1\,p_1 = Ox_1$, $y_2\,p_2 = Ox_2$, etc. The distances Oy_1, Oy_2, Oy_3, etc., are called ORDINATES, and may be measured from any point on OX along a line parallel to OY. OX is called the AXIS OF ABSCISSAS, and OY the AXIS OF ORDINATES. Because the axes are drawn at right angles to each other they are called *rectangular coördinate* axes. O is called the *origin of coördinates*. The line Op_3 representing the relations considered is called a GRAPH.

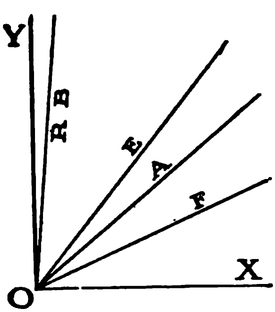


FIG. 4
SLOPE INDICATES VELOCITY

5. What the Slope Indicates. Let us now add to our diagram graphs for two other trains, one of which is a fast freight F traveling uniformly at the rate of 1500 $\frac{cm}{sec}$, and the other an express E at the very high speed of 3000 $\frac{cm}{sec}$. The result is shown in Fig. 4. In what respect are the second and third graphs like A, the first? Which graph has the steepest SLOPE, or in other words which makes the greatest angle with the axis of abscissas? Would the graph for a slow freight traveling at a rate less than 1500 $\frac{cm}{sec}$ have a greater or a less slope than that of the fast freight? Would the graph RB for a rifle ball having a

speed of 80,000 $\frac{\text{cm}}{\text{sec}}$ make a greater or smaller angle with the axis of abscissas than does that for the express? *What characteristic of the motion is indicated by the steepness of the slope?* Can we always find a straight line that will represent a given uniform motion?

6. The Analytical Method. We shall now consider the analytical method of representing the motion of the train. In Art. 2, by an analysis of that motion, we found that *the value of the linear velocity is obtained by dividing the distance by the time.* Hence, if l represent the distance traversed during a number of seconds denoted by t, and if v represent the linear velocity, then,

$$v = \frac{l}{t}. \qquad (1)$$

This is the EQUATION FOR UNIFORM MOTION. It also expresses the average velocity for the time t, if the velocity is changing during that time.

If the right values of l and t, for any one of the three trains mentioned in Art. 5, are substituted in equation (1), will it represent the velocity of that train? How will the numerical value of the ratio $\frac{l}{t}$ for the fast freight compare with that for the express? How will the value of the ratio $\frac{l}{t}$ for the slow freight compare with that for the fast freight? How will the value for the rifle ball compare with that for the express train? It must now be evident that in the analytical method, the velocity of the motion is defined by the ratio of l to t, and that in the graphical method the velocity of the motion is defined by the slope of the graph. *The slope and the ratio* $\frac{l}{t}$, *then, serve the same purpose.*

To get a NUMERICAL MEASURE OF THIS SLOPE of the graph at any point q, draw from this point a parallel to OX, Fig. 5; from any other convenient point p on the graph drop a perpendicular pm to this horizontal line. The angle pqm at the base of the right triangle thus formed is evidently the ANGLE OF INCLINATION OR SLOPE. The ratio of the side opposite this angle to the side

adjacent is constant for all right triangles having this angle; and therefore the numerical value of this constant ratio is an appropriate measure of the inclination of the oblique line to the horizontal. In Trigonometry, this ratio of the side opposite an acute angle in a right triangle to the side adjacent is called the TANGENT OF THE ANGLE; and its numerical value can be found by measuring the lengths of these two lines and dividing the length of the former by that of the latter. Or, if the magnitude of the angle itself has been measured, this value of its tangent can be taken direct from a TABLE OF TANGENTS.

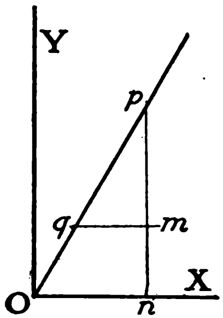


FIG. 5
MEASUREMENT OF SLOPE

7. Increasing Velocity. Thus far we have considered the motion of the train only when it is uniform. What now are the characteristics of the motion just after the engineer has opened the throttle, so that the train is getting up speed; and what of the motion when he has shut off the steam and applied the brakes, so that the train is slowing down?


FIG. 6. READY TO START

Since, now, the velocity is changing at every instant, it can not be measured by the distance traversed in one second. Therefore the VELOCITY AT ANY INSTANT is measured by the distance which would be traversed in one second, provided that throughout that second the rate were to continue the same as it was at the given instant. Suppose now that the train starts from rest, and that at the end of the first second it has gained a velocity of 50 $\frac{cm}{sec}$, that at the end of the second second its velocity is 100 $\frac{cm}{sec}$, and at the end of the third second 150 $\frac{cm}{sec}$; i.e., suppose that the velocities at the end of successive seconds are as follows:

sec	$\frac{\text{cm}}{\text{sec}}$	sec	$\frac{\text{cm}}{\text{sec}}$
0	0	5	250
1	50	6	300
2	100	7	350
3	150	8	400
4	200	etc.	etc.

Is the change of velocity for any one second the same as for any other second, i.e., is the CHANGE OF VELOCITY constant? If during the interval between the end of the eighth second and the end of the twelfth the velocity changed uniformly from 400 to 600 $\frac{\text{cm}}{\text{sec}}$, what was the *rate of change of velocity*, i.e., the change of velocity for any one second?

8. Decreasing Velocity. Again, let us suppose that when the train is slowing down, its velocity changes in the first second from 2500 to 2400 $\frac{\text{cm}}{\text{sec}}$, and that at the ends of the successive seconds the velocities are as follows:

sec	$\frac{\text{cm}}{\text{sec}}$	sec	$\frac{\text{cm}}{\text{sec}}$
0	2500	5	2000
1	2400	6	1900
2	2300	7	1800
3	2200	8	1700
4	2100	etc.	etc.

What is now the rate of change of velocity? Since this rate of change is the *ratio of the change of velocity to the time*, it is expressed as a number of $\frac{\text{cm}}{\text{sec}}$ *per second.* Thus if the rate of change of velocity is such that 75 $\frac{\text{cm}}{\text{sec}}$ is gained or lost each second, then this rate of change is expressed as 75 *centimeters per second per second.* It is customary to write this 75 $\frac{\text{cm}}{\text{sec}^2}$.

9. The Name Given to Rate of Change of Velocity is Acceleration. When the velocity is increasing, the acceleration is *positive;* and when the velocity is decreasing the acceleration is *negative.* When the acceleration is *constant,* as in the examples just given, the motion is called UNIFORMLY ACCELERATED MOTION.

It is to be noted that *the expression for linear acceleration is obtained by dividing a number of units of velocity by a number of units of time.* Since velocity is length divided by time, it is plain that acceleration is length divided by the square of time. Hence the symbol for the unit of acceleration is $\frac{\text{cm}}{\text{sec}^2}$.

10. The Analytical Expression for Acceleration. The analytical expression for acceleration may be found as in Art. 8, except that we now represent the related quantities by letters instead of by numbers. Thus, if a represent the acceleration, V the velocity at the end of a number of seconds denoted by t, and v_0 the velocity at the beginning of this time, then the acceleration is $a = \frac{V - v_0}{t}$. This equation is simply the definition of acceleration written in algebraic shorthand.

It is often necessary to find the change in velocity in terms of the acceleration and the time. In order to do this, we multiply both members of our equation by t, thus obtaining the result $V - v_0 = at$, i.e., the change of velocity is equal to a, the rate of that change, multiplied by t, the time.

If we wish to find the value of the final velocity V when the other quantities are known, we add v_0 to both members of this equation, which gives us

$$V = v_0 + at \qquad (2)$$

i.e., *the final velocity is equal to the initial velocity plus the change in velocity.*

11. Relation of Distances to Times. It will be interesting to know what sort of lines we shall get if we plot graphs that represent the relations of distances to times while our train is starting and stopping. In order to do this we must first know the distance of the train from a given point at the end of each second.

At the beginning of the first second, since the train is at rest, the velocity is zero: and the final velocity is this initial velocity plus the change, or $V = v_0 + at$, as stated in equation 2, Art. 10. Now a, the acceleration, is $50 \frac{\text{cm}}{\text{sec}^2}$; hence the final velocity for the time 1 sec is $V = 0 + 50 \times 1 = 50 \frac{\text{cm}}{\text{sec}}$. Since the velocity begins at 0 and

ends at $50\,\frac{\text{cm}}{\text{sec}}$, it must, during that first second, have all values from zero up to $50\,\frac{\text{cm}}{\text{sec}}$. Which of these values may we use in calculating the distance traversed in that second? Since according to our supposition the velocity increases uniformly, the train will traverse in a given time with the uniformly accelerated motion *the same distance that it would have traversed during that time with uniform motion at the average speed.* The average or mean velocity, then, is that which we must choose.

Since the velocity changes at a uniform rate, the average velocity may easily be found by taking the *arithmetical mean* of the initial and final velocities; and therefore, for the first second, if we represent this mean velocity by v, we have $v = \frac{0 + 50}{2} = 25\,\frac{\text{cm}}{\text{sec}}$.

Solving equation (1) for l, we have $l = vt$; and substituting, we get $l = 25 \times 1 = 25$ cm, the distance of the train from the starting point at the end of the first second. Similarly for the time two seconds (since for each time period we must consider the motion *from the beginning,* in order to get the average velocity), the initial velocity is 0; and the final velocity is $V = 0 + 50 \times 2 = 100\,\frac{\text{cm}}{\text{sec}}$. Whence the average velocity $v = \frac{0 + 100}{2} = 50\,\frac{\text{cm}}{\text{sec}}$; and the whole distance traversed up to the end of the second second is again the mean velocity multiplied by the time, i.e., $l = vt = 50 \times 2 = 100$ cm. In like manner, for 3 sec, we get $V = 0 + 50 \times 3 = 150\,\frac{\text{cm}}{\text{sec}}$, and $v = \frac{0 + 150}{2} = 75\,\frac{\text{cm}}{\text{sec}}$, therefore $l = vt = 75 \times 3 = 225$ cm.

By the same method of calculation, we find that the distances for the first eight seconds are as follows:

sec	cm	sec	cm
0	0	5	625
1	25	6	900
2	100	7	1225
3	225	8	1600
4	400	etc.	etc.

12. The Graph for Distance and Time. We now have the data that we need, and can proceed to construct our graph. Let

us choose our scales so that for the abscissas 1 cm represents 1 sec, and for the ordinates 1 cm represents 100 cm. We locate the point corresponding to each second (Fig. 7) and find them to be O for the beginning of the first second, p_1 for the end of the first second, p_2 for the end of the second second, p_3 for the end of the third second, and so on. If, as before, we should connect the points in succession by straight lines, would the resulting line be straight? Does the velocity of our train *change abruptly* at the end of each second or is it *increasing uniformly at every instant?* Does the broken line connecting O, p_1, p_2, etc., change its slope at every point or only at the points that we located?

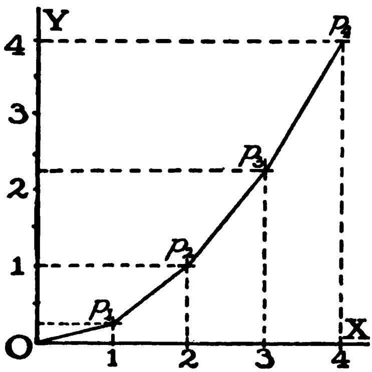


Fig. 7. Graph for Single Seconds

Then does such a line properly represent the uniformly accelerated motion of the train?

It ought now to be clear that *the graph must change its slope at every intermediate point* as well as at the few points that we located, *if it is to represent properly the uniformly increasing velocity of the train.*

If we should locate the points for the intermediate half seconds, in addition to the points already placed, thus reducing our time interval to 0.5 of its former value, and if we should connect all the points successively as before, would the broken line thus obtained more nearly fulfill the condition of changing its slope at every point?

Suppose now that we were to reduce the time interval to 0.2 sec and to plot the corresponding broken line (Fig. 8); would this line approach more nearly than did the other to the line that would represent exactly the uniformly increasing velocity of the train? It must be clear that by continually diminishing our time intervals we shall get broken lines that more and more nearly fulfill the condition of changing slope at every point, and thus more and

more nearly approach to the graph that we want. It is obvious, however, that, in a practical problem, *it is useless either to carry the subdivision of the time interval beyond the point at which the difference between the broken line and a smooth curve is no longer perceptible in the drawing, or to use smaller time intervals than we are able to measure by means of the timepiece used in making our observations.*

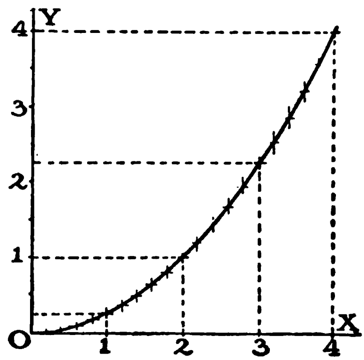


FIG. 8
GRAPH FOR FIFTHS OF A SECOND

In general when we wish to make a graph that corresponds to a series of observations, we locate the points corresponding to each of these observations, and then *draw the smooth curve that most nearly passes through all of the points.*

13. Slope of a Curved Graph. As long as the line is a broken one, the slope of the portion between any two consecutive points is that of the straight line joining those points; but when we pass to a graph that changes its slope at every point, it must be evident that the SLOPE AT ANY POINT is approximately that of a straight line joining the given point with a nearby point. The nearer we take this point to the given one, the more nearly does the slope of the straight line represent that of the curve.

Since, for this graph, the ordinates and the abscissas represent respectively distances and corresponding times, just as they did in the graphs for uniform motion, *the slope at any point of this graph must represent the velocity at the corresponding instant of time,* just as it did in their case.

14. The Train is Stopping. In order to construct the graph that will represent the relation between distance and time when the train is slowing down, we must again calculate the distances of the train, at the ends of the successive seconds, from the point

at which the engineer applies the brakes. The velocity at this instant is the initial velocity and is 2500 $\frac{\text{cm}}{\text{sec}}$ (*cf.* Art. 8). As the speed is decreasing, the acceleration is negative; and so (*cf.* Art. 8), its value is $a = -100\frac{\text{cm}}{\text{sec}^2}$. Hence the final velocity for the first second is $V = v_0 + at = 2500 - 100 \times 1 = 2400\frac{\text{cm}}{\text{sec}}$; and the average velocity $v = \dfrac{2500 + 2400}{2} = 2450\frac{\text{cm}}{\text{sec}}$. Multiplying the average velocity by the time as before, we get for the distance traversed in the first second, $l = vt = 2450 \times 1 = 2450$ cm. Likewise for the second second we get $V = 2300$, $v = 2400$; so that $l = 4800$ cm. The values for succeeding seconds are as follows:

sec	cm	sec	cm
0	0	5	11 250
1	2450	6	13 200
2	4800	7	15 050
3	7050	8	16 800
4	9200	etc.	etc.

15. Graph for Negative Acceleration. Choosing scales such that for the abscissas 1 cm represents 10 sec, and for the ordinates 1 cm represents 10,000 cm, and plotting precisely as before, we obtain the graph shown in Fig. 9. In what way is this *graph for the case of negative acceleration* like that for the case of positive acceleration (Fig. 8)? In what way do these graphs differ? At the end of what second does the train come to rest? Assuming that the train then remains at rest, add to the diagram the points corresponding to the next five seconds. At the end of what second does the graph become parallel with the axis of abscissas? What, then, is the slope at the end of the 25th second? At the end of the 28th? the 30th? What velocity is represented by a slope of zero?

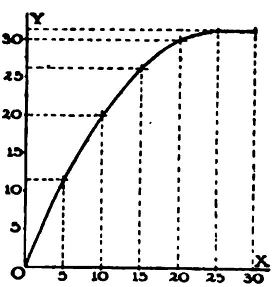


FIG. 9
THE TRAIN IS STOPPING

16. The Entire Motion Represented. We have now the graphs for the uniform motion of the express train going at full

speed, and for the uniformly accelerated motion with positive and negative accelerations while getting up speed and slowing down. In order that all of these motions may be represented by a single diagram that will go on a page, a smaller scale must be used. The complete graph appears in Fig. 10 (1 cm = 30 sec, 1cm = 30,000 cm). *Describe in succession the changes of slope.*

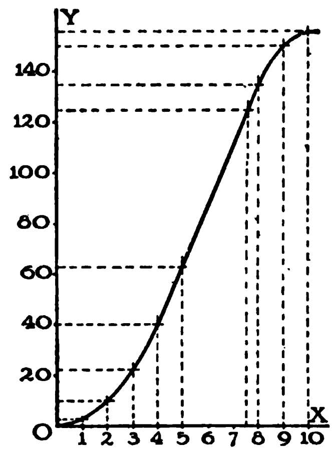


FIG. 10. THE COMPLETE GRAPH

17. Equations for Uniformly Accelerated Motion. Passing now to the analytical method of representing uniformly accelerated motion, let us develop an algebraic expression that will generalize the calculations of Art. 11 and Art. 14. If v_0 represent the initial velocity, a the acceleration, t the time, V the final velocity, and l the distance, then by equation (2), Art. 10,

$$V = v_0 + at$$

Also, the average velocity, v is found by taking half the sum of the initial and final velocities; therefore, $v = \frac{v_0 + (v_0 + at)}{2} = v_0 + \frac{at}{2}$.

On multiplying this average velocity by the time t to get the distance l, we have

$$l = v_0t + \frac{at^2}{2}. \qquad (3)$$

Equations (2) and (3) are the equations for uniformly accelerated motion.

THE LAWS OF UNIFORMLY ACCELERATED MOTION expressed by these equations may be stated as follows:

1. *The final velocity is equal to the initial velocity plus the product of the acceleration and the time.*

2. *The total distance traversed is equal to the product of the initial velocity and the time, plus half the product of the acceleration and the square of the time.*

18. When the Moving Body Starts from Rest. In the cases thus far considered the initial velocity was zero. On substituting this value in the general expression, the term involving v_0 vanishes and the equations become $V = at$ and $l = \frac{at^2}{2}$, which express the relations when the moving body has started from rest.

It should not be forgotten that *when the velocity is decreasing, a, the acceleration, is negative.*

19. Acceleration is Not Necessarily Uniform. Throughout the preceding discussion we have assumed that the acceleration of the train was constant. In reality the case is not quite so simple, because the engineer at first puts on the steam pressure gradually, and because the acceleration is diminished by the resistance of the air, which increases very rapidly when the speed is increased. The acceleration which we assumed to be uniform was the average acceleration during the time considered.

20. Determination of Acceleration. The actual experiment of determining acceleration is made by observing distances and corresponding times, substituting their values in equation (3), and solving for a.

FIG. 11. TRANSLATION AND ROTATION

21. Translatory and Rotary Motions. Thus far we have considered only motion in a straight line. We are now ready to define motion in general, and to distinguish between translatory and rotary motion.

A body is said to be IN MOTION with reference to a given point when it is changing either its distance or its direction from that point.

When a rigid body moves in such a way that all its points describe equal and parallel paths, its motion is called TRANSLATION.

When the motion of a body is such that its points describe circumferences about some point or line, its motion is called ROTATION. The point or the line about which the body rotates is called the center or the AXIS OF ROTATION. The planes in which the particles move are all parallel to one another, and the axis is necessarily perpendicular to these parallel planes.

A sled going down a hill has translatory motion only, provided there are no turns in the road; for then all of its points describe equal and parallel paths. The same is true of a sail boat when it is making a straight course. On the other hand, the buzz saw and the grindstone are familiar examples of bodies that have rotary motion only. Every point on the grindstone, for example, describes a circle about a point in the center of the axle on which the stone is mounted. The centers of all the circles described by the points lie on a line which is perpendicular to the planes of all the circles. When an automobile is traveling along a straight road, the body of the car has translatory motion only, while the wheels, considered with respect to their axles, have rotary motion only; but the wheels have both translation and rotation with reference to a point on the road.

SUMMARY

1. The units of length and of time are the centimeter and the second. Their symbols are cm and sec.

2. Motion may be either translatory or rotary.

3. Linear velocity is the rate of change of distance in a given direction.

4. Uniform linear velocity is measured by the distance traversed in one second. Its symbol is $\frac{\text{cm}}{\text{sec}}$.

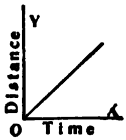
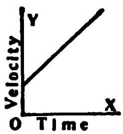
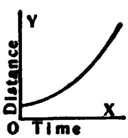
5. Acceleration is the rate of change of velocity.

6. Uniform linear acceleration is measured by the change of velocity in one second. Its symbol is $\frac{\text{cm}}{\text{sec}^2}$.

7. Acceleration may be either positive or negative.

8. The distance traversed by a body having uniformly accelerated motion is found by multiplying the average velocity by the time.

9. The three methods of representing these relations are:

Graphical	Analytical	Verbal
	$v = \frac{l}{t}$ (Equation 1)	Uniform or average velocity equals distance divided by time.
	$V = v_0 + at$ (Equation 2)	With uniform acceleration, final velocity equals initial velocity plus acceleration multiplied by time.
	$l = v_0 t + \frac{at^2}{2}$ (Equation 3)	Distance traversed with uniform acceleration equals initial velocity multiplied by time plus half the acceleration multiplied by time squared.

QUESTIONS

1. Define the scientific unit of length, and give its symbol. Define the unit of time and give its symbol.

2. Define the term, linear velocity. What is meant by a constant linear velocity? What two things must be stated in order that the velocity of a moving body may be fully described?

3. How is the numerical value of a uniform velocity found? What is the unit of velocity? What is its symbol?

4. Explain how to represent a constant linear velocity by the graphical method. In connection with the diagram, point out and name the coördinate axes, the coördinates, and the origin.

5. What characteristic of the motion of a body is shown by the slope of the graph that represents it?

6. When the velocity of a moving body is changing, how can we express numerically the velocity that it has at any instant?

7. Define acceleration, and illustrate by a numerical example.

8. When is an acceleration positive, and when negative? What is meant by uniformly accelerated motion?

9. Draw the graphs that represent the relation of distance to time for a positive and a negative acceleration. In these graphs, what does the slope represent?

10. What changes of slope occur in the graph when the acceleration is positive? When the acceleration is zero? When it is negative?

11. When a graph is curved, what line will represent approximately the direction of its slope at any point?

12. Define motion, and distinguish between translatory and rotary motion, illustrating by examples.

PROBLEMS

NOTE. 1 m = 100 cm = 39.37 inches.

1. A runner passes over 100 yards in 10 sec; what is his speed $\frac{\text{cm}}{\text{sec}}$?

2. What is the speed of a race horse that covers a mile in 2 minutes (1) in $\frac{\text{miles}}{\text{hour}}$? (2) in $\frac{\text{cm}}{\text{sec}}$?

3. What is the speed of an automobile that runs a mile in 55 sec. (1) in $\frac{\text{miles}}{\text{hour}}$? (2) in $\frac{\text{cm}}{\text{sec}}$?

4. Sound, at 0° Centigrade, travels 1090 feet in one sec. What is the speed of sound in $\frac{\text{cm}}{\text{sec}}$? How many seconds would it take to traverse 1000 m?

5. What was the average speed of a railroad train that traveled 134 miles in 115 minutes (1) in $\frac{\text{miles}}{\text{hour}}$? (2) in $\frac{\text{cm}}{\text{sec}}$?

6. Express the velocity of 1 $\frac{\text{mile}}{\text{hour}}$ in $\frac{\text{feet}}{\text{sec}}$, and in $\frac{\text{cm}}{\text{sec}}$.

7. A sled, started from rest and going down a hill of uniform slope, traverses 900 cm from the starting point in 3 sec. What is the acceleration, and what the final velocity?

8. A wagon starts down a hill with a velocity of 30 $\frac{\text{cm}}{\text{sec}}$ and its acceleration down-hill is 80 $\frac{\text{cm}}{\text{sec}^2}$? What is its velocity at the end of 5 sec? What is the total distance traversed in the same time?

9. A wheelman, starting from rest, had attained at the ends of the first three seconds the following distances: 1 sec, 90 cm; 2 sec, 360 cm; 3 sec, 810 cm. Supposing the acceleration to remain constant during that time, what is (1) the acceleration? (2) the velocity at the end of 6 sec? (3) the distance traversed at the end of 5 sec? (4) the distance traversed during the 5th sec? [For (4), subtract the distance attained in 4 sec from that attained in 5 sec.]

10. The results of experiments show that the acceleration of a body allowed to fall freely is 980 $\frac{\text{cm}}{\text{sec}^2}$. (*a*) Calculate the distances attained by the falling body when given an initial velocity of 10 cm vertically downward, making a table of distances and times up to 10 sec. (*b*) Choosing a convenient scale, plot a graph representing the motion.

11. Calculate the velocities of the falling body for the times given in problem 10.

SUGGESTIONS TO THE STUDENTS

1. Which of you can make the longest list of the motions with which you are familiar, classified under the headings: Uniform, Uniformly Accelerated Positive, Uniformly Accelerated Negative, Translatory, Rotary?

2. Mark your height on a door-post; measure it in inches and in centimeters. From these measurements can you find out how many centimeters are contained in one inch?

3. In your debating society, choose for a question the following: Resolved: That the general adoption of the metric system of weights and measures is advisable. For data write to the National Bureau of Standards, Washington, D. C.

4. Which of you can find out the most interesting facts about Galileo and his knowledge of falling bodies?

CHAPTER II

MASS AND ENERGY

22. The Production of Acceleration. In the preceding chapter, we attempted to get clear notions about uniform and accelerated motions, without considering the factors upon which their production and variation depend. What are the relations that determine whether the motion shall be uniform or accelerated? What relations determine the amount of the acceleration? We can most easily find the answers to these questions by again studying the train.

Let us first suppose that the train drawn by the engine consists of six cars all alike. Let us also suppose that the engine, using its full power, can impart to this train an acceleration of $50 \frac{\text{cm}}{\text{sec}^2}$. Now, if this engine be replaced by a smaller one having less power, will the acceleration that this smaller engine can impart to the same train be greater or less than $50 \frac{\text{cm}}{\text{sec}^2}$? Must the engine that can impart to this train an acceleration of $60 \frac{\text{cm}}{\text{sec}^2}$ have greater or less power than the first engine?

Those who can not answer these questions from observations made upon the train itself, will readily answer them by inference from similar cases. Thus, everybody knows that more force is required for imparting to a ball a great velocity in a given time than for imparting to it a small velocity in the same time, that two oarsmen can impart to a boat a greater velocity in a given time than can one, that greater effort is required by a bicyclist to attain a great velocity in a given time than to attain a small velocity in the same time. Observation and experience lead us habitually to associate a greater acceleration with a greater effort or FORCE.

23. Acceleration and Force. Although common experience gives us this general information, it does not give us the specific numerical relations. This information can be obtained only by making careful measurements of the quantities involved.

Thus, if we measure the pulls of different sized engines, having different powers, and observe the corresponding accelerations, and if we make proper corrections for friction of the moving parts and for air resistance, we shall find that the numbers representing the pulls are directly proportional to the numbers representing the accelerations imparted to the train.

Many experiments of this sort have been devised and carried out in physical laboratories to test the validity of this conclusion, and they all tend to establish the truth of the general principle that *when different accelerations are given to the same body, the ratio of the numbers by which we express the forces to those by which we express the corresponding accelerations is constant.*

FIG. 12. EIGHT-OARED SHELL

Another illustration will help to make this clear. When only two of the crew of an eight-oared shell row, they can impart to the boat a certain acceleration. After making proper allowance for the increased resistance of the water and air, it will be found that when four row, they can impart an acceleration twice as great, six an acceleration three times as great, and so on.

24. Different Bodies Having the Same Acceleration. We have thus far considered how the forces vary when different accelerations are given to the same body. Let us now consider how the forces vary when the same acceleration is given to different bodies.

If an acceleration of say $50 \frac{\text{cm}}{\text{sec}^2}$ can be given by a certain engine to a train of five empty cars, must the engine that can give the same acceleration to a train of ten similar cars be more or less powerful than the first? Again, if the same acceleration is to be

given to the train of five cars *loaded with passengers*, can the same engine do the work?

Common experience again gives us qualitative answers; for everybody knows that an engine that can easily move a short train may fail to move a long one, so that another engine must be added. Likewise it requires greater effort on the part of a bowler to give a large ball a certain velocity than to give a small ball the same velocity in a given time; and it requires the efforts of more oarsmen to give a certain acceleration to a big boat than to a little one. The student can recall many similar facts from his daily observation.

FIG. 13. SMALL ENGINE: SHORT TRAIN

It appears, then, that the more we increase the size of a body, the substance remaining the same, the greater is the force required to give it a certain acceleration.

25. Mass. In order to get quantitative relations, experiment is necessary. If we measure the pull of an engine when it is imparting an acceleration of 50 $\frac{\text{cm}}{\text{sec}^2}$ to a train of five empty cars, and again when it is imparting the same acceleration to a train of ten similar cars, we shall find that the pull in the second case is twice that in the first. Likewise we shall find that the pull for a train of fifteen cars is three times that for five cars, and so on; i.e., when the acceleration is the same, the numbers representing the forces are directly proportional to the corresponding numbers of cars.

The matter appears very simple as long as the cars are empty and all alike. But although we know from experience that more force is required to impart a given acceleration to a loaded train than to an empty one, yet it is impossible to determine how much force, until we have adopted a means of comparing the loaded train with the empty one.

These differences in the make-up of the trains, whether in the number of cars or in the load, are differences in MASS.

26. Masses Compared by Forces. It is easy to see that when two trains consist of precisely similar cars, all of them empty, the train of ten cars has twice the mass of the train of five cars, because it is made up of just twice as many units of the same kind. But it has just been shown that to impart a certain acceleration to a train of ten empty cars the force is twice as great as that for a train of five empty cars; so that we may compare the masses of the two trains not only by the numbers of cars, but also by the forces required to give them the same acceleration.

When the differences in the trains are differences in the loads, we can not compare their masses by comparing the number of cars, because the units are not alike. Therefore we must resort to the other method, that of comparing the masses by the forces that can impart to them the same acceleration. This method is applicable to all bodies, whether composed of like or unlike kinds of matter. Therefore, in general, *two masses are equal when, under the same conditions, equal forces can impart to them equal accelerations.*

Applying this method to the cars, it appears that when an engine can give two empty cars the same acceleration that it can give to a single loaded car, the combined mass of the single car and its load is equal to that of two empty cars; and therefore in this case the mass of the load is equal to the mass of one of the cars.

For a given acceleration, then, since the masses are equal when the forces are equal, it follows that if one of the masses be doubled, the corresponding force is doubled; if the mass is made three times as great, the force is also three times as great; and so on. In general, then, *when the acceleration is constant, the forces are proportional to the masses.*

27. Force, Mass, Acceleration. Since we have shown in the preceding paragraphs that when the mass is constant, the force varies directly as the acceleration, and also that when the acceleration is constant, the force varies directly as the mass, it follows that, in

general, the force must vary directly as the product of the mass and the acceleration. If we choose our units of force appropriately, and if we let f represent the force, m the mass, and a the acceleration, we may write

$$f = ma \qquad (4)$$

This equation defines the force in terms of mass and acceleration.

In connection with this equation, it is to be noted that if m and f are constant, a must be constant also; i.e., if a body be acted on by a single or an unbalanced constant force, its motion will be uniformly accelerated.

Besides magnitude, every force has two other characteristics, namely, its direction, and its point of application. When these three characteristics are specified, the force is fully described.

28. Unit Mass. Thus far we have taken an empty car as the unit of mass; but it is manifest that accurate measurement necessitates the establishment of a unit that is fixed and at the same time more convenient. Therefore, just as we have a standard of length, the meter, we have also a standard of mass. The INTERNATIONAL STANDARD OF MASS is a certain piece of platinum which is carefully preserved at Paris along with the standard meter, and is called the *kilogram*. The unit of mass employed by all scientists is the *gram*, which is the one-thousandth part of the mass of the standard kilogram. The abbreviation for gram is gm. To express very large masses, the kilogram is a more convenient unit. The abbreviation for kilogram is Kg.

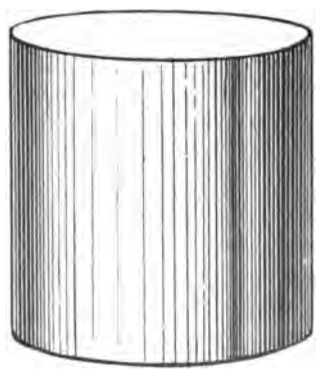


FIG. 14. STANDARD KILOGRAM: ACTUAL SIZE

Since we are now able to express both mass and acceleration in terms of grams, centimeters, and seconds, we may also express force in terms of these same fundamental units. Thus in the equation

$f = ma$, if we substitute $m = 1$ gm, and $a = 1\,\frac{\text{cm}}{\text{sec}^2}$, we obtain $f = 1 \times 1 = 1\,\frac{\text{gm cm}}{\text{sec}^2}$, *which defines the scientific unit of force as that force which can impart an acceleration of one centimeter per second per second to a mass of one gram.* This unit of force is called the DYNE. Note that the number of units of force is obtained by multiplying together the numbers representing mass and acceleration. This operation gives us gm $\times\ \frac{\text{cm}}{\text{sec}^2}$. Hence the symbol for the dyne is $\frac{\text{gm cm}}{\text{sec}^2}$.

By definition (*cf.* Art. 10), $a = \dfrac{V - v_0}{t}$, therefore $f = ma = \dfrac{m\,(V - v_0)}{t}$. The product $m\,(V - v_0)$, or mass $\times$ change of velocity, is called CHANGE OF MOMENTUM; $\dfrac{m\,(V - v_0)}{t}$ is therefore the rate of change of momentum; and since it is equal numerically to ma, it is also a measure of the force to which it corresponds.

29. Weight. Since we have learned to state the relation of force to mass and acceleration, we are in a position to get some definite ideas concerning a subject about which there are many common misconceptions. We all learned in early childhood that bodies, including ourselves, fall to the earth when unsupported. We are accustomed to associate this motion with a force called gravity, which we conceive acts so as to attract all bodies toward the earth. The attraction between the earth and any particular body is called its WEIGHT, and tends to give the body an acceleration vertically downward. This fact is also a familiar one, for everybody knows that a body falling from a great height acquires a greater velocity than does one falling from a less height. It is our knowledge of this fact, acquired from very early experience, that impels us to avoid a high fall.

Now, what is the relation between the weights of bodies and their masses? Equation (4) will give us the answer. Thus, the weight f, in dynes, of any body whose mass is m, is equal to this mass multiplied by a, the acceleration that this weight will give it if it is allowed to fall freely; i.e., $f = ma$. Similarly, the weight f' of any other body having a mass m', and receiving an acceleration a', is $f' = m'a'$. In order to find the ratio of the two

weights in terms of their corresponding masses, we must divide one of these equations by the other, thus: $\frac{f'}{f} = \frac{m'a'}{ma}$. We therefore see that if both bodies have the same acceleration when falling freely, i.e., if $a' = a$, then their weights are proportional to their masses.

30. Galileo's Experiment. The question to be answered now is, When two bodies have different masses, does the attraction of the earth give them equal accelerations? From the time of Aristotle to the end of the sixteenth century, this was a much disputed question. Aristotle (384–322 B.C.) taught that if two bodies of unequal mass were dropped from the same height at the same instant, the heavier body would reach the earth first; and his followers defended this opinion by his authority and by arguments based upon what they thought ought to be the nature of things. Galileo (1564–1642 A.D.) was the first to recognize that *such a dispute can be settled only by experiment.* Accordingly, about the year 1590, he performed the experiment of dropping at the same instant a small cannon ball and a large bomb from the top of the Leaning Tower of Pisa. They reached the ground at very nearly the same instant; so he came to the conclusion that if it were not for the resistance of the air, they would have fallen in exactly the same time. The fact still remained, however, that a body with a large surface in proportion to its mass, such as a feather, was known to fall very much more slowly than

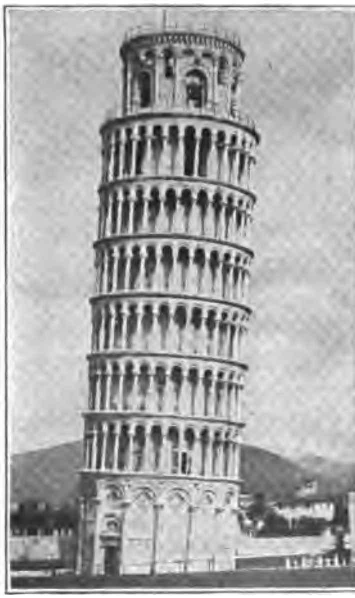


FIG. 15. LEANING TOWER OF PISA

a piece of metal. After the invention of the air pump, in 1660, it became possible to settle the dispute finally. This was done by showing that when a feather and a coin were dropped simultaneously in a long tube from which the air had been pumped, they fell side by side and reached the bottom at the same time.

31. The Relation between Weight and Mass. Reasoning from these experiments by means of equation (3), Art. 17, it follows that the accelerations of all freely falling bodies are equal. For if two bodies fall simultaneously through a distance l in time t, then for the first, since the weight and hence the acceleration is constant, $l = \frac{at^2}{2}$; and likewise for the second, $l' = \frac{a't'^2}{2}$. But since the distance is the same for each, as is also the time, $l = l'$ and $t = t'$, whence $a = a'$.

Thus it has been proved that *at any given place, the acceleration due to the earth's attraction is the same for all bodies and that therefore, so long as they are compared at the same place, the weights of all bodies are proportional to their masses.*

Galileo, wishing to prove this statement with greater accuracy, devised experiments with pendulums of different mass. These experiments verified more accurately the same conclusion. Repeated with greater refinement by Sir Isaac Newton and others, they have given convincing evidence of the truth of this statement.

From what has just been stated, it follows that *we can compare masses by comparing their weights.* This is the method in common use; but it must be noted that, since the attraction of the earth for a given body is different at different places, the weights of the two masses that are to be compared must in general be determined at the same place. For the comparison of masses by means of their weights, the equal arm balance is generally used.

32. Density. In connection with the masses of different bodies, we have seen that bodies having equal volume may differ greatly in mass. Thus, one cubic centimeter of lead has a much greater mass than has one cubic centimeter of water; while the latter has a greater mass than has one cubic centimeter of wood. The

appropriate measure of the DENSITY of any substance is the mass in unit volume at a temperature of zero degrees Centigrade. Thus, if the mass of a specimen of a certain kind of glass is found to be 25 gm, and its volume 10 cm^3, the average density of the glass is 0.1 of 25, or 2.5 grams per cubic centimeter.

If D represent the density, m the mass, and V the volume, these relations are stated analytically by the equation

$$D = \frac{m}{V}.$$

As defined by this equation, *the density of a substance is its mass per unit volume.* The unit of density is one gram per cubic centimeter, and its symbol is $\frac{\text{gm}}{\text{cm}^3}$. Since the gram was intended to be the mass of 1 cm^3 of water, and since it is so, very nearly, the number of cm^3 in the volume of a quantity of water is the same as the number of gm in its mass. The density of water, therefore, may be taken as 1 $\frac{\text{gm}}{\text{cm}^3}$.

33. Work. In Chapter I we have studied the motion of a railroad train and seen how that motion is produced by the engine. *Why does the engine move at all?* Must more steam be used to move a train of large mass than to move a train of small mass? Must more steam be used to move a given train over a long distance than over a short one? Other conditions being the same, does it require a larger amount of coal to generate a larger amount of steam? The student probably knows the answers to these questions, and also in a general way that, other things being equal, the amount of coal required is proportional to the amount of WORK to be done. Since *most kinds of work, like that done by the locomotive, consist in putting bodies into motion, and in maintaining their motions in opposition to resistances of some sort,* and since somebody always has to pay for getting work done, it becomes necessary to know definitely just what an amount of work depends on, and to have a unit in terms of which all kinds of work may be measured.

34. Work, Force, Distance. If an engine or a horse or a man is doing any kind of work it is evident that, other things being

equal, the amount of work done is directly proportional to the push or pull, i.e., to the force exerted by the agent that does the work. Thus, if each of two engines pulls its train on a straight and level track for the distance of a mile, and if the second engine has to pull with twice the force of the first, it is clear that the second engine must do twice the work that the first does. Again, suppose that the second engine pulls for one mile, and then continues to pull with the same force for another mile, it must again be clear that in pulling the train two miles it does twice the work that it did in pulling it one mile. It follows, then, that if the second engine, exerting twice the force of the first, and hence doing twice as much work *per mile* as does the first, should continue pulling with this force through a distance of two miles, it would do four times as much work as the first engine did in pulling its train one mile.

FIG. 16. PLOWING
Work is proportional to force and to distance.

Since, then, the amount of work done by an agent is directly proportional to the force and also to the distance through which the agent acts, and since the amount of work depends on these two factors only, it follows that when the units are properly chosen, *the measure of the work done is the product of the numbers representing the force and the distance.* In symbols, if f represent the force of the agent, and l the distance through which it acts, and if W represent the work done, then

$$W = fl. \qquad (5)$$

This is the EQUATION FOR WORK.

35. Unit Work. The unit in terms of which work is measured may easily be defined with the help of the equation $W = fl$, for if $f = 1$ dyne and $l = 1$ cm, we have $W = 1 \times 1 = 1$.

Therefore, since the equation gives unity for the work when the force is one unit and the distance one unit, it is most convenient to *define the unit of work as the amount of work that is done when a force of one dyne acts through one centimeter.* This is the unit of work adopted by physicists, and it is called the ERG.

Since the symbol for the dyne is $\frac{\text{gm cm}}{\text{sec}^2}$, and since the number of ergs is obtained by multiplying together the number of dynes and the number of centimeters, it follows that the symbol for the erg is $\frac{\text{gm cm}}{\text{sec}^2} \times \text{cm}$, or $\frac{\text{gm cm}^2}{\text{sec}^2}$.

36. Energy. We now come to the question of the relation between the amount of coal burned and the amount of work done. It is generally recognized that a water wheel, in order to move machinery continuously, must be continuously supplied with water, which must be allowed to fall upon it from a higher level; that a windmill will not continue to pump water unless the wind continues to blow against its blades, that a horse or a man can not continue to do work unless he regularly consumes food, and that an engine of any sort must continuously consume coal in order that the steam may be kept up at the necessary pressure while it is doing its work.

FIG. 17. HAYING
A man can not work unless he consumes food.

For centuries the most careful thought of philosophers and the greatest genius of inventors were employed in trying to think out and construct some device for obtaining perpetual motion, i.e., a device which would continue to move indefinitely without a continuous external supply of energy. Since every such attempt has been unsuccessful, scientists have become convinced that a PERPETUAL MOTION MACHINE is impossible. Thus, if any machine be

at rest, it can not start itself; and if it be in motion, the greater the friction of its moving parts the sooner will it stop. If it be harnessed to other bodies and made to do work in moving them, it will come to rest all the sooner. It can be made to work continuously only by supplying it continuously with ENERGY from some external source. *Energy, then, represents ability to do work.* In the case of the water wheel the energy is derived from the motion of a mass of water; in the case of the windmill, from the motion of a mass of air; in the case of the horse or man, from the consumption of a quantity of food; and in the case of a steam or gas engine, from the consumption of a quantity of fuel.

FIG. 18. THE WINDMILL
It will not go when there is no wind.

Thus it becomes evident that to do work energy must be expended, and that to store up this energy work of some sort must have been done. Now, many careful experiments with all forms of energy have shown that a given amount of energy always corresponds to the same amount of work, whether that energy be expended in doing the work, or the work be done in storing the energy.

37. Energy Measured by Work. *Since the energy of a body is equivalent to the work it can do, and also to the amount that had to be done on it in order to impart the energy to it, we may measure this energy by measuring either of these amounts of work.* Sometimes one of these methods is more convenient, sometimes the other. For example, let us consider the energy necessary to run an eight-day clock. Such a clock is usually operated by a spiral spring, or by a weight which is raised by winding up the cord upon which it hangs. Suppose that the weight has a mass of 5000 gm. Then, since $f = ma$, the force with which it pulls on the cord is ma, or 5000 gm multiplied by the acceleration that it would have if allowed to fall freely in consequence of the earth's attraction. This

acceleration, which we have learned is the same for all bodies, is found by experiment to be, at sea level and in the latitude of New York, $980 \frac{\text{cm}}{\text{sec}^2}$. Therefore, the force with which we must pull in order to lift this mass is $5000 \times 980 = 4,900,000$ dynes. To avoid the repetition of zeros it is convenient to write this 49×10^5.

If the distance through which the mass is lifted is 100 cm, then from Art. 34, $W = fl = 4\,900,000 \times 100 = 49 \times 10^7$ ergs. Since this is the amount of work done in winding up the clock, it represents the *energy stored in the lifted weight.* Likewise when the weight descends, *it does work in running the clock* and this work is again $f \times l = 5000 \times 980 \times 100 = 49 \times 10^7$ ergs. Since this is the work done by the energy stored in the lifted weight, it also is a measure of that energy. Thus, in general, if we can measure or compute the work done on a body in imparting energy to it, or the work that it does when it parts with its energy, we can determine the amount of energy that it had.

FIG. 19. HOISTING COAL
Work equals force times distance.

It will be noted that in the example just given a small amount of USELESS WORK, done in overcoming friction while winding the clock, was neglected. *In every case in which energy is transformed or transferred some of this useless work is done.* If the amount of useless work is at all comparable with that of the useful work, allowance must be made for it. The ratio of the useful work done to the total amount of energy expended is called the EFFICIENCY of the machine by which the transformation or transference is accomplished.

Suppose now that in the clock just considered we were to replace the weight by a *spiral spring.* How much energy must the spring have when wound up in order that it may be able to run the clock

for as long a time as the weight ran it? How much work would have to be done in winding up the spring?

38. Energy is Potential or Kinetic. In the cases that we have considered, energy has been stored in a lifted weight, a coiled spring, unburned coal, and unconsumed food. *Energy of this kind that a body has because of its position or internal condition, so that it tends to move and do work, is called potential energy.* In the case of the windmill or the water wheel, the energy of the air or water is due to the fact that it is in motion. Likewise a base ball or cannon ball does work while it is being stopped. Hence it also possesses energy; and it must be quite clear that it has this energy because of its motion. *The energy that a moving body has because of its motion is called kinetic energy.*

FIG. 20. PILE DRIVER
The weight has potential energy when it is raised, kinetic when it strikes.

39. The Kinetic Energy of a moving body being evidently due to its mass and velocity, it is often more convenient to measure it in terms of these quantities than in terms of work done. This may readily be done with the help of equations (3), (4), and (5). Thus $W = fl$, in which f is the average force used in imparting to the moving mass its velocity, and l the distance through which this force acted. Also this force $f = ma$, in which m is the mass of the body, and a its acceleration while acquiring its full velocity. Therefore the work done in giving the body its kinetic energy is $W = fl = mal$. Again, by equation (3), $l = \frac{1}{2}at^2$, and by equation (2) $V = at$, in which l is the distance traversed in the time t while acquiring the velocity V with an acceleration a. Since we do not

care to know the time t, we may eliminate it by substitution. Thus, from (2) $t = \frac{V}{a}$; then $t^2 = \frac{V^2}{a^2}$. Substituting this value for t in equation (3), we have $l = \frac{aV^2}{2a^2} = \frac{V^2}{2a}$. Finally, by substituting this value for l in the equation $W = mal$, we find that the energy is $e = W = \frac{maV^2}{2a}$. Simplifying, we have

$$e = \frac{mV^2}{2} \text{ ergs.} \tag{6}$$

Since the symbol for mass is gm, and that for velocity is $\frac{\text{cm}}{\text{sec}}$, the symbol for kinetic energy is $\frac{\text{gm cm}^2}{\text{sec}^2}$. Note that this is the same symbol as that for the unit of work, as it should be, because energy is measured by work.

The advantage of deriving the equation $e = \frac{mV^2}{2}$ is manifest when we apply it to the case of throwing a ball or firing a shot. For while it would be very difficult to measure t and a corresponding to the distance l through which the force of the hand or the powder was exerted, it is not so very difficult to measure V. Hence it is often desirable to have an equation in which a and t are not involved. In many cases, however, the kinetic energy of a body can be measured with convenience *by the work that it does when it gives up its energy in stopping.*

40. Newton's Laws of Motion. From all that has been said it must be apparent that a body can not of itself start, or stop, or otherwise change either the rate or the direction of its motion. This fact is often expressed by saying that every body has INERTIA.

The relations of the phenomena with which we have become familiar in this chapter were described tersely by Sir Isaac Newton in the following statements, which first appeared in his celebrated *Principia* in 1687. They are known as Newton's Laws of Motion.

1. Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by force to change that state.

2. Change of motion is proportional to the force impressed

and takes place in the direction of the straight line in which the force acts.

3. To every action, there is always an equal and contrary reaction, or the mutual actions of any two bodies are always equal and oppositely directed.

41. Illustrations. All these laws are illustrated in the train. The train can not start unless pulled by the engine, which exerts force upon it, i.e., imparts energy to it. Once started, the train can not stop itself. If brought to rest it gives up its energy in overcoming the resistance of the air, the friction of the moving parts, and the friction of the brakes when they are applied to the wheels. If there were no such resistances, the train, once set in motion with a certain velocity, would continue to move without change either of speed or direction. Again, while starting, if there were no friction, and if a constant force were applied, there would be an increase of velocity the same for each second, i.e., *the rate of change of motion*, as measured by the product of the mass and the acceleration, is proportional to the force; and it is in the direction of the straight track along which the engine pulls.

When the brakes are applied, their force, and therefore the corresponding acceleration, is in a direction opposite to that of the motion; and if this force remains constant, there is a decrease of velocity the same for each second. Here again the total change of motion is proportional to the force impressed and takes place in the direction in which this force acts.

But why is it that the train when under full head of steam does not continue with uniformly accelerated motion, and therefore increase its speed indefinitely, instead of reaching a certain speed, which it can not surpass? The answer is that the resistance of the air increases very rapidly, and therefore the engine soon has to use all its energy against air resistance and internal friction; so that there is no excess left to do the work of increasing the motion of the train. The total external force opposed to the motion of the train is exactly equal to the total external force urging it forward; and therefore *the result is the same as if no force were acting at all*—namely uniform motion in a straight line.

Now where are we to look for *the application of the third law?* We have seen that the engine can not move the train if the driving wheels slip; therefore it appears that the force of the engine is applied at the place where the drivers bear upon the track. The engine, then, tends to push the track backward, and would do so if the track were free to move. But the track is made fast to the earth, and therefore the engine tends to push the whole earth backward. The force of the engine, the action, is equal to ma, i.e., to the total mass of the engine and train multiplied by the acceleration that they acquire. Also the resistance of the earth, the reaction, is equal to $m'a'$, i.e., to the mass of the whole earth multiplied by the acceleration that it receives. This force and acceleration are oppositely directed with respect to those of the train. Why does not the earth move? The answer is that *it does move*, but so little that the motion is imperceptible. This will easily be understood when we remember that the two forces are equal, i.e., $m'a' = ma$. But, dividing both members of the equation by $m'a$, so as to get the ratio of the accelerations, we have $\frac{m'a'}{m'a} = \frac{ma}{m'a}$, whence $\frac{a'}{a} = \frac{m}{m'}$, which tells us that when the forces are equal the accelerations are inversely as the masses. Since the mass of the earth is very large compared with that of the train, it is evident that the acceleration of the earth must be very small. If the masses were more nearly equal, the accelerations would be more nearly equal. Every one knows that when he dives or jumps from a small boat it has a perceptible acceleration in the direction opposite to that in which he jumps; and if he jumps from a larger boat, the accelera-

FIG. 21. DIVING
The boat has an acceleration in the opposite direction.

tion of the boat is smaller; while if he jumps from a big ship, the acceleration of the ship is imperceptible. So it is with the engine and the earth.

42. Rate of Doing Work. In connection with work and energy there remains another important question to be considered. How are we to measure the rate at which energy is supplied; or, what comes to the same thing, how are we to measure the rate at which work is done? With the units we have adopted this is very simple. We have only to calculate the number of ergs of work done per second. Thus, if 120,000,000 ergs are done in 60 sec, then in 1 sec there will be performed one sixtieth of 120,000,000 or 2,000,000 ergs. The rate, then, is 2,000,000 $\frac{\text{ergs}}{\text{sec}}$. The rate at which any agent does work is called its POWER OR ACTIVITY, and the power is measured by the work that it can do in one second; i.e. $\text{Power} = \dfrac{\text{ergs}}{\text{seconds}}$.

43. Engineering Units. For measuring force, work, energy and power, engineers use a system of units based on the pound weight, the foot, and the second. These units are not nearly so convenient as those based upon the centimeter, the gram, and the second; but since they are so widely used in engineering practice they are here described for reference. Those students who expect to prepare themselves for engineering, should master these definitions and be able to apply them in numerical problems.

Since the foot is equal to 30.48 cm, the numerical value of the acceleration of gravity in this system is $\dfrac{980}{30.48}$, or $32.2 \dfrac{\text{ft}}{\text{sec}^2}$ (nearly). This quantity is usually denoted by g.

Instead of deriving their unit of force from a unit of mass and a unit of acceleration, as the physicists do, *engineers use as their unit of force the weight of a pound mass at sea level and in the latitude of New York, and call it the pound-force.*

Whenever in an engineering equation the mass of a body appears—as in the case of kinetic energy—it should be noted that we must eliminate it from this equation with the help of equation (4),

which expresses the relation between the mass of a body and its weight. Thus, $f = ma$, whence $m = \frac{f}{a}$. But f is expressed in pounds-weight, and a, the acceleration in this case, is $32.2 \frac{\text{ft}}{\text{sec}^2}$; therefore the mass of a body $m = \frac{\text{pounds-weight of the body}}{32.2}$, which expression must be substituted for the mass in the given equation.

The amount of work done or of energy expended when a pound-force is exerted through the distance of 1 *foot, is called one foot-pound. By equation* (5), Art. 34:

W (in foot-pounds) $= fl =$ pounds-force $\times$ feet.

To get the measure of the kinetic energy of a body, in terms of its weight and velocity, we must resort to the equation, $e = \frac{mV^2}{2}$. Since in engineers' units the mass m of the body is $\frac{\text{pounds-weight}}{g}$, and since the velocity V is expressed in feet per second, the equation becomes,

$$e = \frac{\text{pounds-weight}}{g} \times \frac{(\text{feet per second})^2}{2}, \text{ or}$$

$$e \text{ (in foot-pounds)} = \frac{\text{pounds-weight} \times (\text{feet per second})^2}{32.2 \times 2}$$

The engineers' unit for power or activity is the horse-power, which is the rate at which work is done or energy expended when 550 *foot-pounds of work are done in each second.* Hence

$$\text{Horse-power} = \frac{\text{pounds-force} \times \text{feet}}{550 \times \text{seconds}}.$$

One horse-power is found to be equal to $746 \times 10^7 \frac{\text{erg}}{\text{sec}}$.

On the continent of Europe, engineers use a system of units based upon the kilogram, the meter, and the second. These units are defined or derived in a similar manner.

Thus, the KILOGRAM-METER is the work done, or energy expended, when a force that is equal to the weight of a kilogram mass is exerted through the distance of 1 meter. Hence,

W (in kilogram-meters) $= fl =$ kilograms-force $\times$ meters.

One kilogram-meter equals 980×10^5 ergs.

Since g, the acceleration of a freely falling body, expressed in meters and seconds, is $9.8 \frac{\text{m}}{\text{sec}^2}$, the equation for kinetic energy in kilogram-meters is

$$e = \frac{mV^2}{2} = \frac{\text{kilograms-weight}}{g} \times \frac{(\text{meters per second})^2}{2}, \text{ or}$$

$$e \text{ (in kilogram-meters)} = \frac{\text{kilograms-weight} \times (\text{meters per sec.})^2}{9.8 \times 2}.$$

To solve problems in which the relations are expressed by these equations, it is necessary only to substitute the known values for the quantities represented in the equations, *each expressed in its appropriate units;* and then the unknown quantities can be found, provided, of course, that in the statement of the problem, one equation can be formed for each of the unknown quantities.

SUMMARY

1. To describe a force completely we must state: 1, its point of application; 2, its direction; 3, its magnitude.

2. Two bodies are said to have equal masses if equal forces give them equal accelerations.

3. The unit of mass is the gram, and its symbol is gm.

4. The unit of force is the dyne and its symbol is $\frac{\text{gm cm}}{\text{sec}^2}$.

5. Force is measured by the product of the mass and the acceleration, i.e., $f = ma$.

6. If a body that is free to move be affected by an unbalanced constant force, the motion will be uniformly accelerated.

7. At any given place, all freely-falling bodies have the same acceleration. At sea level in the latitude of New York this acceleration is $980 \frac{\text{cm}}{\text{sec}^2}$; therefore, since $f = ma$, a mass of 1 gm has a weight of $1 \times 980 = 980$ dynes.

8. At any given place the weights of bodies are proportional to their masses; therefore the masses of two bodies may be compared by comparing their weights.

9. The density of a substance is its mass per unit volume. Its symbol is $\frac{\text{gm}}{\text{cm}^3}$.

10. When masses are moved, or when their motions are changed,

work is done; and the measure of the work done is the product of the force and the corresponding displacement, i.e., $W = fl$.

11. The unit of work is the erg, and its symbol is $\frac{\text{gm cm}^2}{\text{sec}^2}$.

12. Scientists are convinced that a perpetual motion machine is impossible.

13. The doing of work implies the transfer of energy, and in every such transfer two bodies are equally and oppositely affected.

14. The energy transferred is measured by the work done, i.e., $e = W$.

15. The ratio of the useful work done to the total amount of energy expended is the efficiency of the machine.

16. Energy is either potential or kinetic.

17. Kinetic energy may also be measured in terms of mass and velocity, i.e., $e = W = \frac{mV^2}{2}$.

18. Activity is the rate of doing work, and is measured by the number of ergs done per second.

19. Engineering units are, for force, the pound-force; for work or energy, the foot-pound; and for activity, the horse-power.

QUESTIONS

1. In what two ways may bodies differ in mass?

2. When different forces act on the same mass, what is the relation of the forces to the corresponding accelerations? Illustrate by examples.

3. When the same acceleration is imparted to different bodies, what is the relation between the forces and the corresponding masses? Illustrate.

4. What kind of motion results from the action of a single or an unbalanced constant force?

5. Define the cm-gm-sec unit of force and give its name and symbol.

6. What is meant by the weight of a body?

7. What is the use of the equal arm balance, and why can it be employed for this purpose?

8. Of what does work consist, and what is the numerical measure of an amount of work? Write the equation for work.

9. Name and define the cm-gm-sec unit of work. Give its symbol.

10. When is a body said to possess energy?

11. What is meant by the term perpetual-motion machine? What reason have we for believing that no such machine can be made?

12. What are some of the sources from which our supplies of energy ordinarily come?

13. When a body possesses energy, is all of it available for the doing of useful work? Illustrate by some examples.

14. Define the terms kinetic energy and potential energy, and give some examples of each kind.

15. What is the advantage of having an equation for kinetic energy in terms of mass and velocity?

16. Explain the application of each of Newton's laws to the cases of a runner, a bicyclist, or an automobile.

17. If all the moving bodies on the earth, such as railroad trains, steamships, and animals, were to travel eastward at the same time, and continue to do so indefinitely, what would be the ultimate effect upon the eastward velocity of the earth's rotation?

18. Define the engineer's units of force, work, energy, and activity.

19. What expression should be substituted for mass when engineer's units are employed?

PROBLEMS

1. The masses of two loaded cars are 40,000 and 50,000 lb. respectively. If a locomotive engine, exerting 2000 pounds-force on the first car, gives it an average acceleration of 2.0 $\frac{\text{ft}}{\text{sec}^2}$, what acceleration would it give to the second car? What force would give the second car the same acceleration as was given the first? Note: Friction is not here considered. Each car would require a certain force to overcome this, in addition to that required for the acceleration.

2. Five men, rowing a boat, give it in 30.0 sec a velocity of 15.0 $\frac{\text{ft}}{\text{sec}}$. What is the average acceleration? All other things remaining the same, what velocity would be given the boat by three men? What is the average acceleration in this case? What in each case is the distance traversed in the 30 sec?

3. A base ball has a mass of 140 gm, and is thrown from home base to first, a distance of 2743 cm, in 0.90 sec. What is its velocity? If the catcher applied the throwing force during 0.10 sec, what was the average acceleration during that time? What was the force in dynes? If it was stopped by the first baseman in 0.05 sec, what then was the amount and sign of the acceleration? What force did it exert on his hands?

4. What velocity and acceleration are given to a mass of 500.0 gm by a force of 50,000 dynes applied for 10.00 sec? Through what distance does the force act? How many ergs of work are done? How many ergs of kinetic energy are stored in the moving mass?

5. A base ball, mass 140 gm, was thrown vertically upward and is caught by the thrower at the end of 5.8 sec. Find the height to which it rose, the velocity with which it was thrown, its weight in dynes, the work done on it, and the energy stored in it. If the force of the thrower was applied during 0.05 sec, what was its amount, exclusive of that required to overcome the weight?

6. A block of marble, $1.00 \times 0.50 \times 3.00$ m, has a mean density of 2.70 $\frac{\text{gm}}{\text{cm}^3}$. What is its mass? Express its weight in kilograms, and in dynes. Express in ergs and in kilogram-meters the amount of work that would be needed to lift it 5 m from the ground.

7. A brass cylinder has a mass of 122.50 gm, a diameter of 1.90 cm, and a length of 5.10 cm. What is its density?

8. What is the volume of a copper ball whose mass is 130.0 gm, and whose density is 8.87 gm?

9. A pound = 2200 gm. How many dynes does its weight equal? Find the weight of a 130 lb. boy in grams and in dynes.

10. To how many dynes is the weight of a kilogram equal? How may ergs equal a kilogram-meter?

11. The pile driver, Fig. 20, has a mass of iron weighing 3500 lb. This mass is raised by a steam hoisting engine to a height of 45 ft. and dropped upon the head of a pile. Calculate the work in foot-pounds required to raise the mass of iron to position. How much potential energy has it when lifted, and how much kinetic energy when it strikes? How much work does it do? If the pile is driven 6 inches, with how many pounds-force did it resist? Calculate the time of falling and the final velocity of the iron mass. From the weight and velocity, calculate the kinetic energy when striking, and compare the result with that calculated from the weight and the height. Which method of calculation for the energy would you choose if both weight and height were given, as in this problem? Which if the velocity were known, but not the height?

12. How many pounds of water can be pumped per minute from a mine 600 ft. deep by a 75 horse-power pump?

13. It is desired to raise ore from a mine 550 feet deep at the rate of 3 tons per minute; what horse-power must the hoisting engine be able to develop? How many foot-pounds of work would it do per ton?

14. An automobile weighing with its load 2000 lb., starting from rest, requires 22 seconds to attain a speed of 88 $\frac{\text{ft}}{\text{sec}}$, when it continues at uniform speed. Calculate its kinetic energy. What average horse-power was used in putting it into motion? What other work had to be done? How was the energy being expended after the speed became constant?

15. How many pounds of water must go over a fall each second in

order to furnish 25 horse-power, if the fall is 10 ft. high and all the power is to be used? How many cubic feet of water were used each second if 1 cu. ft. weighs 62.5 lb.? What must be the cross-sectional area of the stream at the fall, if the speed of the water there is $3\,\frac{\text{ft}}{\text{sec}}$?

SUGGESTIONS TO STUDENTS

1. Consult the libraries on the life of Sir Isaac Newton, and prepare a brief paper containing the facts that most interest you. This paper may be read before the Physics class, published in the school magazine, or offered as a theme in the English class.

2. Repeat Galileo's experiment, by throwing a block of wood and a brick from a third story window.

3. How high can you throw a base ball? Take the time with a stop watch, or with an ordinary watch, as accurately as you can; and use equation (3). Plot a graph for the complete motion of the ball, working out the distances and times for each of the seconds by equations (2) and (3). Let the best throwers plot on the blackboard, to the same scale, the graphs for their throws. Let the class compare and interpret the changes of slope.

4. Get the necessary data by trial; and calculate your horse-power (*a*) when going upstairs as fast as you can comfortably without a load; (*b*) when carrying the greatest load that you can.

5. Devise a method of measuring on the wall of the house the greatest height to which your lawn hose can throw water. Observe with a watch the number of seconds taken by it to fill a gallon jar. Allowing 8 lb. to the gallon, calculate the number of pounds of water thrown out in one sec. From this and the height, calculate the horse-power that this stream of water could be made to furnish to a small water motor.

CHAPTER III

COMPOSITION AND RESOLUTION OF MOTIONS

44. Up Grade. In Chapter II we have learned that a train moves because the driving wheels of the engine are turned; and we have studied the motions when the track is straight and level. There still remain, however, many questions that need consideration. Why must the engine work harder in ascending a grade? How can we find the amount of this extra work? What is the relation between the extra pull of the engine and the weight of the train?

In all our previous study we have considered motions along a straight line, i.e., in one direction or dimension only. The questions just asked lead us to the consideration of what takes place when a body has at the same time two or more different motions. Since these motions may or may not be in the same direction, the resultant motion may take place in two or three dimensions, i.e., the path of the motion may be a plane curve, or it may be twisted like the thread of a screw.

45. The Composition of Motions. One of the simplest cases of two simultaneous motions of the same body is that of a man walking lengthwise in a car that is moving uniformly on a straight, level track. If the car is moving northward at the rate of $600\frac{\text{cm}}{\text{sec}}$, and the man walks in the same direction at the rate of $150\frac{\text{cm}}{\text{sec}}$, how far does the man travel northward in one second? In three seconds? If the man faces about and walks southward in the moving car at the rate of $150\frac{\text{cm}}{\text{sec}}$, how far northward will he travel in one second? In ten seconds?

From these examples it must be evident that in considering motions we must take account of two characteristics of the motion, namely, direction and magnitude. For this reason it is very convenient to represent a motion by a straight line whose length

and direction correspond to the direction and magnitude of the motion. Thus, for the first case just considered, let *ab*, Fig. 22, represent the motion of the car northward: *cd*, which has the same direction and is one-fourth as long, will then represent the motion of the man with reference to a point in the car; and *ad*, which is obtained by adding together *ab* and *cd*, will represent the resultant motion of the man in both direction and magnitude.

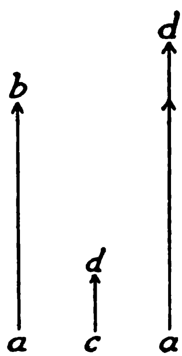


FIG. 22. VECTORS

Similarly, in the second case, if *ab*, Fig. 23, represent the motion of the train, then *ef*, which has the opposite direction and is one-fourth as long, will represent the motion of the man with reference to a point in the car. Hence *af*, which is obtained by adding together the two oppositely directed lines, will represent the resultant motion of the man in both direction and magnitude.

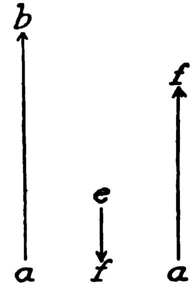


FIG. 23

It is to be noted that in both cases *this addition of the lines is performed by drawing the first line with its proper direction and magnitude, and then from the end of the first line drawing the second with its proper direction and magnitude. Then the line drawn from the beginning of the first line to the end of the second represents the resultant motion.*

46. Vectors. In order to indicate clearly the direction that such a line represents, it is usually *tipped with an arrow point* as in the figures. A line may be used in this manner not only to represent motions, but also to represent any sort of physical quantity that has both direction and magnitude. A line that is used to represent both the direction and magnitude of a physical quantity is called a VECTOR.

47. The Motions are at Right Angles. Suppose now that instead of walking northward or southward in the moving car,

the man walks eastward across it. If the velocity of the car is 600 $\frac{cm}{sec}$ northward, and that of the man 150 $\frac{cm}{sec}$ eastward, what is the resultant motion during two seconds? Simple arithmetic can not give us a solution that will determine the resultant both in direction and magnitude. Therefore let us see what the graphical method will do for us.

In Fig. 24 let the distances northward be represented by the ordinates, and the distances eastward by the abscissas, the scale being 1 cm = 200 cm for each motion. Plotting the graph in accordance with the method learned in Chapter I, we find that p_1 and p_2 represent the positions of the man at the ends of the first and second seconds respectively. Will the points that represent his position at the end of 0.5 sec and 1.5 sec also lie on the line Op_2? Will this line include the points corresponding to his position at the end of 0.1, 0.3, 1.9 sec, etc.? If we further subdivide the time unit and locate points corresponding to any of the hundredths of a second, will these lie on the line Op_2? Is it necessary to subdivide the time unit further in order to show that the line Op_2 represents the path of the man's motion as accurately as is possible in the drawing?

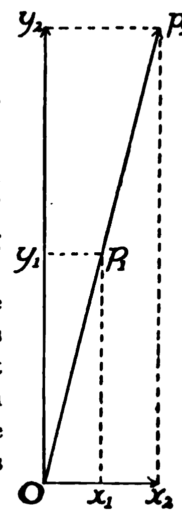


FIG. 24
PARALLELOGRAM OF MOTIONS.

Fig. 24 gives us the clew to an easy method of finding the resultant of any two uniform motions; for it is clear that the resultant is represented by the concurrent diagonal Op_2 of the parallelogram $Oy_2\,p_2\,x_2$, whose adjacent sides Ox_2 and Oy_2 represent the two component motions in both their directions and their magnitudes.

48. The Motions are not at Right Angles. Furthermore, a little careful thought will make clear the fact that *whatever may be the angle between the component motions, the resultant is completely represented by the concurrent diagonal of the parallelogram whose adjacent sides represent the two component motions both in direction and magnitude.* Thus, in Fig. 25, Ox represents one

of two uniform motions, Oy the other, and the diagonal Op the resultant. This construction is called the PARALLELOGRAM OF MOTIONS.

49. A Shorter Method. It may already have occurred to the reader that the process of finding the resultant may be very much abbreviated. For it is evident that we can determine the resultant Op_2 (Fig. 24) just as definitely by means of the triangle Ox_2p_2 as by the whole parallelogram. In order to do this we have only to draw the vector Ox_2, representing the first motion, and from its end x_2 to draw the vector x_2p_2 representing the second motion; and then the line Op_2, which joins the beginning of the first vector with the end of the second, is the vector that represents the resultant.

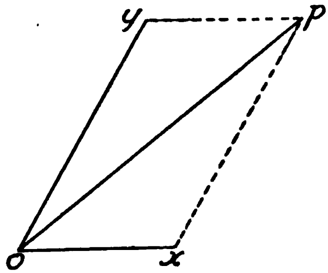


FIG. 25

This method of construction is called the VECTOR METHOD. Fig. 26 is the diagram for a problem similar to that just considered. Since the vector method is simpler than the parallelogram method and is employed by physicists and engineers, it will be used in the discussions that follow. When we have found the vector that represents the resultant, the actual magnitude of the resultant can readily be found either from the diagram or by the analytical method. Thus, in Fig. 24 we can measure the resultant vector Op_2 and we find its length to be 6.15 cm (nearly), and since in this case 1 cm of the vector represents 200 cm traversed, the resultant distance is $6.15 \times 200 = 1230$ cm, *the result by construction and measurement.*

FIG. 26

50. The Analytical Solution. To obtain the analytical solution we note that, since the two component motions are at right angles to each other, the resultant is the hypothenuse of a right triangle; and therefore, since the square of the hypothenuse is equal

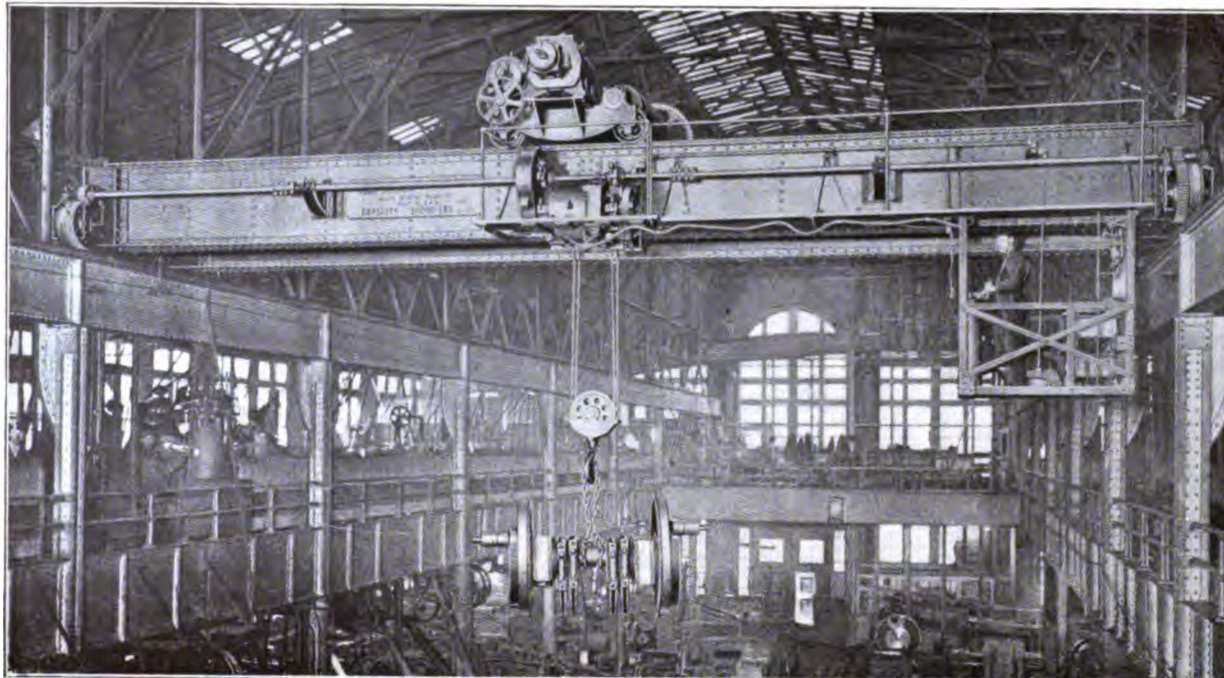


Plate II. A Traveling Crane

to the sum of the squares of the other two sides, the square of the resultant is equal to the sum of the squares of the two components. Hence, in the example represented in Fig. 24, since one component is 1200 cm, and the other, at right angles to it, is 300 cm, the resultant $= \sqrt{1200^2 + 300^2} = 1236$ cm, *the result by the analytical method.* In general, if R represent the resultant and A and B the two components, then $R = \sqrt{A^2 + B^2}$, provided that the components are at right angles with each other.

Since the two numerical results just obtained represent the same distance, why are they not identical? Would the agreement be closer if the diagram were constructed more carefully and on a larger scale?

51. When the Angle between the Components is Oblique. In this case the magnitude of the resultant can be obtained graphically by the addition of vectors in the way just described. Having drawn the resultant vector, we measure it in centimeters, and multiply its length by the number of units that 1 cm represents on the scale used in the diagram.

When the vector triangle is oblique, a purely analytical solution is impossible without the use of the elements of trigonometry. With a very little knowledge of trigonometry the solution is simple, but those who have not this knowledge can always find the resultant by construction and measurement. In fact, it is generally more convenient to get the resultant in this way; so that this method of solution is very generally used by engineers.

52. Traveling Crane. The composition of three motions is illustrated by a device used in shops where heavy castings or other weights have to be lifted and carried from one position in the shop to any other. This device, Plate II, is called a TRAVELING CRANE and consists of a steel bridge whose ends rest on little motor cars which run on tracks supported by the side walls of the shop, so that the crane can traverse the shop from one end to the other. The bridge also carries another track along which another motor car can run across the shop from one side to the other, while the weight to be carried may be lifted to any desired height by means

of a pulley hanging from the bottom of this car. The motor cars and pulley are operated by electricity, steam, or compressed air, and the operator controls them by levers so that the car carrying the pulley may be made to move either across or along the shop while the weight is being raised or lowered by means of the pulley. Thus the weight may move vertically while the car carries it horizontally across the shop, or the crane may also at the same time carry the weight horizontally along the shop. Therefore, with this device it is possible to combine motions in three directions at right angles to each other.

53. Resolution of Motions. We have just seen how two component motions may combine to make a single resultant motion. In obtaining the solutions of engineering problems it is often convenient to conceive that an observed motion is the resultant of two other motions that may have combined to produce it. Thus, when by means of the traveling crane a casting is made to move diagonally across the shop, it is clear that its actual motion is the resultant of two motions, one across and the other along the length of the shop. In a similar way the motion of a railroad train up a grade may be conceived as the resultant of two component motions, one horizontal and the other vertical.

This separation of the actual motion into two conceived motions leads us at once to the solution of several interesting problems connected with the motion of bodies "up hill"; for let us suppose that a train, running with uniform speed, is just beginning to ascend a grade. How much more work must the engine do in pulling the train up the grade than in pulling it for the same distance and at the same speed along the level track? It is evident that the amount of this extra work depends only on the steepness of the grade. Suppose that the grade is 1: 10, i.e., for every 100 cm measured along the track, the track rises 10 cm. In order to calculate the amount of extra work done in pulling the train up the grade, we shall, as stated, conceive the motion along the track as the resultant of two component motions, one horizontal and the other vertical. In Fig. 27, *ac* is the vector representing the motion of the train while passing over 100 cm of track, and *ab*

and bc are the vectors representing respectively the horizontal and vertical motions of which we conceive ac to be the resultant. Now, since ab is horizontal, no extra work is done by the engine in imparting to the train the motion represented by that vector. But bc is vertical, and it is clear that the engine can not impart a vertical motion to the train without doing the work of lifting the weight of the train. Hence the extra work done by the engine in pulling the train up grade is the work done in imparting to the train the motion represented by the vector bc. But since the grade is 1 : 10, the work done when the train traverses 100 cm of track is that of lifting the train through a vertical height of 10 cm. Thus, if the mass of the train is 2×10^8 gm, then, since the acceleration of gravity is $980 \frac{\text{cm}}{\text{sec}^2}$, we find by equation (4) that the weight of the train is $f = ma = 2 \times 10^8 \times 980 = 196 \times 10^9$ dynes. Since the vertical displacement is $l = 10$ cm, we have from equation (5) for the work done $W = fl = 196 \times 10^9 \times 10 = 196 \times 10^{10}$ ergs. This, then, is the extra work done on the train by the engine for every 100 cm up grade along the track.

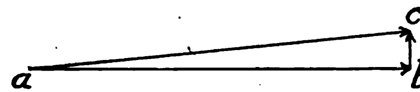


FIG. 27. RESOLUTION OF MOTIONS

54. The Engine is Stalled. If the engine is not able to supply the extra energy necessary to do this amount of work, it will be stalled on the grade. Let us suppose that this has just happened. What tendency to motion down grade has neutralized that due to the engine pulling up grade? How does the magnitude of this tendency depend upon the steepness of the grade?

55. Force Vectors. In these questions we are dealing with forces, not motions; but forces have both direction and magnitude; and therefore they can be represented by vectors, provided they act at the same point. Thus, in Fig. 28 let the vector Om represent the weight of the train, *which acts vertically downward.* This weight produces both a pressure against the track, and a tendency to move downward along the incline. Hence to answer our questions, we conceive the vector Om to be resolved into two components, one perpendicular to the track and the other parallel

to it. The first vector Op will then represent the pressure against the track, in both direction and magnitude, and in like manner the vector pm will represent the tendency to move down the incline. Now, since the pressure of the train on the track produces no motion in the direction of the vector Op, it must be evident that this pressure is balanced by an equal and opposite pressure. This equal opposing pressure will at once be recognized as the reaction of the track and earth. But if the component represented by the vector pm were not balanced by an opposing force, the train would move down the incline, i.e., in the direction of pm. At the instant when the train is stalled, it is not moving either upward or downward along the incline, and therefore what must be the direction and magnitude of the opposing force that prevents the downward motion? How should the vector representing this opposing force be drawn in the figure?

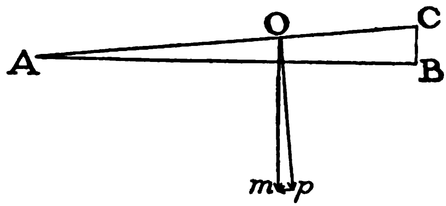


FIG. 28. RESOLUTION OF FORCES ON AN INCLINED PLANE

56. Balanced Forces. When two or more forces act simultaneously on a body in such a way that no motion results, these forces are said to be in EQUILIBRIUM. When forces are in equilibrium the vectors that represent them in the vector diagram, when added together, form a CLOSED FIGURE. Thus, in the example just discussed there are three forces acting, namely, the weight of the train, represented by Om (Fig. 28), the resistance of the track, represented by pO, and the pull of the engine, represented by mp. If these three vectors be added by laying off one from the end of another, each with its proper direction and magnitude, and in any order, they form a closed triangle as in Fig. 29.

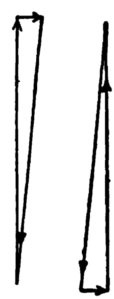


FIG. 29 FORCES IN EQUILIBRIUM

On the other hand, and in general, if we have any number of forces *not* in equilibrium acting on a body, and wish TO FIND THE FORCE THAT WILL HOLD THE SYSTEM IN EQUILIBRIUM,

we add successively the vectors that represent the given forces, and draw a line *from the end of the last vector to the beginning of the first.* This line will then be the vector that represents the force sought, in both direction and magnitude. Thus, in our example, if we know the weight of the train and the resistance of the track, and if we wish to find by this method the pull of the engine which will hold the train stationary on the grade, we add together the vectors *rs* and *st* which represent the two known forces as in Fig. 30; then the line *tr* is the vector sought.

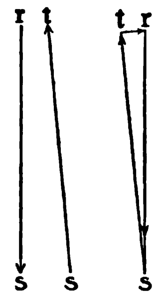


FIG. 30

This method of finding the force that is able to hold a system of other forces in equilibrium is very useful in engineering practice in connection with the design of bridges, roof trusses, and other structural work in which it is necessary to determine how strong a beam or tie must be in order to resist the given stresses and hold them in equilibrium.

57. The Pull of the Engine and the tendency down the incline are in equilibrium; therefore we can determine the magnitude of either of them, either graphically or analytically. Thus, since the mass of the train is 2×10^8 gm, the vector Om (Fig. 28), 2 cm long, represents the weight, namely, 196×10^9 dynes; and hence 1 cm in the diagram represents 98×10^9 dynes. The length of the vector pm is found by measurement to be 0.2 cm, and hence it represents a force of $0.2 \times 98 \times 10^9 = 196 \times 10^8$ dynes, which is the magnitude sought.

58. To Get the Analytical Solution we must notice that the triangles ABC and Omp (Fig. 28) are similar. (Why?) Therefore $\frac{pm}{Om} = \frac{BC}{AC}$. (Why?) But since the grade is $\frac{BC}{AC} = \frac{1}{10}$ by hypothesis, it follows that $\frac{pm}{Om} = \frac{1}{10}$. (Why?) Whence $pm = \frac{1}{10}\, Om$. Since $Om = 196 \times 10^9$, $pm = \frac{1}{10} \times 196 \times 10^9 = 196 \times 10^8$ dynes, as in the preceding paragraph.

59. Less Force: Greater Distance. Now, we have learned in Art. 53 that the extra work done in pulling the train 100 cm along the incline is the work done in lifting the train through a vertical height of 10 cm, i.e., in the case there considered, it is $W = fl = 196 \times 10^9 \times 10 = 196 \times 10^{10}$ ergs. But we have just seen that the pull of the engine is 196×10^8 dynes; and therefore, when this pull is exerted through a distance of 100 cm, the work done is $W = f'l' = 196 \times 10^8 \times 100 = 196 \times 10^{10}$ ergs, as it should be. It will be noted, however, that although the amount of work is the same as that previously calculated from the vertical lifting of the train, the *force* of the engine is only $\frac{1}{10}$ of that which would be required to lift the train vertically through the 10 cm. The advantage of using AN INCLINED PLANE is therefore apparent, since we see that by means of it we can do a given amount of work with a smaller force than would be required without it. Hence such an inclined plane is said to furnish a MECHANICAL ADVANTAGE. *This mechanical advantage is defined as the ratio of the resistance overcome to the effort applied.* In the case of the inclined plane, when the effort is applied parallel to the length of the plane, the measure of the mechanical advantage has been shown to be the ratio of the length to the height.

Thus, in general, if h represent the height of the plane, l its length, R the vertical resistance to be overcome, and f the force exerted parallel to the plane (*cf.* Art. 58 and Fig. 28), then

$$\frac{R}{f} = \frac{l}{h}.$$

This is the analytical expression for the mechanical advantage of the inclined plane when the effort is applied parallel to its length. It may also be written $Rh = fl$, which expresses analytically the fact that *the amount of work done by the force applied parallel to the plane is the same as that which would be done if the body were lifted vertically through a distance equal to the height of the plane.*

It has probably occurred to the reader to ask, Since the engine is stalled part way up the grade because its pull is no greater than the pull of the train down grade, why does the train ascend the grade at all? The answer is that when the train reached the grade it

was moving with a uniform velocity; hence it had kinetic energy whose amount is determined by equation (6) as $e = \frac{1}{2}mV^2$. It was this kinetic energy that did the work of lifting the train; and when this energy was expended, the unaided force of the engine could carry the train no farther.

60. Definitions. Some of the ideas considered in this chapter occur so frequently that we shall do well to frame definitions for them.

The single motion that will produce the same effect as that produced by two or more motions is called a RESULTANT MOTION.

FIG. 31. INCLINED RAILROAD, PIKE'S PEAK

The several motions that combine to produce the resultant are called COMPONENT MOTIONS.

The process of finding the resultant of two or more motions is called the COMPOSITION OF MOTIONS.

The process of finding the components when the resultant is known is called the RESOLUTION OF MOTIONS.

By substituting the word force wherever the word motion is used, we can frame a similar set of definitions for the composition and resolution of forces.

The single force that will hold two or more others in equilibrium is called their EQUILIBRANT. *The equilibrant of any set of forces is equal in magnitude to their resultant, and opposite in direction. The point of application of the resultant is identical with that of the equilibrant.*

61. The Problem of the Resolution of a Motion, or of a force acting at a given point, into two components is indeterminate unless something more than the resultant is given. Stated

geometrically, the problem is: given one side of a triangle, to find the other two. Evidently, we can construct any number of triangles that will satisfy this condition.

A little attention to the geometry of the triangle shows that in addition to the direction and magnitude of the resultant, we must know of the components either (1) both magnitudes (three sides); or (2) both directions (a side and two adjacent angles); or (3) one magnitude and one direction (two sides and an angle).

SUMMARY

1. Any linear motion may be represented in both direction and magnitude by a straight line called a vector.

2. The vector of a resultant motion is found by adding the vectors of the component motions.

3. If two component motions are at right angles to each other, the resultant motion is numerically equal to the square root of the sum of the squares of the two component motions.

4. Any motion may be resolved into two or more component motions.

5. In order to resolve a motion into two components, we must know of the components either (1) both directions; or (2) both magnitudes; or (3) one direction and one magnitude.

6. The mechanical advantage of an inclined plane is equal to the length of the plane divided by its vertical height.

7. The work done in moving a body up an inclined plane is equal to the work done in lifting the same body vertically through a distance equal to the height of the plane.

8. Forces that act at a given point may be represented by vectors.

9. When the vectors that represent any set of forces in equilibrium are added together in any order, they form a closed polygon.

10. The vector that represents the resultant of a number of forces not in equilibrium is found by adding in any order the vectors of these forces, and drawing a straight line from the beginning of the first vector to the end of the last.

11. The equilibrant of any set of unbalanced forces is equal to their resultant in magnitude, but opposite in direction.

QUESTIONS

1. Explain what a vector is, and how it may represent completely any physical quantity that has direction and magnitude.

2. Explain how vectors may be added in order to find the resultant of two motions when these two components have: 1, the same direction; 2, opposite directions; 3, directions that are neither the same nor opposite. How is the magnitude of the resultant motion found after the resultant vector has been drawn?

3. Explain the manner in which the analytical expression for the resultant of two motions may be found when the components have directions at right angles to each other.

4. Describe a traveling crane, and explain how with it a body may be given two or three different motions at the same time.

5. With the aid of a vector diagram, explain how the motion of a body up or down an inclined plane may be conceived as made up of two components, one horizontal and the other vertical.

6. When the weight of a body and the vertical height of an inclined plane along which it is to be lifted are known, what is the amount of work done in lifting it along the plane?

7. How does it follow from the vector diagram in question 5 that the work done in lifting the body up the incline is equal numerically to the weight of the body multiplied by the vertical distance through which it is lifted?

8. Show by a vector diagram that the weight of a body is to the force necessary to hold it in equilibrium on an inclined plane as the length of the plane is to the height.

9. Write an equation which expresses this relation. How does this equation show that the inclined plane furnishes a mechanical advantage?

10. By means of this equation, show that the work done by a force pushing the body upward along the incline is equal to the work that would be done if the body were lifted vertically through a distance equal to the height of the plane.

11. Show how the vector for the resultant of any set of forces acting at a point may be found.

PROBLEMS

1. A man rows a boat with a velocity of 200 $\frac{\text{cm}}{\text{sec}}$ southward in a stream that has a velocity of 100 $\frac{\text{cm}}{\text{sec}}$ southward. Find the resultant velocity of the boat by the vector method, and also by calculation.

2. Find the resultant velocity by both methods when the boat is rowed northward, the speeds remaining the same.

3. Find the resultant velocity by both methods when the man keeps the boat headed due westward, and does not try to resist the current, but rows with the same speed as before.

4. A boy rows a boat with a velocity of 3 $\frac{\text{miles}}{\text{hour}}$, keeping it headed across the stream, and not attempting to resist the current. The velocity of the current is 4 $\frac{\text{miles}}{\text{hour}}$. Find the resultant velocity of the boat by both methods.

5. Suppose that the width of the stream in problem 4 is $\frac{1}{3}$ mile, how many minutes will it take to cross the stream? How far will the boat drift down stream? How far will it actually travel along the resultant path?

6. The boy wishes to cross the stream in the same time as in problem 5, but intends to land at a point directly opposite the starting point. Show by vectors the direction in which he must keep the boat headed. By both methods find the speed with which he must row in order that the boat may move in a straight line from the starting point to the landing point. How far up stream would his row have taken him if there were no current?

7. If the traveling crane, Plate II, carries the pair of wheels across the shop at the rate of 1.2 $\frac{\text{ft}}{\text{sec}}$, while it moves along the shop at the rate of 1.6 $\frac{\text{ft}}{\text{sec}}$, find the resultant velocity by both methods.

8. Suppose that in addition to the other two motions of problem 7 the crane pulley rises vertically at the rate of 0.5 $\frac{\text{ft}}{\text{sec}}$, what is the final resultant velocity of the pair of engine wheels?

9. A trolley car weighs 10 tons and moves 1000 ft. along a grade that rises 1 ft. in every 100; how much work must the motor do? What is the mechanical advantage of the plane? What is the amount of the force that moves the car up the grade?

10. If in problem 9, the speed was 50 $\frac{\text{ft}}{\text{sec}}$, what was the horse-power?

11. The height of an inclined plane is 2 m and its length 10 m; the weight of a barrel that is rolled up this plane is 150 Kg; required the mechanical advantage of the plane, the number of kilograms-force exerted, and the number of kilogram-meters of work done.

12. A ball rolls down a smooth inclined plane whose length is 10^{10} cm and whose height is 10^4 cm. If the ball had fallen vertically, its acceleration would have been 980 $\frac{\text{cm}}{\text{sec}^2}$. Conceive this acceleration to be made up of two components, one along the plane, and the other perpendicular to it. Determine both graphically and by calculation the acceleration of the ball down the incline.

13. The weight of a kite is 2×10^5 dynes; the pull on the string is 4×10^5 dynes and makes an angle of 60° with the vertical. Find the resultant pull on the kite. What must be the direction and magnitude of the force that keeps the kite in equilibrium?

14. In Fig. 32, *ab* represents the direction of the keel of a boat, and the line *d* the direction of the sail, and *f* is a vector that represents the effective pressure of the wind, 200 kilograms-force. By the vector method, find the force that urges the boat forward, and also that which urges it sideways. How is sideways motion prevented?

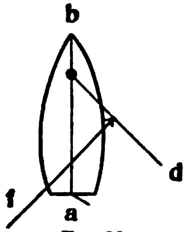


FIG. 32

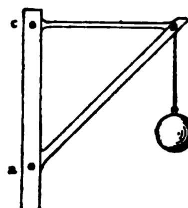


FIG. 33

15. By the vector method, find the number of kilograms-force with which the beam or strut *ab*, Fig. 33, must push, and that with which the tie rod *cd* must pull in order to keep the 40 Kg ball in equilibrium.

16. A ball is thrown upward with a velocity of 4900 $\frac{\text{cm}}{\text{sec}}$. For how many seconds will it rise before its velocity is reduced to zero by the negative acceleration of 980 $\frac{\text{cm}}{\text{sec}^2}$? What is the distance to which it rises in this time? How long will it take to reach the starting point? Calculate the velocity at the instant of reaching the starting point and compare this with the velocity with which it is thrown.

17. Calculate the distances traversed by the ball of problem 12 at the ends of the successive seconds, and plot the graph for its motion. Describe the changes of slope. What is the slope at the maximum distance, or highest point? Compare this graph with the path described by a body thrown obliquely upward.

SUGGESTIONS TO STUDENTS

1. Point your lawn hose at an angle of 45° elevation; note the path of the drops of water. Assuming 1000 $\frac{\text{cm}}{\text{sec}}$ as the initial velocity of the water, find its vertical and horizontal components. Assume that the horizontal velocity is uniform and that the vertical velocity has a negative acceleration of 980 $\frac{\text{cm}}{\text{sec}^2}$. Calculate the distances traversed vertically and horizontally at the end of each fifth of a second. Plot a graph with the vertical distances for ordinates, and the horizontal distances for abscissas. Is this graph the same sort of curve as the actual path of the water? Are you justified in inferring from the comparison that the vertical velocity was uniformly accelerated and the horizontal velocity uniform in the case of the water jet?

2. Bring a toy sail-boat to the class room to illustrate problem 14. With the aid of vectors, can you find an explanation of how such a boat can "beat against the wind"?

3. Bring in sketches or photographs which show struts and ties used in ways similar to that mentioned in problem 15. You will find them on electric light poles, supporting signs, in the frames under cars, in roofs, in bridge trusses, in jib cranes, in locomotive cranes, in bicycle frames, etc. Try to draw the vector diagrams for each case brought in.

4. How high can you throw a ball? Note with a watch the total time taken by the ball in rising and falling. Also calculate the initial velocities (*cf.* problems 16 and 17). Place on the blackboard the names of the best throwers, with velocities and distances attained.

CHAPTER IV

MOMENTS

62. How Rotation is Caused. Thus far, we have considered motion of translation only. We are now ready to take up some of the conditions under which rotary motion may occur; and the railroad train furnishes us with several questions whose answers will help us to describe accurately some relations about which we already have some general ideas. How is the translatory motion of the piston converted into rotary motion of the drivers? And why are the drivers of the fast passenger engine made large, while those of the freight engine are made small?

In order to find the answers to these questions, let us consider the diagram, Fig. 34. When the connecting rod pushes on the

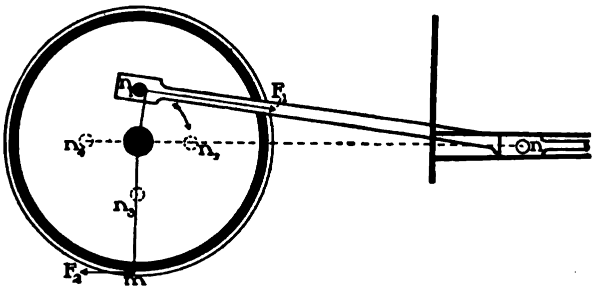


FIG. 34

crank pin at n_2, or pulls at n_4, it is evident that it can not cause the wheel to rotate, but produces only a useless strain on the moving parts. When, however, the crank pin is anywhere above or below the line nn_4, the pull or push of the connecting rod will cause the wheel to revolve. Furthermore, common experience tells us that the force of the connecting rod is more and more effective as the distance from the center of the wheel to that rod increases. There is, then, some relation between the effectiveness of a force in pro-

ducing rotation, and the distance from the axis of rotation to the line of direction in which the force acts. How shall we measure the effectiveness of a force for producing rotation?

Let us suppose that the board in Fig. 35 is supported at the middle. It will then balance, so that its weight may be left out of the problem. Suppose that a boy, whose weight is 20 kilograms, sits 100 cm from the axis. If a girl is seated 100 cm from the axis on the other side, the boy's weight can just hold the girl's in equilibrium, provided her weight is also 20 kilograms. Now, if the boy's weight is 25 Kg and his distance from the axis is 100 cm, he can balance another at 100 cm whose weight is 25 Kg; and so on. Thus in general it appears that the effectiveness of a force at a constant distance from the axis of rotation, is directly proportional to the magnitude of the force.

FIG. 35. THE MOMENTS ARE BALANCED

Again, suppose that a boy's weight is 20 Kg, and that he is distant 200 cm from the axis. He can now balance two children at 100 cm, each having the weight of 20 Kg (Fig. 36). If the 20 Kg boy is distant 300 cm from the axis, his weight will be as effective in turning the board as is a weight of 60 Kg at 100 cm; and so on. Thus, in general, if the distance from the axis varies, while the force remains constant, the effectiveness

FIG. 36. MOMENT EQUALS FORCE × ARM

of the force in producing rotation about that axis is directly proportional to the ARM OF THE FORCE, i.e., to the perpendicular distance between the axis and the line of direction of the force.

63. Moment of Force. The effectiveness of a force in producing rotation about an axis is called the MOMENT OF THE FORCE about that axis.

Since we have seen that the moment of a force is directly proportional to the magnitude of the force when the arm is constant, and directly proportional to the arm when the force is constant, it is clear that the *appropriate numerical measure of the moment of a force is the product of the force and its arm with respect to the given axis.*

Returning to the case of the locomotive drivers (Fig. 34), we see that the measure of the turning effect is the force F_1, applied to the crank pin, multiplied by the perpendicular distance from the center of the wheel to the middle line or axis nn_1 of the connecting rod.

Since now we know how to calculate the moment of a force, we shall be able to consider a few problems that will lead us to the statement of some very important principles, and will also enable us to answer the questions that were raised concerning the relative sizes of driving wheels for passenger and freight engines.

64. The Lever. Suppose that the man in Fig. 37 is to do the work of lifting, with the lever, a stone which weighs 100 Kg. He pushes vertically downward at one end with a force which we will call f. The fulcrum, i.e., the axis p (Fig. 38) about which the lever turns, is distant 40 cm from the center of the stone and 200 cm from the man's hands. The moment of f with respect to the fulcrum is $f \times 200$, and that of the stone's weight is 100×40. If the moment of f is just sufficient to keep that of the stone's

FIG. 37. THE LEVER

weight in equilibrium, then $f \times 200 = 100 \times 40$. Whence, finally, $f = 20$ kilograms-force $= 20 \times 1000 \times 980 = 196 \times 10^5$ dynes.

The force that will move the stone must, of course, be somewhat greater than this, because some unbalanced force is required to produce the acceleration.

The equation may be written:

$$\frac{100}{f} = \frac{200}{40} = \frac{5}{1},$$

which states that the mechanical advantage of this lever is 5 (*cf.* Art. 59).

65. The Work Done by the Lever is easily calculated. When the lever is moved, Fig. 38, the point *s* describes an arc with a radius of 40 cm, and moves, say, from *s* to *s'*, while the point *h* describes a similar arc with a radius of 200 cm, going from *h* to *h'*. Suppose the vertical distance *sm* through which the stone is lifted is 10 cm. The effort, at the same time, acts through the vertical distance *hn*. If the stone weighs 100 Kg, or 98×10^6 dynes, calculate how many ergs of work are done in lifting it through 10 cm.

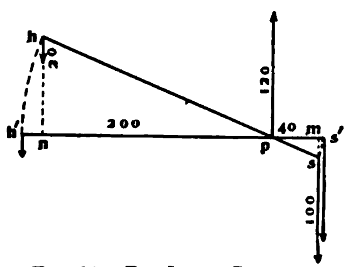


FIG. 38. THE LEVER DIAGRAM

Now, since the right triangles *msp* and *nhp* are similar (Why?), $\frac{hn}{sm} = \frac{200}{40} = \frac{5}{1}$. Since $sm = 10$ cm, what is the value of hn? Thus it appears that, although by means of this lever we are able to do the work of lifting a stone with a force that is only one-fifth of the weight of the stone, this force must be exerted through a distance or displacement five times as great as that through which the resistance is moved.

The work done by f is f multiplied by its displacement, or $(196 \times 10^5) \times 50 = 98 \times 10^7$ ergs. How does this amount of work, done by the man, compare with that done on the stone as previously calculated?

A lever is often used in another way, as in Fig. 39, when the fulcrum is at one end, and the resistance between,—the effort being applied at the other end as before. In this case the application of the principle is entirely similar; but the possible mechanical advantage is greater, because the lever arm of the effort is longer. The moment of the effort with respect to the fulcrum is now $f \times 240$, and that of the resistance is 100×40 as before, the mechanical advantage is therefore found from the equation $f \times 240 = 100 \times 40$. Whence $f = 100 \times \frac{40}{240} = 16.66$ Kg-force. The mechanical advantage in this case, therefore, is $\frac{240}{40}$, or 6. The geometrical construction by which the number of ergs of work are found and proved equal is much like the preceding, except that the similar triangles are differently placed. It is easily seen from the figure that $\frac{h'n}{s'm} = \frac{240}{40} = \frac{6}{1}$, and that the effort $\times$ 60 = the resistance $\times$ 10.

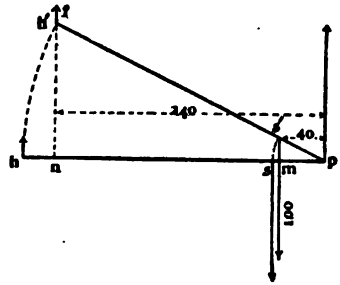


FIG. 39. ANOTHER LEVER DIAGRAM

66. The Lever Principle. In the examples just worked out, we have learned four things about the lever. Other problems involving levers can be solved in a similar manner. The four things that we have learned are:

1. *The lever is in equilibrium when the moment tending to turn it in one direction is equal to that tending to turn it in the opposite direction.*

2. *The mechanical advantage of a lever is equal to the effort arm divided by the resistance arm.*

3. *The mechanical advantage of the lever may also be obtained by dividing the displacement of the effort by the displacement of the resistance.*

4. *The work done by the effort is equal to the work done on the resistance.*

These statements may all be verified by very simple experiments in which the forces and distances are measured when various kinds of levers are in equilibrium. In such experiments and problems, it must be noted that whenever the weight of the lever itself is at all comparable in magnitude with the other forces involved, it also must enter into the calculation.

67. Equilibrium of Parallel Forces. Another important fact concerning the lever (Fig. 38) is sufficiently obvious without argument. The two downward forces must produce a downward pressure on the fulcrum; this downward pressure is their resultant, and is equal in magnitude to their sum. Hence it is evident that the fulcrum must exert an upward resistance which is the equilibrant of this resultant, and which is therefore equal in magnitude to the sum of the downward forces. It is also clear from Art. 65 that the point of application of this equilibrant divides the line joining the points of application of the two forces into segments that are inversely proportional to the magnitudes of the forces. Therefore when a system of parallel forces in one plane acts on a body, the condition that must be fulfilled in order that no translatory motion may take place is that the sum of the forces acting in one direction be equal to the sum of those acting in the opposite direction.

Similarly, the condition that must be fulfilled in order that no rotation may take place is that there be no resultant moment, i.e., that the sum of the moments tending to turn the system in one direction about any point be equal to the sum of the moments tending to turn it in the opposite direction about the same point.

It will easily be understood that these conditions for equilibrium which we have seen apply in the case of three parallel forces, must hold for any number of such forces; for clearly if there is no unbalanced force, there can be no translation; and if there is no unbalanced moment, there can be no rotation.

68. Illustration by a Problem. If we wish to determine the single force that will hold a system of known parallel forces in equilibrium, we can do so with the help of these principles. For

example, suppose it is required to lift the shaft with its pulleys, Fig. 40, by applying a single vertical force in such a way that the shaft will remain in a horizontal position as it rises. How great a force will be necessary, and at what point must it be applied? The indicated weights of the wheels and the shaft are the known forces, and their respective distances from A, the end of the shaft, are the known arms. We may assume that the bar is uniform, so that its weight acts at its middle point, as shown in the figure. The lifting force f and its arm r are to be determined. From the dimensions on the diagram we see that the sum of the downward forces is $50 + 60 + 30 + 80 = 220$. The required upward force, therefore, must be equal to this sum, or $f = 220$ Kg-force.

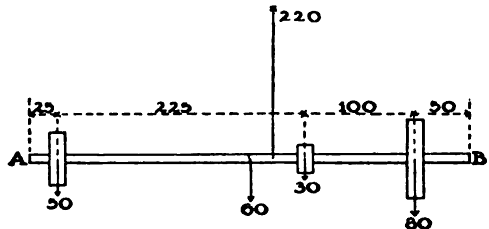


FIG. 40. THE SHAFT REMAINS LEVEL

Since the condition for no rotation is that the moments, taken with respect to any point, be balanced, we may select the left end of the shaft as the most convenient point of reference. The sum of the moments with respect to this point is, then, evidently $(50 \times 25) + (60 \times 200) + (30 \times 250) + (80 \times 350) = 48750$. This moment must be counterbalanced by that of the upward force of 220 having the unknown arm r. Hence, $(220 \times r) = 48750$. Whence $r = 221.6$ cm, i.e., the force necessary to hold the shaft in equilibrium is 220 Kg-force; and it must be applied at a point 221.6 cm from the left end of the shaft. Of course some additional force will be required to produce the acceleration when the shaft is moved.

69. The Equilibrant of Any Number of Parallel Forces may be determined in a manner similar to that used in the example just given. Since we can form two equations in which all the forces appear, we may determine either one force and one arm, as in the example, or two forces whose arms are known.

70. The Locomotive Drivers. Let us apply the principles of the lever to the driving wheels of the locomotive. Let F_1 represent the pull of the connecting rod nn_1 (Fig. 34); and let r_1, which is the perpendicular distance from the center of the wheel to nn_1, represent the arm of this force. Also let F_2 represent the push mF_2, exerted by the rim of the wheel along the track; and let r_2, the radius of the wheel, which is the perpendicular distance from its center to mF_2, represent the arm of the push F_2. Then from the equation for the mechanical advantage of the lever, $\frac{F_2}{F_1} = \frac{r_1}{r_2}$. This equation shows that the horizontal push at the rim of the driver is less than that exerted on the crank pin, in the same proportion as the distance r_1 of the crank pin from the center is less than the radius r_2 of the wheel.

FIG. 41. A FAST ENGINE

Now, an engine with large driving wheels can develop greater speed than can one with smaller ones, because the circumferences of the drivers are large; and the engine will go farther, for each stroke of the piston. But in this case our equation shows us that, other things being equal, the push that can be exerted on the track is proportionately less; because r_2, the radius of the driving wheel, is increased in the same proportion as is the circumference.

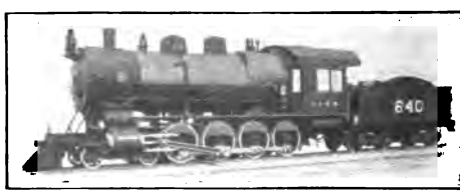


FIG. 42. A POWERFUL ENGINE

On the other hand, an engine which is to haul a long and massive freight train, must be able to exert a very great horizontal push on the track, and therefore r_2 must be made smaller in proportion to r_1. This necessitates smaller driving

wheels, giving less speed. Also, since the aim is to get as much power as possible, the engine must not only have large and powerful steam cylinders, but must also be very heavy, so as to exert sufficient pressure on the track; otherwise the driving wheels will slip, and the engine will not be able to move the train.

71. Weight and Center of Mass. Some very important applications of the principles pertaining to parallel forces are found in the action of gravity on bodies. For gravity tends to pull each particle of a body toward the center of the earth; therefore, the gravity forces that act on all the particles of a body are practically parallel, and their resultant is the weight of the body.

FIG. 43. THE SWING

Now, when any body is acted on by a system of forces affecting all its particles, and all in the same direction, there is a point so situated that the moments of all those forces will be balanced about any axis that passes through this point. This point, which is evidently the point of application of the resultant of all the parallel forces acting on the particles of the body, is called the CENTER OF MASS. The center of mass of a body is, therefore, the point of application of its weight, and hence it is often called the center of gravity.

72. Equilibrium. *If a force acting vertically upward, and equal to the weight of a body, be applied so that its line of direction passes through the center of mass, the body will be in equilibrium under the action of this force and its weight.*

Thus suppose that c (Fig. 44) is the center of mass of a body suspended at a point s, about which it is free to rotate, as is the case with swing (Fig. 43). Then if the body has been slightly displaced

from the position wherein c is vertically below s, there is a moment which is equal to the product of its weight w and the distance sb, and which will return it to that position. In what position will such a suspended body be in equilibrium? The vase and the pitcher, Fig. 45, are in equilibrium; what moment tends to return each of them, when it is slightly tilted?

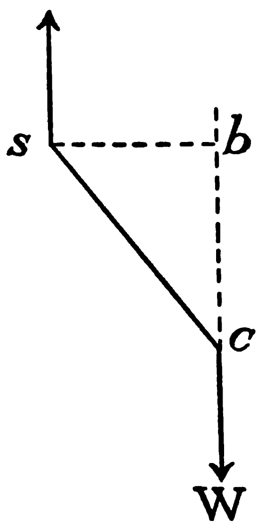


FIG. 44. THE SWING DIAGRAM

73. The Stability of a Body like the vase or the pitcher, which rests on a BASE, is measured by the amount of work that must be done in overturning it. A little consideration will show that the amount of this work may be determined as follows: With a as a center and a radius equal to ac, describe an arc. This arc is the path that the center of mass c will describe when the body is overturned about the point or axis represented by a. When the center of mass c is in the vertical line that passes through a, the body will be in unstable equilibrium, and the smallest further displacement will overturn it. From c draw a horizontal line intersecting ac' at a point b. Then bc' is the vertical distance through which the center of mass must be raised in order to overturn the body; and the work done is found by multiplying the weight by this vertical distance.

Other things being equal, if the base of the body were smaller, or if the center of mass were higher, as in the case of the vase, what would be the effect on bc', and on the work done in overturning the body?

Answers to questions like these lead to the general conclusion that, *other things being equal, the larger the base of a body, and the lower its center of mass, the greater is its stability.*

Fig. 46 represents a sphere of uniform density, whose center of figure is therefore its center of mass. Show that it is in equi-

librium in any position on a level plane. Show also that when the plane upon which it rests is slightly tilted, there is a component

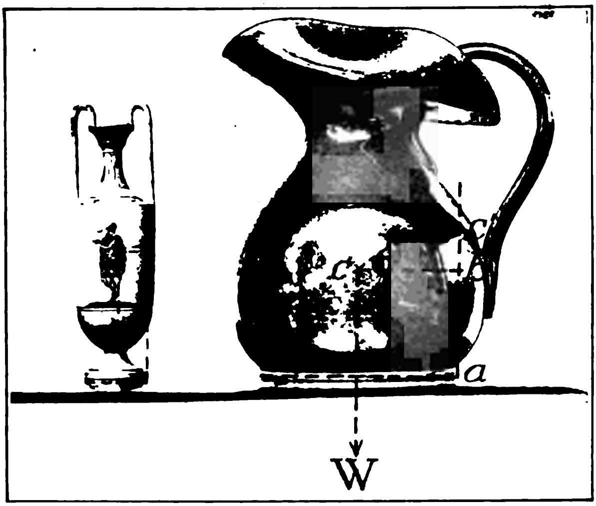


FIG. 45. STABILITY IS MEASURED BY WORK

of force urging it down the plane, and also a moment of force tending to rotate it.

74. Determination of Center of Mass. The foregoing principles of equilibrium enable us to find the center of mass of a body by experiment; for if the body be freely suspended from a point near one of its extremities, it will come to rest in the position wherein the arm of its weight becomes zero (*cf.* Fig. 44). This position is evidently that in which the center of mass is in the vertical line passing through the point of support. If this line be indicated by a plumb line, and marked on the body, we know that it contains the center of mass.

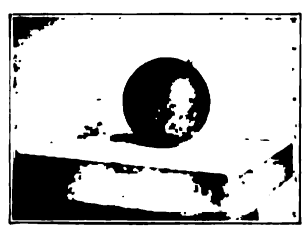


FIG. 46. THE BALL HAS NO STABILITY

If, now, another point of suspension be selected, and a new vertical line marked in the same way, it must be apparent that the center of mass, since it is in both these lines, can be nowhere else than at their intersection.

Another way of finding the center of mass of a flat, thin body, such as a piece of pasteboard, is to balance it flatwise upon a straight-edge, and mark on it the axis upon which it balances. This axis, in accordance with the definition, must contain the center of mass. Therefore, if another axis about which the moments balance be located in the same way, the center of mass is the point in which the two axes intersect, for it will be found that every other axis on which the body will balance passes through this point.

FIG. 47. THE DRIVER HAS A COUNTERPOISE

These experiments are of great convenience in connection with certain engineering problems; for it is often necessary to find the center of mass of a part of a machine, or of a piece of some structural work, in order that it may be designed so as to be in equilibrium under the given conditions.

For example, there must be placed on a locomotive driver (Fig. 47) a counterpoise having a moment exactly equal to that due to the connecting rod or side rod used in turning the wheel. This is because if the moments of all the rapidly rotating parts are not thus accurately balanced against each other, the system will wabble and produce a wasteful and even destructive strain on its axis of rotation. In order to place the counterpoise properly, its center of mass must be known; and since it is not a regular body, this determination can not easily be made by geometry. The usual practice, therefore, is to cut out a pasteboard model to a certain scale, and experimentally determine the center of mass of this model. The center of mass of the real object is then easily located, for it is the point of the real object that corresponds to the center of mass of the model.

The same method is used in order to get the position of the center of mass of half of a stone arch, so that the moment due to its weight can be calculated.

75. Mechanical Advantage of a Composite Machine. Before leaving the study of the applications of the lever principle, let us consider how we can find the mechanical advantage of a contrivance like that in Fig. 48, in which the lever principle and that of the inclined plane are used simultaneously.

In pulling the safe up the inclined plane whose height is 100 cm and whose length is 400 cm, the mechanical advantage obtained by means of the plane (*cf.* Art. 59) is $\frac{l}{h} = \frac{400}{100} = 4$; i.e., the weight of the body that can be moved along the incline is four times the pull of the rope. Furthermore, the effort, which is applied to the end of the crank, has a greater lever arm than has the pull of the rope. The effort arm in this case is the length of the crank, and the resistance arm is the radius of the axle. Therefore we obtain by this device a mechanical advantage (*cf.* Art. 65), which is equal to the ratio of the length of the crank to the radius of the axle. If these two lengths are 50 cm and 10 cm respectively, then $\frac{f_2}{f_1} = \frac{50}{10} = 5$, i.e., the pull on the rope is five times as great as the effort applied at the crank handle.

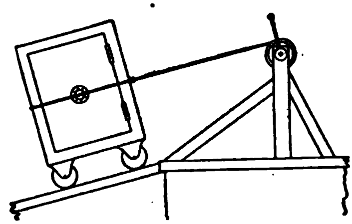


FIG. 48. COMPOSITE MACHINE
A windlass and an inclined plane.

Now, if the man applies to the crank handle a force equal to the weight of 40 Kg, it is clear that since the mechanical advantage of the windlass is 5, the pull on the rope is equal to 40 × 5 kilograms-force. Furthermore, since the mechanical advantage of the inclined plane is 4, the weight that can be lifted along the incline by this pull of 40 × 5 is, $(40 \times 5) \times 4 = 40 \times 20 = 800$ kilograms-weight. Thus we see that the mechanical advantage of the combination is $\frac{\text{resistance}}{\text{effort}} = \frac{800}{40} = 20$. This mechanical

advantage of the combination can be quickly obtained by multiplying together the mechanical advantages of the elementary parts; *e.g.*, $5 \times 4 = 20$.

Similar reasoning applied to other problems shows that this method of procedure will give the mechanical advantage of any composite machine, no matter how complicated it may be. Hence in general *we can find the mechanical advantage of any composite machine by multiplying together the mechanical advantages of the several elementary machines of which it is composed.*

76. The Law of Machines. We have seen that for the inclined plane and for the lever, the work done by the effort is equal to the work done on the resistance. Let us now see if this is true for the combination of these two devices. In the case just discussed, the effort was supposed to be 40 kilograms-force $= 40 \times 1000 \times 980 = 392 \times 10^5$ dynes. The resistance was 800 kilograms-weight $= 800 \times 1000 \times 980 = 784 \times 10^6$ dynes. The distance l_1 through which the effort acts in one revolution of the handle is $l_1 = 2\pi \times 50$ cm; and the vertical distance l_2 through which the weight is lifted may be found as follows: For one turn of the crank, the rope is drawn up a distance equal to the circumference of the axle, or $2\pi \times 10$ cm. Evidently the safe moves the same distance up the incline. But since the height of the plane is one-fourth of its length, the vertical distance through which the safe is lifted is one-fourth of the corresponding distance that it moves along the incline; i.e., $l_2 = \dfrac{2\pi \times 10 \text{ cm}}{4} = 2\pi \times 2.5$ cm.

The work done by the effort, therefore, is found to be $f_1 l_1 = (392 \times 10^5) \times (2\pi \times 50) = 392 \times 10^7 \times \pi$. That done on the resistance is $f_2 l_2 = (784 \times 10^6) \times (2\pi \times 2.5) = 392 \times 10^7 \times \pi$. Thus the two amounts of work are equal.

Similar reasoning proves that this principle, which we have demonstrated in the cases of an inclined plane, of a lever, and of a combination of the two, applies to all machines whatsoever. It is usually called the LAW OF MACHINES, and is stated as follows: *The product of the effort and the distance through which it acts is*

equal to the product of the resistance and the distance through which it is overcome; or, the work done by the effort equals that done on the resistance. In symbols,

$$f_1 l_1 = f_2 l_2. \qquad (7)$$

It should be noted that the *distances l_1 and l_2 must always be measured in the directions of the corresponding forces.* Also, in applying this statement to any particular case, it should be borne in mind that *the total work done invariably includes some useless work against such resistances as friction, rigidity of parts, inertia, reaction, and resistance of the air;* so that in order to make the statement precise and perfectly general, this useless work must be understood to be added in with the useful work. When this has been done, it is invariably found that the work done is the exact equivalent of the energy expended upon the contrivance (*cf.* Art. 36).

77. Efficiency. As the cost of the energy used in a manufacturing plant or a system of transportation is a very large part of the operating expense, the efficiency of the machinery used is a feature of great importance. It often proves to be very poor economy to buy machinery of low efficiency simply because it is cheaper.

Since some useless work is always done, no machine has an efficiency of 100%. No machine can create energy (*cf.* Art. 36); it can only transfer or transform energy that is supplied to it from some external source. It enables the user to apply his energy in more convenient ways than would be possible without it; but the user is always taxed, as it were, a certain per cent of the energy for the convenience thus obtained.

78. Mechanical Advantage from Law of Machines. The law of machines enables us to find the mechanical advantage of any machine. For we may write equation (7) (Art. 76) in the form $\frac{f_2}{f_1} = \frac{l_1}{l_2}$, i.e., the mechanical advantage of any machine is equal to the ratio of the displacement of the effort to that of the resistance. It is often more convenient to find the mechanical

advantage of a composite machine by measuring these distances than it is to calculate it by multiplying together the mechanical advantages of the parts. This is the case with the screw.

79. The Screw. The thread of the screw is an inclined plane wrapped around a cylinder. Fig. 49 shows how the screw would look if one turn of the thread were unwrapped.

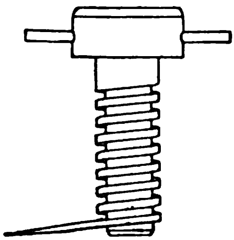


FIG. 49. SCREW THREAD UNWRAPPED

In order to turn the screw about its axis, a force f_1 is applied at the end of the lever, or at the circumference of the head; and its displacement l_1, for one turn, is the circumference described by the point of application of this force.

When the screw is rotated, either the screw itself or the nut in which it turns, moves in a direction parallel to the axis. Thus when the jack screw (Fig. 50) is turned once around, the stone, or whatever rests on the head of the screw, is lifted through the distance l_2 between two adjacent turns of the thread. This distance, measured parallel to the axis, is called the PITCH of the screw. Finally if f_2 represent the resistance to be overcome, we have from equation (7), $\frac{f_2}{f_1} = \frac{l_1}{l_2}$, which tells us that the MECHANICAL ADVANTAGE OF THE SCREW is numerically equal to the circumference through which the effort is applied, divided by the pitch of the screw.

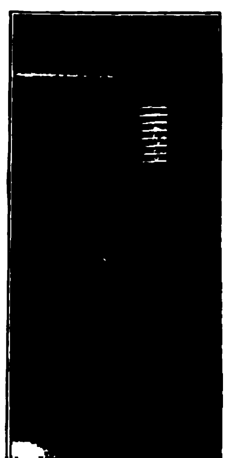


FIG. 50. JACK SCREW

Since the lever or head of the screw may be made very large, and the pitch very small, this equation shows that the mechanical advantage may be enormous, and is limited only by the strength of the materials used in the construction of the screws. Thus, wagons, locomotives, and even large buildings are lifted by means of jack screws.

Fig. 51 shows how a large house was lifted up a hill 100 ft. high with the help of such screws. They may be seen in the picture between the timbers and the house.

80. The Equal Arm Balance, which is generally used for comparing masses, is another important application of the law of moments. If we wish to weigh a certain quantity of some substance, for example a pound of sugar, the mass of the sugar in one

FIG. 51. JACK SCREWS IN ACTION

pan is assumed to be one pound when its weight balances a standard pound weight on the other pan. For if the balance comes to rest with the pointer at zero the opposing moments are equal.

Hence if f_2 represent the weight of the sugar, and f_1 that of the standard pound mass, and if r_2 and r_1 represent the corresponding arms, then the equation for the balanced moments is $f_2r_2 = f_1r_1$. But $r_2 = r_1$, therefore $f_2 = f_1$, i.e., the weights are equal. This will be true if the arms are exactly equal, and if the balance comes to rest with the pointer at zero under no load.

And since it was shown in Art. 31 that the weights are proportional to the corresponding masses, it follows that if the weights are equal the masses are also equal. Thus the law of moments shows us that we are correct in our habitual assumption that we can compare masses correctly by the process of weighing. Of course the

accuracy of the comparison is dependent on the accuracy of the balance and of the masses in the set employed as standards.

81. Looking Backward. We have now arrived at a place in our studies in Physics where it will be well to pause and review what we have learned. The principles are really very few, although, as we have begun to see, their applications to everyday life and to the devices of modern civilization are countless.

First, we have learned how uniform and uniformly accelerated motions may be accurately and concisely described, and especially so by the graphical and analytical methods. We then endeavored to gain clear notions of the relations of mass and acceleration to work, energy and activity, or power; and we found that by means of concise equations in which these relations are expressed, many important practical problems may be solved.

We then considered the behavior of bodies in motion and of bodies in equilibrium when acted on simultaneously by two or more forces; and we found that the resultant motions and the resultant forces can be represented with great ease and clearness by means of vectors. Further, we learned that many complicated machines are made up by combining the principles of two or more of the elementary mechanical devices known as the inclined plane, the lever, the pulley and the screw; and that for each of these devices a simple numerical relation between the effort and the resistance can be established. This is done by compounding or resolving forces or motions with the aid of vectors, or by taking the moments of all the forces with respect to some conveniently chosen axis, and forming the equation for their equilibrium.

In conclusion, we found that whenever energy is expended upon any kind of machine for the purpose of doing work, the sum of the useful work and the inevitable useless work is the exact equivalent of the energy expended—no more, no less.

82. Our Future Study of the several forms of energy will show us that there is nothing in the study of Physics but the accurate description of relations that occur when energy is transferred from one portion of matter to another, or changed from one form

into another form. *We can gain knowledge of these changes only through observation and experiment.*

Phenomena are thus learned and grouped into classes. The conditions under which they occur and their relations to each other are described in concise statements called LAWS. With the aid of the reasoning powers and the trained imagination, HYPOTHESES are framed for the explanation of these laws. The hypotheses are then tested by deducing from them relations which follow as necessary consequences. Careful experiments are then devised and carried out in order to determine whether or not the relations thus deduced are verified—that is, whether they are true or not.

When a hypothesis is found competent to explain every known fact that must follow as a consequence of it, and is verified by every appropriate experiment that is made in order to test it, it takes rank as an established THEORY.

By deducing from a hypothesis or theory certain consequences, and then testing these deductions experimentally, most of the great *scientific discoveries* have been made.

The method of study here outlined is called the SCIENTIFIC METHOD. Since the history of great scientific discoveries, and of the inventions which have always followed in their wake, has plainly shown that this method is the only one by which such advances have been made, the great advantage of a study like Physics is manifest. It is only by training the powers of observation and reasoning, and by developing the scientific imagination in as many people as possible, that individuals can be produced who shall continue the progress in discovery and invention which is now going on. For discoverers and inventors must not only be born and educated, but they must be supported materially, and encouraged by an intelligent interest on the part of the great body of people among whom they live and work.

SUMMARY

1. In order that a body may be made to turn about an axis, it must be acted on by a force whose line of direction does not pass through the axis.

2. The effectiveness of a force in producing rotation is its moment. Moment of force = force × arm of force.

3. The mechanical advantage of a lever may be found by equating the opposing moments taken with respect to the fulcrum. It is equal to the ratio, $\frac{\text{effort arm}}{\text{resistance arm}}$.

4. The mechanical advantage of a lever is also equal to the ratio, $\frac{\text{displacement of effort}}{\text{displacement of resistance}}$.

5. The resultant of two parallel forces having the same direction is equal to their sum; it has the same direction as the two forces; its point of application lies on the line joining theirs, and divides that line into segments that are inversely as the magnitudes of the two forces.

6. In order that any system of parallel forces may be in equilibrium, the sum of the forces in one direction must be equal to the sum of the forces in the opposite direction; also the sum of the moments tending to turn the system in one direction about any point or axis must be equal to the sum of those tending to turn it in the opposite direction about the same point or axis.

7. The equilibrant of any number of parallel forces may be fully determined by means of equations formed in accordance with this statement.

8. The center of mass of a body is the point of application of the resultant of any set of forces that act in the same direction equally on all its particles.

9. The center of gravity of a body is the point of application of its weight, and is identical with its center of mass.

10. The stability of a body is measured by the work that must be done in overturning it.

11. The mechanical advantage of a composite machine is equal to the product of the mechanical advantages of all its elementary parts.

12. In the case of every mechanical contrivance, the work done by it is equal to the work done upon it, or $f_1 l_1 = f_2 l_2$.

13. The work done by every machine includes some useless work.

14. The mechanical advantage of any machine is also equal to the ratio, $\frac{\text{displacement of the effort}}{\text{displacement of the resistance}}$, both displacements being measured in the directions of their corresponding forces.

15. The mechanical advantage of the screw is equal to the ratio, $\frac{\text{circumference described by the effort}}{\text{pitch of the screw}}$.

16. The equal arm balance is used for comparing masses by means of their weights.

QUESTIONS

1. How must force be applied to a body in order to make it rotate about a given axis?

2. What is meant by the moment of a force with respect to a given axis? What is its numerical measure?

3. Show how, by equating the moments about the fulcrum of a lever, we can find the equation for its mechanical advantage.

4. Show by geometry that for a lever the displacements are proportional to the corresponding arms. What, then, is the relation between the displacements and the corresponding forces?

5. Show in the case of the lever that the work done by the effort equals that done on the resistance.

6. What are the conditions that must be satisfied in order that any system of parallel forces may be in equilibrium? What kind of motion will result if each of these conditions is not satisfied? If neither is satisfied?

7. Explain what the center of mass of a body is.

8. Explain why the weight of a body may be supposed to be a single force, acting at its center of mass.

9. What is the measure of the stability of a body?

10. Show how the stability of a body may be determined graphically.

11. Show by a diagram that the stability of a suspended body is increased by increasing the distance between its center of mass and its point of suspension; and vice versa.

12. Show by diagrams that the stability of a body supported on a horizontal plane is increased (*a*) by increasing the area of the base, (*b*) by lowering the center of mass.

13. Explain how the principles of stability are applied practically in the construction and loading of buildings, wagons, and cars.

14. Explain why a man on a step ladder is more easily overturned the higher he ascends, unless the feet of the ladder are put proportionately farther apart.

15. In what two ways may the mechanical advantage of a composite machine be determined?

16. State the law of machines, and write the equation that expresses it analytically.

17. What kinds of useless work are done by a machine?

18. Of what commercial importance is the efficiency of machines?

19. What are some of the uses of the screw? How may its mechanical advantage be determined?

20. From the law of moments, show why we can correctly compare masses by means of the equal arm balance.

PROBLEMS

1. A workman applies 75 Kg-force at one end of a crowbar 200 cm long; what weight may be lifted at the other end distant 25 cm from the fulcrum? What is the mechanical advantage? With the same ratio of the arms, what effort will be required to overcome a resistance of 800 lb.? In each case what was the amount and direction of the pressure exerted by the fulcrum?

2. A safety valve lever, Fig. 52, has its fulcrum at a, and is to push down on the valve rod at c, which is 2 cm from a. What must be the weight of the ball, if it is to be applied at notch 5, which is 6 cm from a, and exert at c a 5 kilograms-force? What force will be exerted at c if the ball weighs 2 Kg and is placed at notch 10, which is 12 cm from a?

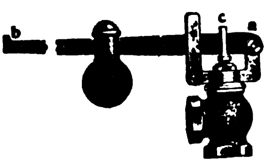


Fig. 52

3. Suppose that a trunk 0.9 m long, 0.6 m high, and weighing 120 Kg is to be tipped over on one end by lifting at the other. The weight is uniformly distributed: how much force is necessary to start it? Will this force increase or diminish as the trunk approaches the upright position? Represent graphically the stability of the trunk, and express the value of the stability in kilogram-meters of work.

4. Devise a scheme for weighing a turkey, using only a stick of uniform density and cross-section, and 50 cm long, a cm scale, some strong cord, and a flatiron known to weigh 2.73 Kg (6 lb.). Be sure that your scheme provides for eliminating the weight of the stick. Illustrate your method by working out a numerical example.

5. A bridge whose weight is 3×10^5 lb. rests on abutments 75 ft. apart. Assuming that the weight of the bridge is uniformly distributed, what part of this weight is supported by each abutment? What

additional pressure is applied when an engine that weighs 12×10^4 lb. stands with its center of mass 25 ft. from one end of the bridge?

6. With a single fixed pulley (Fig. 53), what pull on one end of the cord will support a weight of 100 Kg at the other? Use the lever principle, assuming the radius of the pulley to be 5 cm. In this case what is the pull on the support if the pulley itself weighs 2 Kg? When the resistance is overcome through 1 m, through what distance does the force act? What is the mechanical advantage?

7. A movable pulley is arranged as shown in Fig. 54. With what force f_1, and in what direction, must you pull in order to support a weight f_2, of 150 Kg? Would the force be the same if you pulled at f, making use of the fixed pulley? Use the lever principle, taking q for one fulcrum and i for the other. Can you obtain the same result from the law of machines [equation (7), Art. 76]? Express the mechanical advantage in terms of the lever arms, and also in terms of the distances.

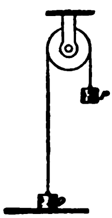


FIG. 53

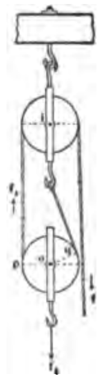


FIG. 54

8. Show that if the arrangement of pulleys in Fig. 54 were turned end for end, the mechanical advantage would be 3 for a pull at f. In arrangements of this sort, what is the relation between the mechanical advantage and the number of parts of the cord that pull against the resistance? Is there any useless work done by the pulleys, so that the mechanical advantage actually obtained is less than that given by the calculation?

9. The screw of a cider press has a pitch of 0.5 cm, and is turned by a lever 50 cm long. What is its mechanical advantage? What pressure will be exerted on the apples when you apply a 30 Kg-force at the end of the lever?

10. Make a diagram of a combination of any two of the machines mentioned in this list of examples. Find the mechanical advantage of the combination (*cf.* Art. 75), and the effort necessary to overcome a resistance of 400 Kg-force.

SUGGESTIONS TO STUDENTS

1. Measure the lever arm and the pitch of the screw of a vise, and find its mechanical advantage.

2. If you are interested in turning lathes, examine one in a shop, and see how many of its parts have mechanical advantages. How is more force and slower speed obtained by shifting the belt from one pair of pulleys to another? When a screw is to be cut, how do you

make the cutting tool travel along the lathe bed at the desired rate;—for example, to cut twice as many threads to the inch as there are on the lead screw? What other examples of the composition of motions and of the lever principle does the lathe furnish?

3. What kinds of lever can you find in a sewing machine? in a typewriter? in a bicycle? Consult a book on physiology and see if you can find the lever principle in the human arm, foot, jaw, etc.

4. Can you solve the lever problems presented in rowing a boat? in using a nut-cracker? in the sugar tongs? in the scissors? in the gas tongs? in the claw hammer?

5. With a set of pulleys like Fig. 54, determine by experiment the number of gms-weight at f that will just lift a given weight at W with uniform speed. Measure the distance l through which f moves, while W is being lifted a distance $h = 10$ cm. Calculate $f \times l$, the work done by the effort, and $W \times h$, the work done on the resistance; also calculate the efficiency, $\frac{W \times h}{f \times l}$. Now, by taking off weight at f, find the number of gms-wt at f that will just allow W to descend with uniform speed; and also find the efficiency in this case as you did in the first. Take the average of these two efficiencies as the mean efficiency for the given load. In the same way, find the mean efficiencies for, say, 9 other loads, and choosing a convenient scale, plot a graph with efficiencies for ordinates, and loads for abscissas. Does the efficiency increase with the load? In direct proportion, or according to some other law? A set of pulleys can be bought cheaply at a hardware store, and will be all the more interesting if not too good. How can you determine the number of gms-force of the friction?

6. If mechanically inclined, you may find mines of interesting information about all sorts of mechanical devices, their mechanical advantages and efficiencies in Perry's *Applied Mechanics* (Van Nostrand, N. Y.), and in Pullen's *Mechanics* (Longmans, N. Y.). There is much in these books that you may not be able to understand; but you can read without difficulty enough to increase immensely the knowledge that you have thus far acquired. Lodge's *Mechanics* (Macmillan, N. Y.) is easier reading, and will also interest and help you.

Plate III. The Big Flywheel Steadies the Motion

CHAPTER V

ROTATION

NOTE. The authors recommend that this chapter be used for informal discussion on the first reading. If time is limited it may be omitted.

83. Flywheels. In the preceding chapter we learned that, in order to cause rotary motion, an unbalanced moment of force is required.

Another important case of the conversion of translatory motion into rotary motion is that of a stationary engine and its flywheel, Plate III. Here the relations involved in producing the rotary motion are the same as in the locomotive and its drivers. But the flywheel is large and has a very massive rim, and is designed to produce an effect which is not necessary in the locomotive drivers.

This effect is that of steadying the motion; for it is clear from what has preceded that the moment of force acting on the wheel is different in different positions of the crank pin, and this will cause a jerky motion of the machinery. But the big flywheel receives and stores up energy of rotation when the crank pin is in the favorable positions, and faithfully pays it back again when the crank pin is in the unfavorable positions. Thus it prevents the sudden jerks which would be injurious to both the engine and the machinery which it runs.

Why is it that the flywheel has a massive rim and large diameter? How does this distribution of the mass make it more effective in storing up energy and paying it out again?

84. Angular Measures. These questions can not be answered by expressing the relations in terms of linear velocity, because it is clear that different portions of the mass, being at different distances from the axis of rotation, have different linear velocities. Therefore we must have some other means of measuring this velocity. Now, it is evident that every spoke of the wheel, and

in fact every radius, sweeps over the same angle in the same time, and therefore all the particles of the wheel have the same ANGULAR VELOCITY. How, then, is angular velocity measured?

A convenient way to measure an angle is to find the number of times that the radius is contained in the corresponding arc; i.e.,

$$\text{angle} = \frac{\text{length of arc}}{\text{length of radius}}.$$

If in this equation we make length of arc equal to length of radius, we have, angle $= \frac{r}{r} = 1$. Hence, the appropriate unit angle is that angle which corresponds to an arc whose length equals that of the radius. This unit angle is called the RADIAN, and by a simple calculation is found to be equal to 57°.27, Fig. 55.

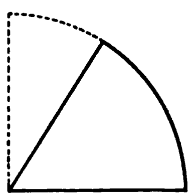


FIG. 55. ONE RADIAN

It should be noted that since the numerical value of an angle is simply the number of times that the radius is contained in the corresponding arc, it is not expressed in grams, or centimeters, or seconds; and hence angle has no symbol in terms of these fundamental units.

Now, since the angular velocity is the ratio of the angular space described to the time in which it is described, and since the unit angle is the radian, angular velocity is measured in radians per second; and unit angular velocity is the angular velocity of a body which rotates at the rate of one radian per second. Since unit angle = 1, the symbol for unit angular velocity is $\frac{1}{\text{sec}}$.

If the angular velocity varies, there will be a rate of change of angular velocity, i.e., an ANGULAR ACCELERATION; and the measure of this angular acceleration is, of course, the change in angular velocity per second. Since

$$\text{angular acceleration} = \frac{\text{angular velocity}}{\text{time}}$$

the unit angular acceleration is one $\frac{\text{radian}}{\text{sec}}$ per second, and its symbol is $\frac{1}{\text{sec}^2}$.

85. What Corresponds to Mass? We have now learned that when we are dealing with rotation instead of translation, we must consider moment of force instead of force, angle instead of distance, angular velocity instead of linear velocity, and angular acceleration instead of linear acceleration. But what, in the former case, corresponds to mass in the latter?

The answer to this question may be obtained by considering the case of a small boy swinging on a gate (Fig. 56). Suppose that the gate is open, and that a boy is perched on it at a distance of 100 cm from the hinge, or axis of rotation. For simplicity let us leave out of consideration the moment of force necessary to close

FIG. 56. A MOMENT OF FORCE PRODUCES ANGULAR ACCELERATION

the unloaded gate, and ask how much must be the extra moment of force necessary to give the gate a certain angular acceleration when there is a second child on the gate with the first (Fig. 57). If the masses of the two children are equal, then, since all the conditions are the same as before except that the mass has been doubled, it is evident that the required moment of force must be twice as great for two children as for one, three times as great for three children as for one; and so on.

Hence, in general, it appears that when the arm of the mass is constant, the moment of force necessary to impart to a mass a given angular acceleration is directly proportional to the mass.

Again, let us ask how the moment of force required to give the

gate a certain angular acceleration is affected by a change in the distance of the boy from the axis.

Suppose that the one boy is now 200 cm instead of 100 cm from the axis (Fig. 58). The required moment of force will now be

FIG. 57. GREATER MASS: GREATER MOMENT OF FORCE

much greater. But how much greater will it be? The mass of the boy now has twice the arm, and therefore his moment, with

FIG. 58. MOMENT OF FORCE IS PROPORTIONAL TO (ARM OF MASS)^2

respect to the axis, is doubled, and the moment of force required to produce the given angular acceleration would be doubled on this account alone. But since the mass must now move through an arc twice as long in the same time as before, it follows that

the moment of force required would be doubled for this reason as well. Therefore this moment must be $2 \times 2 = 2^2 = 4$ times as great as before. Similar reasoning shows that if the boy were 300 cm from the axis the required moment is $3 \times 3 = 3^2 = 9$ times as great as when the boy is 100 cm from the axis. In general, then, we find that the moment of force required to produce the given angular acceleration is proportional not only to the mass, but also to the square of its distance from the axis.

Furthermore, if we wish to shut the gate more quickly, all other conditions remaining the same, a greater moment of force is required to do it. It can be shown by experiment that the moment of force must be increased in the same proportion as is the angular acceleration that it is to produce.

Therefore, since the moment of force required to put the mass into rotation about an axis is directly proportional to the mass, to the square of the distance of the mass from the axis, and to the angular acceleration; and since it depends on these quantities alone, it follows that we may write as the equation for rotary motion,

$$\text{moment of force} = \text{mass} \times (\text{arm of mass})^2 \times \text{angular acceleration}$$

Since for translation

$$\text{force} = \text{mass} \times \text{linear acceleration},$$

we see that the quantity that stands in the same relation to moment of force and angular acceleration as does mass to force and linear acceleration is mass $\times$ (arm of mass)^2.

This quantity is called the MOMENT OF INERTIA of the mass about the given axis and is generally denoted by I.

86. Rotation vs. Translation. These relations which we have only roughly illustrated in the case of the boys on the gate have all been verified by careful experiments and shown to be true in all cases; so that in general when we are considering

ROTATION	instead of TRANSLATION, we must consider
Moment of force	" " Force,
Angle	" " Distance,
Angular velocity	" " Linear velocity,
Angular acceleration	" " Linear acceleration,
Moment of inertia	" " Mass.

Furthermore, we may obtain the relations that hold in cases of rotary motion from those for the corresponding cases of translatory motion simply by making the substitutions according to the table just given. For example, we have learned (*cf.* Art. 39) that the kinetic energy of a body in translatory motion is equal to

$$\frac{\text{mass} \times (\text{velocity})^2}{2}.$$

whence the kinetic energy of a body in rotary motion is equal to

$$\frac{\text{moment of inertia} \times (\text{angular velocity})^2}{2}.$$

87. Determination of Moment of Inertia. The determination of moment of inertia is usually a difficult problem, because different particles of the mass are generally at different distances from the axis, so that the quantity (arm of mass) is different for different particles. Hence, in order to get the value of the moment of inertia of any rotating mass, we must first consider the moments of inertia of the single particles and then sum them all up.

Thus for the rim of the flywheel, Fig. 59, the moment of inertia of a particle on the outside of the rim is the mass of this particle multiplied by the square of the outer radius of the wheel, and therefore that of the layer of particles in the outer rim is the mass of that layer multiplied by the outer radius squared, because the arm of each of these particles is equal to this radius. Likewise the moment of inertia for the layer of particles in the inside of the rim is the total mass of the particles in that layer multiplied by the square of the radius of the inside of the rim. Now the remainder of the mass of the rim may be conceived to be made up of similar layers of particles, with radii that are intermediate in length between the inner and the outer radius.

FIG. 59. MOMENT OF INERTIA EQUALS MASS × (RADIUS OF GYRATION)^2

Accordingly the total moment of inertia of the rim is the sum of all the products obtained by multiplying the mass of each layer by the square of its radius. But since the sum of the masses of the layers is the total mass of the rim, and since the radii are all intermediate in value between the outer and the inner radius, there must be some radius intermediate in value between the inner and the outer such that multiplying the total mass by the square of this intermediate radius will give the same result as would the summing up of all the separate products obtained by multiplying the masses of the several layers by the squares of their respective radii. This intermediate radius is called the RADIUS OF GYRATION.

The determination of the total mass of the rim is an easy geometrical problem, but the calculation for the radius of gyration requires the higher mathematics. In cases like that of the rim of the flywheel, unless great accuracy is required, we may assume that the radius of gyration is equal to half the sum of the inner and the outer radii of the rim.

88. Effectiveness of Flywheels. The effectiveness of the flywheel in steadying the motion depends on the magnitude of its moment of inertia, and since this magnitude is proportional not only to the mass of the wheel, but also to the square of the radius of gyration, it becomes apparent that not only must the mass be large, but that also this mass must be placed as far as practicable from the axis. In fact, we see that a wheel of large radius of gyration is just as effective as one of half the radius of gyration and four times the mass.

From this it might seem that we could increase the effectiveness of the flywheel indefinitely by increasing its radius of gyration without correspondingly increasing its mass. There is, however, a limit which can not be passed with safety, because the flywheel may burst. A little careful reasoning with the assistance of our algebra and geometry will enable us to see why this is true, and also to arrive at a very important general principle.

89. Conditions for Circular Motion. Let the circle $p\ c\ q$, whose center is o, Fig. 60, represent a circle on the rim of the fly-

wheel, and p the position of a small portion m of its mass at a given instant. Suppose that the wheel is rotating so that the linear speed along the arc is uniform.

According to the first law of motion, m, if not a part of the rigid wheel, would move in the direction of the tangent to the circle at p; but since it is constrained to remain on the arc, it will at the end of a very short time t arrive at some point c on the circumference. Since t is very small, the arc pc will be so nearly equal to the chord that we may without appreciable error regard the arc and chord as identical. We may now let pc be the vector that represents the motion along the arc. As in the case of the inclined plane, we may resolve this motion into two components, one in the direction of the tangent and the other in that of the radius po. The lines pb and bc will be the two component vectors.

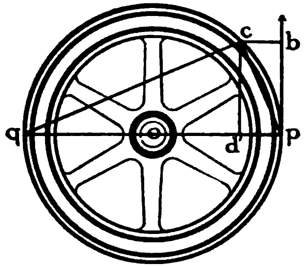


FIG. 60. THE ACCELERATION TOWARDS THE CENTER IS $\frac{v^2}{r}$

Now if we extend the radius po to cut this circumference at q, and draw qc, the right triangle, pcq and pbc are similar (Why?); therefore we have $\frac{bc}{pc} = \frac{pc}{pq}$; whence, $bc = \frac{pc^2}{pq}$. But bc represents the distance that m, starting at p, traverses in the direction po in the small time t; and hence it is equal to $\frac{at^2}{2}$, in which a is the linear acceleration of m in that direction. Also pc represents the distance that m traverses in the time t with the uniform velocity along the arc pc; so that if v represent this velocity, $pc = vt$ (Why?). Further, pq is $2r$, i.e., twice the radius of the circle. Therefore, when we substitute these values in the foregoing equation, we have $\frac{at^2}{2} = \frac{v^2t^2}{2r}$. Solving this for a, the acceleration toward the center, we obtain $a = \frac{v^2}{r}$.

The centrally directed force (sometimes called centripetal

force) that causes this acceleration $\frac{v^2}{r}$ must be [*cf.* equation (4), Art. 27] $f = ma = \frac{mv^2}{r}$. This force, of course, must have an equal and opposite reaction, which is often called centrifugal force.

The conclusions expressed by this equation will follow for any other small time t and for any other point of the circumference: hence the equation

$$f = \frac{mv^2}{r}$$

applies to all cases of rotary motion. It means that *a mass, in order to move with uniform linear speed around the circumference of a circle, must be acted on by a constant force whose direction is always toward the center, and whose magnitude is directly proportional to the mass and the square of the linear speed, and inversely proportional to the radius.*

FIG. 61. LOOP THE LOOP
The Centripetal Force is $\frac{mv^2}{r}$.

If m is expressed in gm, v in $\frac{\text{cm}}{\text{sec}}$, and r in cm, then f will evidently be in $\frac{\text{gm}}{\text{cm}} \times \frac{\text{cm}^2}{\text{sec}^2} = \frac{\text{gm cm}}{\text{sec}^2}$, which will be recognized as the symbol for dyne.

90. Why Wheels Burst. It is often convenient to express this relation in terms of angular units instead of linear units. In order to do this we must substitute angular velocity for linear velocity and moment of inertia for mass, according to the table, Art. 86. Thus, if u represent the angular velocity

$$f = \frac{mv^2}{r} = \frac{mr^2u^2}{r} = mru^2. \qquad (8)$$

This equation shows that if we increase the radius of a flywheel indefinitely, while the angular velocity u and mass m remain the same, the force required to hold the parts together will soon become greater than the cohesive force of the particles and the wheel will then burst.

FIG. 62. THE EMERY WHEEL MAY BURST

The equation also shows that for a given wheel, for example, an emery wheel, Fig. 62, the force required to hold the parts together increases as the square of the angular velocity u; and hence if we continue to increase the number of revolutions per second of the wheel, a limit will be soon reached beyond which the angular velocity can not be increased with safety.

91. Distribution of Mass. There is another very important condition which must be observed in the construction of a flywheel, or in fact of any other rotating part of a machine. The equation $f = mru^2$ tells us why this condition must be complied with, for it is evident that for a given body rotating about an axis, all the small masses m have the same angular velocity u; and hence the centrally directed force required to keep each such mass moving in its circle, depends on the value of the quantity mr corresponding to this mass. This latter quantity mr or mass $\times$ (arm of mass) is often called MOMENT OF MASS, just as force $\times$ (arm of force) is called moment of force. Now, unless the central force for any mass m on one side of the axis be balanced by an equal and opposite force on the opposite side of the axis, there will be an unbalanced lateral strain on the axis, and the wheel will wabble.

Hence it is clear that the condition that must be fulfilled to prevent the wabbling is that every moment of mass on one side of the axis must be balanced by an equal moment of mass on the opposite side. In other words, *the moments of mass must be symmetrically disposed about the axis* (*cf.* Art. 74).

92. Railroad Curves. Since the relation $f = \frac{mv^2}{r}$, which we found to hold in the case of the flywheel, applies to every mass that is moving with uniform linear speed in a circular path, it must apply to the case of a railway train when it rounds a curve. Railway curves are usually arcs of circles, and the portions of straight track at the two ends of the arc are tangents to the circle of which the curve is a part.

In accordance with the first law of motion, the car, at the instant when it reaches the curve, tends to continue moving with uniform velocity in the direction of the tangent. For have not all of us had the experience of being apparently thrown against the side of the car when it began to round a curve unexpectedly? The car turns out of its straight path because it is pushed laterally by the track; but the passenger continues in the straight path until he is suddenly pushed into the new direction by the side of the car. It is clear, then, that to keep the car moving in the curve, a horizontal force must continually be exerted by the track; and that this horizontal force must act so as always to be perpendicular to the track at the point where the pressure is exerted. This lateral force exerted by the rails is evidently directed inward toward the center of curvature, i.e., along the radius at the point where the push is exerted; and, as we have seen in the case of the rim of the flywheel, it must be equal in magnitude to $\frac{mv^2}{r}$, where m is the mass of the car, v the uniform speed along the curve, and r the radius of curvature of the track. Now, how must the track be built that it may exert the central force $\frac{mv^2}{r}$ with the minimum strain on both itself and the train? This is accomplished by inclining the road-bed so that the outer rail is higher than the inner.

Let xyz, Fig. 63, represent a cross-section of the roadbed and c the center of mass of the car, where all the forces may be supposed to act. Since the pull of the engine is balanced against the friction and air resistance, these forces may be left out of the problem. The forces that must be supplied by the track are, first, a force vertically upward and equal to mg, the weight of the

train; second, a force horizontally inward toward the center of curvature and equal to $\frac{mv^2}{r}$. These forces are represented by the vectors bp and pc respectively; therefore the resultant F of these two forces is represented by the vector bc. Now, in order to exert this force with the least possible strain on the rails, the roadbed must be perpendicular to bc. When this is so, we see from the geometry of the diagram that $\frac{zy}{xz} = \frac{pc}{bp} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$, i.e., the necessary lateral slope of the track is directly proportional to the square of the velocity and inversely proportional to the radius of curvature. Furthermore, it is not affected by the mass of the car, since the mass cancels out of the expression.

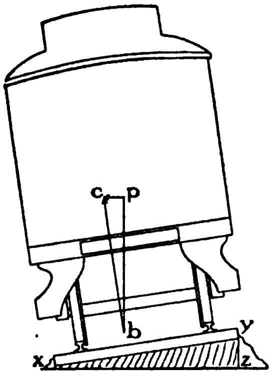


FIG. 63. THE CURVED TRACK IS INCLINED

The student will recall the similar cases of the inclination of a circus ring or race track and the inward leaning of the horse and rider; also the impossibility of making a turn with a bicycle on a slippery pavement without slackening speed.

93. Spinning Tops. Another interesting and important fact about rotary motion is that the inertia of a rotating body shows itself not only in the tendency to continue rotating when started, but also in the resistance which it offers to any force tending to change the direction of its axis of rotation. This is well illustrated in the case of the top, Fig. 64, which will stand on its point, and resist any force tending to overturn it, only so long as it is rapidly spinning. A similar case is that of a rifle ball, which is given a rapid spin about its long-


FIG. 64. THE TOP STANDS ON ITS POINT ONLY WHILE SPINNING

est axis by cutting helical grooves in the rifle barrel. Since this longest axis of the projectile coincides with the path in which it was started, the bullet tends to continue in its path point foremost, and to strike in that attitude.

SUMMARY

1. The unit angle is the radian; it has no symbol in terms of gm, cm, and sec.

2. The value of the radian in degrees is 57°.27.

3. The unit angular velocity is one radian per second. Its symbol is $\frac{1}{\text{sec}}$.

4. The unit angular acceleration is one radian per second per second. Its symbol is $\frac{1}{\text{sec}^2}$.

5. The moment of inertia of a rotating particle is mass × (arm of mass)^2.

6. The relations of moment of inertia to rotation are the same as those of mass to translation.

7. The equations of rotary motion may be obtained from those of translatory motion by substituting moment of force for force, angle for distance, angular velocity for linear velocity, angular acceleration for linear acceleration, and moment of inertia for mass.

8. The moment of inertia of an extended mass is equal to the sum of the moments of inertia of its separate particles.

9. The numerical value of the moment of inertia of an extended mass may be obtained by multiplying the total mass by the square of the radius of gyration.

10. A body will not move in a curved path unless it is constantly acted on by a central force that gives it a uniform acceleration toward the center of curvature of the curved path.

11. The numerical value of the necessary acceleration toward the center in linear units is $\frac{v^2}{r}$, and in angular units it is rv^2.

12. The force f that will keep a body moving uniformly in a circular path is equal to $\frac{mv^2}{r}$.

13. Moment of mass is mass × (arm of mass).

14. In order that a body may rotate smoothly about an axis,

the moments of mass of all its particles must be symmetrically disposed with respect to that axis.

15. A rotating body resists any force tending to change the direction of its axis of rotation.

QUESTIONS

1. What is the use of the flywheel of a stationary engine?

2. What is meant by the angular velocity of a rotating body? Is it the same for all particles of the body?

3. What is meant by the radius of gyration of a rotating body? How can the moment of inertia of an extended mass be calculated when its radius of gyration is known?

4. Why is it important in the case of a flywheel to have a large radius of gyration? How is the wheel made so as to secure this result?

5. How may material be economized in the construction of such a wheel, and why?

6. Explain why a flywheel or an emery wheel will burst if of too large diameter, or if rotated too rapidly.

7. With the aid of a vector diagram, show why a curved railway track must be inclined inward toward the center of curvature. Give some examples of similar cases.

8. Mention two ways in which the inertia of a rotating body manifests itself, and illustrate by examples.

PROBLEMS

1. Since a circumference $= 2\pi \times$ radius, $360° =$ how many radians? How many degrees are there in one radian?

2. An emery wheel makes 2400 revolutions per minute; what is its angular velocity in $\frac{\text{radians}}{\text{sec}}$?

3. A moment of force whose average numerical value is 4×10^6 gives to the flywheel of an automobile an angular acceleration of $2\ \frac{\text{radians}}{\text{sec}^2}$. What is the moment of inertia of the wheel? How many seconds are required to give this wheel an angular velocity of $20\ \frac{\text{radians}}{\text{sec}}$?

4. The rim of a flywheel has a thickness of 20 cm, an inner radius of 90 cm, and an outer radius of 110 cm. What is its volume? If its density is 8, what is its mass? Taking the mean radius as the radius of gyration, calculate the moment of inertia of the rim.

5. How many dynes must act with an arm of 20 cm in order to give the wheel of problem 4 an angular acceleration of $1\ \frac{\text{radian}}{\text{sec}}$?

6. An emery wheel of 15 cm radius is making 20 revolutions per

second. How many dynes are required to keep each gram of emery at the circumference from flying off?

7. In a loop-the-loop, Fig. 61, when the car is at the top of the loop, what will happen unless the centripetal force necessary to keep the car moving in the circular path is equal to or greater than the weight of the car? Let m represent the mass of the car, 980 the acceleration of gravity, v its linear velocity at the top of the loop, and r the radius of the loop, and show that the car will not fall if $m \times 980 = \frac{mv^2}{r}$. Need the mass be considered? If the radius of the loop is 500 cm, what must be the velocity v?

SUGGESTIONS TO STUDENTS

1. See if you can swing a small pail full of water around in a vertical circle without spilling the water. Is this experiment like a loop-the-loop? If your arm is 75 cm long, what must be the number of revolutions per second when the water does not spill?

2. Can you find out how the governor of a stationary steam engine works? What can you find out from the laundryman about centrifugal drying machines?

3. If you have occasion to take the rear wheel off your bicycle, hold it by the step, spin it rapidly, rest the end of the step on your finger, and see what the wheel will do.

4. For interesting information about tops, gyroscopes, and rotation, consult Hopkins, *Experimental Science*, (Munn & Co., N. Y), pages 10-37.

CHAPTER VI

FLUIDS

94. Pumps. In the foregoing chapters, we have found it both convenient and interesting to learn some of the principles of Physics, and some of the methods of investigating physical phenomena, by considering the motion of a railway train. We found that in order to answer only a few of the questions which naturally arise in connection with the motions of a locomotive and its parts, it was necessary to master much of that part of Physics which deals with forces and their effects, and is called Mechanics. When we take up the study of that form of energy called Heat, we shall have frequent occasion to refer to the steam engine; and in fact, if we wished thoroughly to understand the working of every part of the highest type of modern locomotive, and explain all the physical phenomena that occur in connection with it, we should find before we had finished that we needed to know the greater part of what there is to be known about Sound, Light, and Electricity, as well as about Mechanics and Heat. But, as we are now to take up the study of the Mechanics of Fluids, we shall find more obvious relations in that class of machines called pumps; because the specific purpose of every pump is to propel some liquid, like water, or some gas, like air or illuminating gas, and to deliver it under pressure at places where it is to be utilized. Furthermore, we shall gain a much wider view of the value of such knowledge and training as the study of Physics can give us, by finding out something of a few other great inventions that contribute largely to our modern civilization, and are as closely related to our everyday lives as is the locomotive engine.

In Plate IV is shown one of the powerful pumping engines that distribute the water supply of a great city; and Fig. 65 shows a very similar machine designed to distribute compressed air for operating drilling machines and other appliances used in mines and

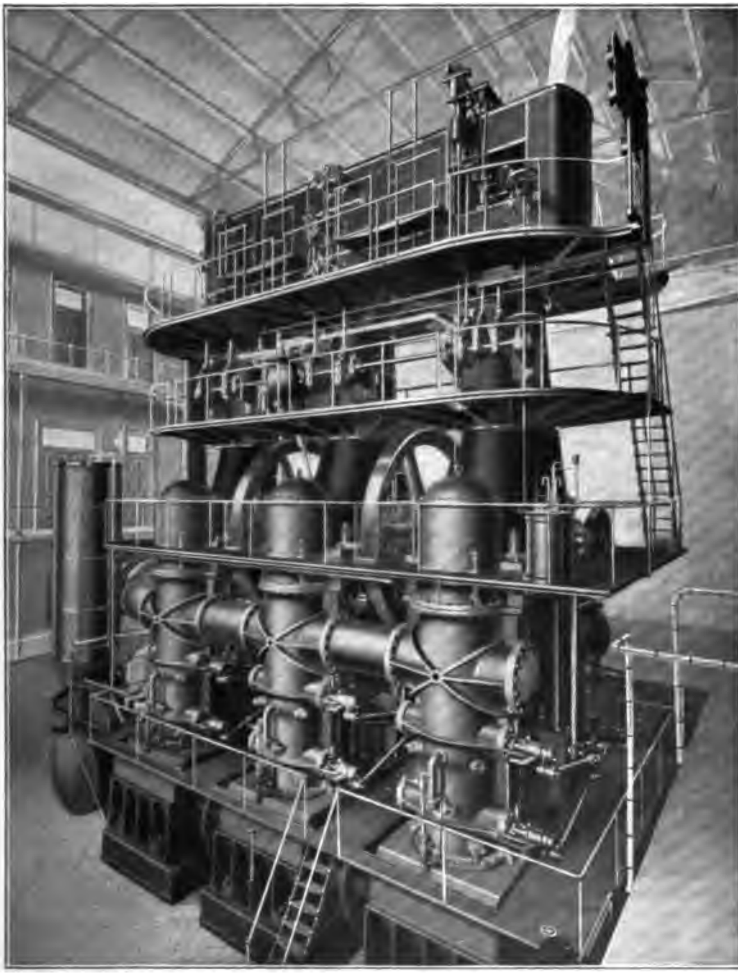


PLATE IV. PUMP IN A LARGE CITY WATER WORKS

This Pump has a capacity of 25,000,000 gallons a day. The pipes in front of the picture contain the valves, the plungers for pumping are in the cylinders farther in the rear.

factories. These mammoth pumps grew out of very small beginnings and were many centuries in coming to their present state of power and efficiency.

95. Lift Pump. The common lift pump was known and used in the time of Aristotle. The diagram, Fig. 66, shows how it is constructed and how it acts. It consists of a hollow cylinder at the bottom of which is a valve opening upward like a trapdoor and called the

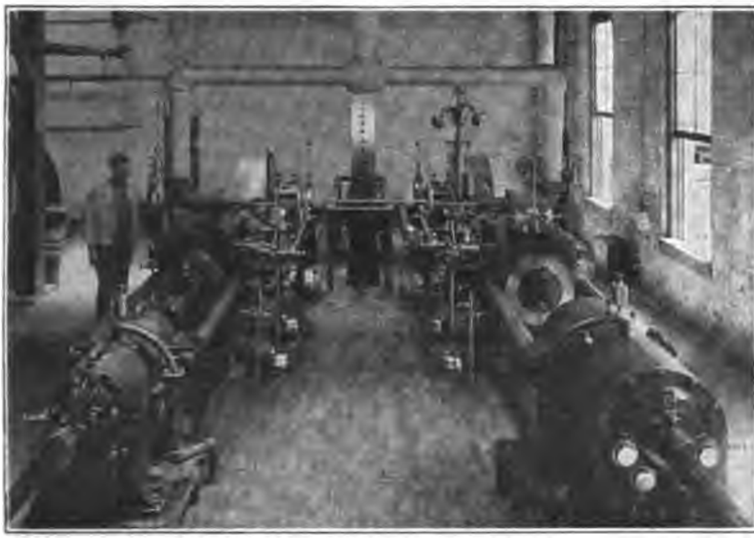


FIGURE 65. AIR COMPRESSOR

inlet valve. Fitting closely into the cylinder is a piston perforated by a hole over which is fitted another valve, also opening upward and called the outlet valve. A long pipe called the suction pipe extends into the water below. When the piston is lifted by means of the piston rod, the outlet valve remains closed, while the inlet valve opens, and air from the suction pipe enters the cylinder. When the piston is pushed down, the inlet valve closes, so that when the piston tends to compress the air in the cylinder, this air, by its reaction, opens the outlet valve and passes through the perforation in the piston. At the end of the stroke the air that had entered the

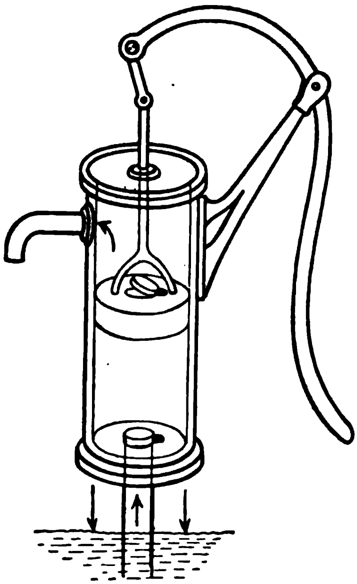


FIG. 66. THE LIFT PUMP

cylinder is above the piston and is lifted out by the piston during the next stroke The next few strokes remove the remainder of the air from the suction pipe; and the water which follows the air up the pipe and into the cylinder, passes through the pump and is pushed out in precisely the same manner as was the air.

But what force is it that pushes the air and the water into the suction pipe, and causes it to lift the inlet valve and flow into the cylinder? Aristotle and his followers offered in explanation the saying that Nature abhors a vacuum; but the reader will recognize that this is not an explanation at all.

96. Force Pump. During the first century A.D., the force pump (Fig. 67) was invented by a philosopher named Ctesibius of Alexandria. This differs from the lift pump only in that the piston is not perforated; and the outlet valve is at the end of the cylinder near the inlet valve. The outlet valve must of course open outward. Ctesibius also invented a double acting force pump for putting out fires, which differed but little from the hand fire engines now used in villages. He knew a great deal about *how* pumps worked; but discovered nothing that enabled him to explain *why* they worked. The idea that Nature abhorred a vacuum seemed sufficient to satisfy the minds of most men until the seventeenth century, when there arose a most

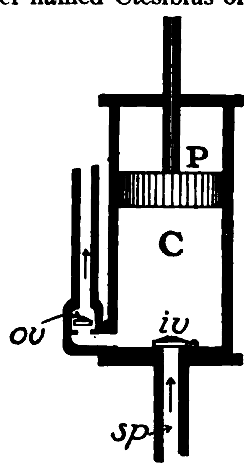


FIG. 67. FORCE PUMP

remarkable group of men, who in their search for truth about the things of the material world began to use the scientific method (*cf.* Art. 82). The result was that they made more discoveries and contributed more to accurate scientific knowledge in a few years than did all the philosophers in all the centuries before them.

97. Air Has Weight. Galileo had proved that air has weight by weighing a glass globe, forcing more air into it, and weighing it again. The difference between the two weights, he rightly ascribed to the weight of the air that had been added. He did not discover that the weight of the air had anything to do with Nature's alleged horror of a vacuum. He was astonished when informed that a lift pump had been made with a suction pipe about forty feet long, and that no amount of pumping would cause the water to rise higher than about thirty-three feet. Since a vacuum remained in the cylinder and upper part of the suction pipe, he was led to remark that the horror of a vacuum was a force that had its limitations and could be measured by the column of water that it would raise.

Galileo's friend and pupil Torricelli (1608-1647), who succeeded him as professor at the Academy of Florence, took advantage of this suggestion, and began a series of experiments which led him to infer that the weight of the water column in the suction pipe was supported by the weight of the atmosphere that rested upon the surface of the water in the cistern. Reasoning that since mercury is 13.6 times as dense as water, the weight of the atmosphere ought to be sufficient to balance that of a column of mercury only about one-fourteenth as long as the water column, he caused two of his pupils to carry out the experiment, which is known by his name.

98. Torricelli's Experiment. A glass tube, about 33 inches long, was closed at one end and completely filled with mercury. When the open end of the tube had been closed by the finger, and the tube inverted, it was supported in a vertical position with the open end in a dish of mercury. On removing the finger, the mercury sank down a little way in the tube, and, after a few oscillations,

came to equilibrium with the surface of the mercury inside the tube about 30 inches (76 cm) above that of the mercury in the dish. In the upper end of the tube was a very nearly perfect vacuum. Torricelli noticed that the height of the mercury column often varied; and he inferred that the variations were due to the changes in the pressure of the atmosphere which was "now heavier and dense, now lighter and thin."

Torricelli's hypothesis as to the pressure of the air was thus confirmed so far as his experiments went, but other experiments were necessary in order to establish its truth.

99. Pascal. When Pascal (1623-1662), who had been studying the phenomena of fluids in equilibrium, learned of Torricelli's experiments he repeated them, and concluded that "the vacuum is not impossible in Nature, and she does not shun it with so great horror as many imagine." Pascal reasoned that if one were to ascend a mountain, the pressure of the air at the greater elevation should be less, because there would be less air overlying the mountain top than there was overlying an equal area of the plain. Accordingly he wrote to his brother-in-law, who lived near the Puy de Dome, an ancient volcano in the Auvergne, France, asking him to ascend the mountain with a Torricellian tube and observe whether the mercury column would not fall because of the diminished atmospheric pressure. The experiment was made; and it was found that the mercury column became three inches shorter during the ascent, but gradually resumed its previous length during the descent to the plain.

Pascal also repeated Torricelli's experiment with wine instead of mercury; and he found as he had inferred, that, since wine is less dense than water, the atmosphere balanced a column of it which was longer than the water column; for of course it would take a longer column of the lighter fluid to make the same weight.

The hypothesis of Torricelli and Pascal as to the pressure of the atmosphere was thus placed upon a firm experimental basis, and was now competent to explain the phenomena of pumps; but it required the evidence of many more experiments to secure its general acceptance.

100. The Mercurial Barometer. The mercurial barometer which is an instrument of great precision, and of inestimable value, is simply a Torricellian tube in which the dish for the mercury is replaced by a flexible bag of chamois skin. The tube and bag are enclosed in a metal case which is fitted with a very accurate scale by means of which the height of the mercury column may be measured.

Since changes in the weather are caused by the passing of AREAS OF LOW PRESSURE, the barometric column falls when one of these areas is approaching, and rises again after the low pressure area has passed and a high pressure area has taken its place.

By means of barometers and other instruments, read simultaneously at scores of stations, the U. S. Weather Bureau officials are able to map the weather conditions of the entire country every eight hours; and thus, as the areas of low or of high pressure travel across the country, taking with them their characteristic weather conditions, the forecast official announces by telegraph the probable time of its arrival and the kind of weather that may be expected to accompany it. These WEATHER FORECASTS and STORM WARNINGS, which would be impossible without the barometer and thermometer, save many lives and much property annually.

The barometer is also much used in MEASURING ELEVATIONS, such as the heights of mountains and the altitudes attained in balloon ascensions. Near sea level, the barometer falls 0.1 inch, or 2.54 mm, for every 80 feet of elevation; but at greater elevations, since the density of the air is much less, the change of elevation corresponding to a barometric depression of 0.1 inch is greater than 80 feet, and increases steadily with the increasing elevation. The reason why the upper layers of atmosphere are less dense than the lower layers is that those upper layers have much less air above them pressing down upon them. With a good barometer a difference of four feet in altitude can be detected.

Since the pressure of the atmosphere on 1 cm^2 exactly balances the weight of a column of mercury having a certain length and equal cross-sectional area, we can calculate this pressure in grams or dynes per square centimeter by calculating the weight of this column. Thus, at sea level, the average height of the bar-

rometer column is 76 cm, and the density of mercury at 0° Centigrade—the freezing point of water—is 13.59 $\frac{gm}{cm^3}$. The volume of a mercury column 76 cm high and 1 cm^2 in sectional area is 76 cm^3. Its mass, therefore, is equal to the product of its volume and its density, i.e., $M = VD = 76 \times 13.59 = 1032.84$ gm.

Its weight in grams is therefore represented by the same number. The average pressure of the atmosphere at sea level is thus found to be 1032.84 $\frac{gm}{cm^2}$.

Let the student substitute 1032.84 for m in equation (4), Art. 27, and 980 for a, and find the average pressure of the atmosphere in dynes per square centimeter.

101. Characteristics of Fluids. We must now return to the researches of Pascal on fluids. Both liquids and gases are classed as FLUIDS, because they both have the property of offering no permanent resistance to forces that tend to change their shape. Any portion of a fluid acted on by forces not equal in all directions flows freely in the direction in which it is urged by the greater pressure. Furthermore, all fluids have perfect ELASTICITY OF VOLUME; that is, if they are compressed ever so much they immediately resume their former volumes when the additional pressure is removed.

FIG. 68. TRANSMISSION OF FLUID PRESSURE

Recognizing these two familiar properties, Pascal reasoned about fluids somewhat after this manner: Let the bottle, Fig. 68, be completely filled with any fluid, and let the little circles represent the elastic particles of the fluid. Suppose the end of the stopper to have an area of 1 cm^2, and let it be pushed in so as to exert a pressure of 1000 dynes. This pressure of 1000 dynes per square centimeter will act directly upon the layer of particles adjacent to it, so that 6, for example, will be pushed downward against 11 and 16, and this pressure will be transmitted without loss through 21, to the bottom. The pressure transmitted by 6 to 7 and 8 will tend to push them to the right and left respectively, so that the

pressure is transmitted undiminished to the sides of the bottle. Furthermore, 8 for example will tend to force 3 upward and 13 downward, thus transmitting the same pressure to those portions of the top and bottom that lie above and below them. Since all the particles will be affected in precisely the same manner, it follows that the pressure will be transmitted not only in the directions mentioned, but also in every other possible direction; and therefore the pressure of 1000 dynes exerted on one square centimeter of the fluid will cause a pressure of 1000 dynes on every other square centimeter of resisting surface with which the fluid is in contact. If the bottle is strong enough to resist this pressure, all parts of the fluid will be in equilibrium; for if it were not, any portions of the fluid affected by the unbalanced pressures would move freely in the directions of the unbalanced pressures until all the pressures were equalized.

The direction of the pressure on any part of the surface must be perpendicular to that part; for if the pressure is not perpendicular to that part of the surface upon which it acts, it may be resolved into two components, one perpendicular to the surface and the other parallel to it (*cf.* Art. 53). This parallel component, if it were exerted, would tend to rotate the bottle about some axis; and since no such tendency has ever been detected, we believe that no such parallel component exists. Therefore the pressures are all at right angles to the surfaces on which they act. It should be carefully noted that the truth of this reasoning does not depend in any way on the shape, size, or number of the particles; and the reader should avoid the notion that the particles are spherical, or that the diagram in any way represents their size or number.

102. Pascal's Principle. Having arrived at these conclusions, Pascal announced them in the following concise statement, which is known by his name: *A pressure exerted upon any part of a fluid enclosed in a vessel is transmitted undiminished in all directions, and acts with equal force on all surfaces of equal area, in directions perpendicular to those surfaces.*

A thorough understanding of this principle enables us to explain all the phenomena of fluid equilibrium.

103. Hydraulic Machines. If, for example, we have a vessel (Fig. 69) consisting of a large and a small cylinder connected by a pipe and each fitted with a water-tight piston, and if the sectional areas of these pistons are 1 cm² and 100 cm² respectively, a force of 1 kilogram exerted upon the smaller piston will produce a pressure of 1 kilogram per square centimeter on the larger piston, or a total force of 100 kilograms. Thus *the force transmitted to any surface by a fluid is directly proportional to the area of that surface.* "Hence," said Pascal, "it follows that we have in a vessel full of water a new principle of mechanics and a new machine for multiplying forces to any degree we choose."

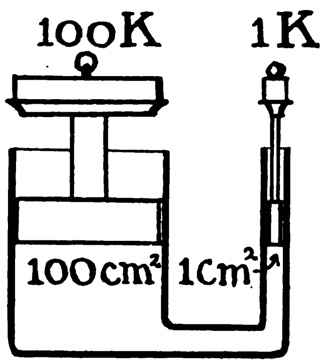


FIG. 69. THE FORCES ARE PROPORTIONAL TO THE AREAS

Since the pressures on the two pistons are proportional to their areas, the MECHANICAL ADVANTAGE OF A HYDRAULIC MACHINE of this sort is equal to the ratio of the area of the larger piston to that of the smaller.

With regard to the WORK done, it should be noted that if in the example just mentioned the small piston is pushed through a distance of 1 cm, the large piston will be displaced through only 0.01 cm, because the liquid forced out of the small cylinder into the larger has to spread over an area 100 times as great. Therefore, if we multiply the forces by the corresponding displacements, we find that the resulting amounts of work are equal. It can easily be shown that this is true no matter what areas the pistons have, and no matter what the force and displacement of the smaller piston is; so that *every hydraulic machine conforms to the* GENERAL LAW OF MACHINES (*cf.* Art. 76).

The principle of Pascal is extensively applied in a class of machines of which the HYDRAULIC PRESS, Fig. 70, is a type. It consists of a large, strong cylinder connected by a pipe with a force pump (Fig. 71). The piston of the large cylinder has an area many times larger than that of the pump. The pump piston is worked

by means of a lever and forces water or oil into the large cylinder, causing the large piston to rise. Thus a bale of cotton, or whatever substance is to be compressed, is squeezed between the pressure head of the large piston and the heavy frame above it. Hydraulic jacks, used for lifting very heavy weights, work on the same principle; and hydraulic elevators are operated by the ordinary pressure in the city water mains. In this case, the large piston is connected with a system of pulleys by which the displacement and speed of the motion are multiplied. The same principle is applied in operating drills and other tools by means of compressed air.

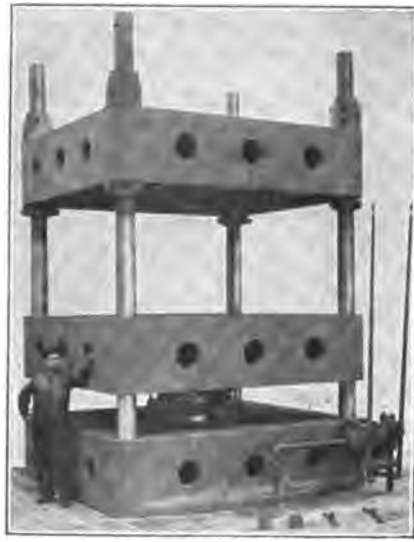


Fig. 70. The Hydraulic Press

104. Pressure Due to the Weight of a Fluid. Since every fluid has weight, it follows that every surface submerged in a fluid in equilibrium is affected by a pressure which is due to that weight alone.

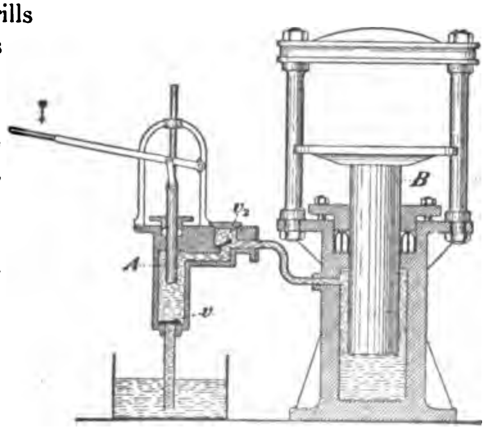


Fig. 71. The Hydraulic Press Diagram

Let us suppose that a very thin plate having an area of 1 cm² is placed horizontally in water at a depth of 1 cm. What is the amount of the pressure on its upper surface?

It may be seen from the diagram, Fig. 73, that the water which rests on this surface is a vertical sided column 1 cm^2 in sectional area; and its volume is 1 cm^3. But since the density of water is 1 $\frac{gm}{cm^3}$, the mass of this cubic centimeter of water is 1 gm, and therefore its weight is 980 dynes. Since this mass rests on the given surface, it must be evident that it exerts a pressure directly

Fig. 72. Mining Coal with a Compressed Air Drill

on it, and that this pressure is nothing more or less than its weight. If the plate is so thin that its thickness may be neglected, its under surface will be affected by an equal upward pressure, because at any given depth the pressure due to the weight of the overlying fluid will be transmitted undiminished in all directions, as stated in Pascal's principle.

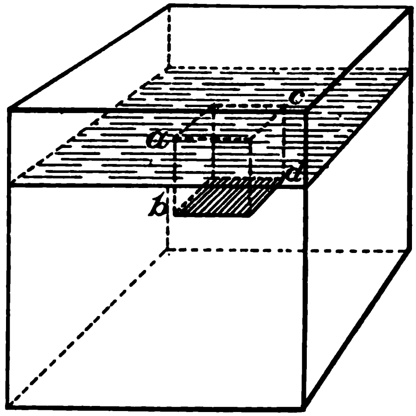


FIG. 73. LIQUID PRESSURE IS PROPORTIONAL TO DEPTH

If the plate were at a depth of 2 cm, the pressure on it would be 2 gm-force or 1960 dynes, because the weight of the overlying water column would be that of a volume of 2 cm^3, and so on.

Again, if the liquid, instead of being water, with a density of 1 $\frac{gm}{cm^3}$, were mercury, which has a density of 13.6 $\frac{gm}{cm^3}$, the weight on each square centimeter would be 13.6 times as great as that of an equal column of water. If the liquid were alcohol, whose density is 0.8 $\frac{gm}{cm^3}$, the pressure that it would exert on the submerged surface would be only 0.8 as great as that exerted by the weight of water at the same depth; and so on for other liquids.

FIG. 74. TALL STANDPIPE: GREAT PRESSURE

Since the pressure due to the weight of the fluid will be transmitted with undiminished force to all equal areas, the total force on any given surface from this cause would be directly proportional to the area of that surface.

Finally, if instead of being horizontal, the surface were vertical or oblique, the total force transmitted to it by the weight of the liquid would be exactly equal to that which would be exerted directly on it if it were horizontal, *provided its center of mass were at the same depth;* because for every small portion of the surface having a depth greater than that of the center, there will be an equal

portion having a depth that is less than that of the center by just the same amount; and therefore, the *mean pressure* on every such pair of small portions of the surface will be equal to the pressure at the center of mass. The total force on the surface will therefore be the same as it would be if the surface were horizontal and at the depth of its center of mass.

Thus the following general statements may be deduced from Pascal's principle:

The force due to the weight of a liquid in equilibrium, exerted on any surface submerged in it, is

(1) *Directly proportional to the depth of the center of mass of the surface, the depth being measured vertically from the center of mass to the level of the free surface of the liquid.*

(2) *Independent of the direction in which the surface is turned, provided its center is kept at the same depth.*

(3) *Directly proportional to the density of the liquid.*

(4) *Directly proportional to the area of the surface.*

These principles enable us to calculate the amount of the force exerted on any surface by the weight of a liquid in which it is submerged; we have only to apply this simple rule:

The force due to the weight of any liquid, exerted on a surface submerged in it, is numerically equal to the product obtained by multiplying together the weight per unit volume of the liquid, the depth of the submerged surface, and its area.

105. Free Level Surface of a Liquid. We can now understand why the free surface of a liquid in equilibrium is level, and why, in a system of communicating vessels, like a teapot and its spout, the liquid stands at the same level in all the vessels. For if the surface were higher at one place than at any other place, the liquid pressure there would be greater, and the liquid therefore would flow from the place of higher level to the places of lower level, until no part of the liquid were higher than any other.

106. Gases. Returning now to gases, we can appreciate that all we have said of liquids applies equally well to gases, with two important exceptions. First, the pressure in a gas is not propor-

PLATE V. THE MAGDEBURG EXPERIMENT

tional to the depth, because gases are easily compressed, so that the lower portions are denser than the upper ones. Second, gases have no such thing as a free level surface, for they tend to expand indefinitely, so as to fill completely the vessels in which they are confined.

107. The Air Pump. The phenomena of atmospheric pressure were very thoroughly investigated by Otto von Guericke (1602-1686), an eminent engineer, and burgomaster of Magdeburg.

Guericke began experiments on the vacuum by filling a cask with water, and then pumping the water out of an opening at the bottom. Finding this method very imperfect, he finally succeeded in inventing a fairly efficient AIR PUMP, which, a few years later, was much improved by Boyle and Hooke, in England.

108. The Magdeburg Hemispheres. Guericke made many experiments, one of the cleverest of which was that of pumping the air out of a pair of hollow iron hemispheres having smooth rims which fitted very accurately together. When the air was pumped out of these, it was found, as Guericke expected, that great force must be exerted in order to separate them; because the pressure due to the weight of the air above them held them together. Since the force required to overcome this excess of external pressure is constant, no matter in what direction the axis of the hemispheres is turned, the experiment proves that *at any given place the pressure of the atmosphere acts with equal force in all directions.*

This experiment was made by Guericke in the presence of Emperor Ferdinand II and the Reichstag, with hemispheres 1.2 feet in diameter. The force of sixteen horses was required to separate them. Of course eight horses would have done as well if he had attached one of the hemispheres to a wall or post, but the dramatic effect might not have been so great. Plate V is a photograph of the picture that appeared in Guericke's book.

109. Density of Air. Guericke was the first to demonstrate that air has weight by pumping some out of a hollow globe instead of forcing it in as Galileo did. He used a vertical tube of water

as a barometer or vacuum gauge. The density of air has been very accurately determined by weighing in accordance with the method of Guericke, and is .001293 $\frac{\text{gm}}{\text{cm}^3}$, at 0° Centigrade and 76 cm barometer pressure.

Since we have learned that the pressure of the atmosphere is about one kilogram-force on each square centimeter, the question naturally arises as to how we are able to withstand so great a pressure on our bodies. The reason is that our blood and tissue cells contain air at the same pressure. The presence of this air can be demonstrated in an experiment with the "hand glass." This is a receiver which fits on the plate of the air pump, and has an opening at the top to which the palm of the hand can be fitted air tight. When the air is pumped out of the receiver, the hand is not only pushed down with great force by the weight of the overlying air, but also the fleshy part of the palm swells out and extends through the opening into the receiver. This is because the pressure of the outside atmosphere has been removed from that part of the hand, and the air within the hand, being now freed from this pressure, expands and distends the cells in which it is confined. It is for this reason that aëronauts and mountain climbers often suffer great inconvenience, for, as they ascend, the pressure of the atmosphere diminishes so rapidly that the blood is forced to the surface by the pressure of the air within their tissues. This unbalanced pressure puts an unwonted strain on the blood vessels, which often causes some of them to burst.

110. Theory of Pumps. We are now in possession of all the information needed for explaining *why* a pump acts, as well as *how* it acts. When the piston *P*, Fig. 75, is withdrawn, it removes the atmospheric pressure from the fluid in the cylinder *C*; and the fluid is pushed into the suction pipe by the atmospheric pressure outside. Therefore the resultant force tending to push the fluid into the cylinder is equal to the atmospheric pressure diminished by the weight of the fluid in the suction pipe. If the atmospheric pressure exceeds this weight, the inlet valve *iv* will be pushed open, and some of the fluid will be pushed from the suction pipe into the cylinder. Meanwhile the pressure due to the

weight of the fluid in the outlet pipe, plus the atmospheric pressure, keeps the outlet valve *ov* closed, and prevents the reëntrance of any fluid from the outlet pipe. When the piston is pushed in, it exerts on the fluid in the cylinder a pressure which closes the inlet valve, opens the outlet valve, and pushes the fluid out through it.

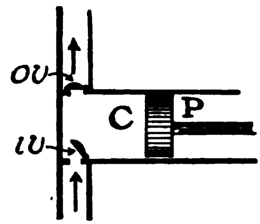


FIG. 75. PUMP DIAGRAM

Thus every double stroke removes a volume of fluid which is very nearly equal to the area of the piston multiplied by the length of the stroke. If the fluid to be pumped is air or any other gas instead of a liquid, and if it is to be pumped into or out of a closed vessel, the pressure in the closed vessel is mainly caused, not by the weight of the confined gas, but by its elasticity.

111. Archimedes's Principle. Another important corollary deducible from Pascal's principle, is known by the name of Archimedes. Let Fig. 76 represent a vessel filled with liquid to the level *ab*, in which is submerged a rectangular solid *cdef*, and let us find the resultant force due to the weight of the liquid and tending to move the solid.

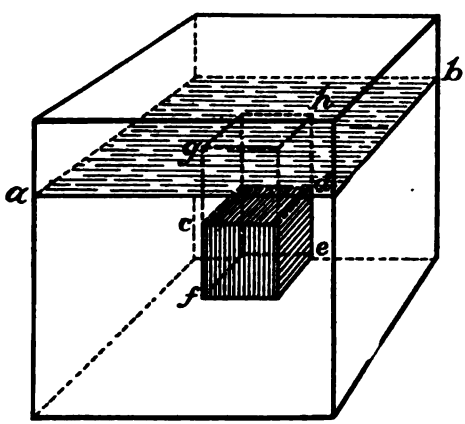


FIG. 76. BUOYANT FORCE EQUALS WEIGHT OF WATER DISPLACED

The resultant of all the horizontal forces is zero, because every such force is opposed by an equal and opposite force at the same depth on the opposite side. But how about the vertical forces? The force on *cd* is equal to the weight of a column of liquid represented by *cdhg*, acting downward; and the force on *fe* is equal

to the weight of the column of liquid *fehg*, transmitted as described by Pascal's principle, and acting upward. The resultant of these two forces, and hence of all the forces, is equal to their arithmetical difference, which evidently is equal to the weight of a volume of the liquid represented by *fecd*. This volume is that of the body, and therefore of the liquid displaced by it.

Archimedes of Syracuse (287?—212 B.C.), who was the greatest natural philosopher that lived before the time of Galileo, must have known much of what we have learned about the equilibrium of liquids, for he discovered the principle that we have just reached and announced it substantially as follows:

A body immersed in a fluid is buoyed up by a force that is equal to the weight of the fluid displaced by it.

The foregoing argument for the principle of Archimedes does not depend for its conclusion on the kind of fluid, nor on the depth to which the body is submerged. It has also been shown to apply to bodies of all shapes and sizes.

112. Floating Bodies. By Archimedes's principle, we can easily predict whether a body will float or sink in any fluid, whether that fluid be a liquid or a gas. Thus, if the weight of the body is greater than that of an equal volume of the fluid, the body will sink to the bottom of the fluid; if the weight of the body is less than that of an equal volume of the fluid, the body, if submerged, will float upward; if the weight of the body is exactly equal to that of an equal volume of the fluid, the body will remain wherever it is placed within the fluid.

From the foregoing principles it follows that if a body which is lighter than the same volume of a given fluid be placed in that fluid, it will rise or sink (depending on where it is placed), and will come to equilibrium when it displaces a volume of the fluid that weighs exactly as much as it does. The buoyant force will then exactly balance its weight. This special case of the application of the principle of Archimedes is known as the PRINCIPLE OF FLOTATION.

The principle of Archimedes describes implicitly the behavior of a boat or a balloon. The more heavily a boat is loaded, the

deeper it will sink. Why? Its gross displacement is the weight of the maximum volume of the water that it can safely displace. The weight of its maximum safe load is evidently the difference between its weight and its gross displacement. A very great load necessitates a correspondingly large displacement, which tends to diminish the speed that it can attain.

FIG. 77. SANTOS DUMONT'S "RUNABOUT"

Boats are made practically unsinkable by reserving a sufficient amount of the interior space for separate water-tight compartments, so that the boat can not sink by taking in water unless a number of the compartments are punctured at the same time. Submarine boats are made to sink by letting water into their compartments, and are made to rise by forcing it out with strong pumps.

The load that a balloon, Fig. 77, can support is equal to the weight of air displaced, diminished by the sum of the weights of the balloon, car, rigging, contained gas, and ballast. If the aëronaut wishes to ascend, he diminishes the gross weight of the balloon by throwing out ballast. If he wishes to descend, he diminishes the volume of the balloon by opening a valve at the top and letting some gas escape. This diminishes the buoyant force. Why?

113. Determination of Density. Among the important applications of the principle of Archimedes are several methods of determining density. The following example illustrates one of these. A piece of rock weighs 25 gm. When suspended from the balance pan so as to be wholly submerged in pure water, it is balanced by

15 gm. The buoyant force of the water on it is equal to its apparent loss in weight, or 10 gm. We have learned in Art. 32 that 1 cm^3 of water has a mass of 1 gm; we know, therefore, that the volume of the 10 gm of water is 10 cm^3. Since the volume of the rock is the same as that of the displaced water, or 10 cm^3, and since the mass of the rock was found to be 25 gm, its density is $\frac{\text{mass}}{\text{volume}} = \frac{25 \text{ gm}}{10 \text{ cm}^3} = 2.5 \frac{\text{gm}}{\text{cm}^3}$.

Archimedes used this method in determining for Hiero, king of Syracuse, whether some gold, which he had furnished to an artisan to be made into a crown, had been partly retained by that worthy, and the weight made up by alloying with baser metal.

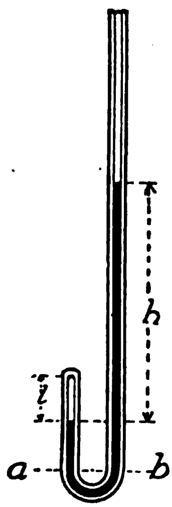


FIG. 78
BOYLE'S TUBE

When we are possessed of the knowledge gained by the experimental researches of Galileo, Torricelli, Pascal, and their eminent contemporaries, it seems easy enough for us to understand and apply the principle of Archimedes; but when we remember that Archimedes lived nineteen centuries before these men, we can not but marvel at the genius which enabled this great philosopher to think so clearly about this principle and its applications to liquids.

114. Boyle's Experiments. Among those scientists of the seventeenth century who contributed so largely to our knowledge of fluids was Robert Boyle of England (1627–1691). His greatest discovery, that of the law that goes by his name, was made during an investigation upon the elasticity of air.

In order to find out what elastic force compressed air was able to exert, and what would be the effect of increased external pressures on the corresponding volumes, Boyle provided a tube, Fig. 78, "which, by a dexterous hand and the help of a lamp, was in such manner crooked at the bottom that the part turned up was almost parallel to the rest of the tube." The shorter leg was closed and the longer open.

He started with the column of confined air 12 inches long, and with the mercury at the same level in both legs of the tube. Since the columns of mercury in the two legs then balanced each other, the pressure on the confined air was simply that of the atmosphere. (Why?) Reading his barometer, he found this atmospheric pressure equal to 29 inches of mercury. More mercury was then poured in, till the column h was 29 inches long; the pressure on the confined air was therefore twice 29 inches. (Why?) When the confined air column was under this pressure, its length e was found to be 6 inches. Thus, when the pressure had been doubled, the volume was reduced to one-half. Another 29 inches of mercury reduced the volume to 4 inches, or one-third; and so on.

115. Boyle's Law. As a result of extended experiments, therefore, Boyle announced the following law: *The volume of a given mass of gas, at a constant temperature, varies inversely as the pressure that it supports.*

In symbols, if V and V' represent any two volumes of a certain mass of gas at a constant temperature, and P and P' the corresponding pressures, Boyle's law is represented by the equation

$$\frac{V}{V'} = \frac{P'}{P} \text{ or } VP = V'P'.$$

The latter form of the equation shows that the products obtained by multiplying together each volume and its corresponding pressure are all equal. Therefore, calling this constant product K, we may represent Boyle's law by the equation $VP = K$ and express it in ordinary language as follows:

At a constant temperature, the product of the numbers representing the volume and pressure of a given mass of any gas is a constant quantity. This law has been verified by a great many experiments, and is found to be approximately true for all gases within certain wide limits; but at certain temperatures and pressures for each gas the law fails, notably when, on account of great compression, or low temperature, or both, the gas is about to liquefy.

Aëriform bodies, like air or steam, are classified as PERFECT GASES when they act in accordance with Boyle's law, and as VAPORS when they do not so act.

The graphical representation of Boyle's law is very interesting, and of great assistance in the solution of problems connected with the elastic pressures of gases in the cylinders of engines using the energy of steam, gas, or compressed air.

SUMMARY

1. The average pressure, at sea level, of the atmosphere balances the weight of a column of mercury 76 cm high. This pressure is equal to 1032.84 grams-weight per square centimeter, and is called one atmosphere.

2. The barometer is used to measure atmospheric pressure. It is applied: 1, in weather observations; 2, in determining elevations; 3, in many experiments with gases.

3. A pressure exerted on any portion of a fluid enclosed in a vessel is transmitted undiminished in all directions, and acts with equal force on all surfaces of equal area, in directions perpendicular to those surfaces. (Pascal's principle.)

4. Pressure due to the weight of a liquid in equilibrium is proportional to its depth and to its density.

5. The pressure due to the weight of a gas is not proportional to its depth.

6. Liquids have a free level surface; gases do not.

7. Gases tend to expand indefinitely.

8. A body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. (Archimedes's principle.)

9. When a body is submerged in water, the number of gm of water displaced by it is equal to the number of cm^3 in its volume.

10. The volume of a given mass of any gas at constant temperature is inversely proportional to the pressure that it supports. (Boyle's law.)

QUESTIONS

1. Describe the simple mercurial barometer, as arranged in the experiment of Torricelli.

2. Why is the mercury column thus upheld shorter than the column of water in the suction pipe of a pump?

3. Is there any air in the space above the mercury in a Torricellian tube?

4. Explain briefly why a falling barometer indicates stormy weather, and a rising barometer, fair weather.

5. Explain how a barometer may be used for measuring altitudes above sea level.

6. Why is it that if a balloon were ascending uniformly a barometer column would fall faster near the surface of the earth than it would at a greater altitude?

7. Show how a mechanical advantage may be obtained from a body of fluid, as in the case of a hydraulic press. Show that a hydraulic machine conforms to the general law of machines.

8. State four facts about the force due to the weight of a liquid in equilibrium, and show how they are deducible from Pascal's principle.

9. Suggest some experiments by means of which these four facts might be verified.

10. State a rule, derived from these facts, for determining the total force exerted by the weight of a liquid on a surface submerged in it. In this rule, how is the depth to be measured?

11. By means of a suitable diagram, explain why the free surface of a liquid in equilibrium is level.

12. Describe the experiment of the Magdeburg hemispheres. Tell what it proves, and how it proves it.

13. Describe Guericke's method of proving that air has weight, and contrast it with Galileo's.

14. Explain why animals can withstand the great crushing force of the atmosphere.

15. Draw a sectional diagram of a force pump, and fully explain both how and why the fluid is propelled through it.

16. Discuss the application of the principle of Archimedes to a boat, and to a balloon.

17. Explain how to calculate the load that a given boat or balloon can support.

PROBLEMS

1. On a mountain the barometer reads 45 cm; what is the pressure of the atmosphere there (*a*) in $\frac{\text{gm}}{\text{cm}^2}$? (*b*) in $\frac{\text{dynes}}{\text{cm}^2}$?

2. When the atmospheric pressure supports a column of mercury 75 cm high, how high a column of water will it support? How high a column of alcohol? Take the densities, as: mercury, 13.6; water, 1; alcohol, 0. 8.

3. A plunger whose cross-sectional area is 4 cm^2, is pushed into a cylinder full of oil with a force of 5×10^6 dynes; what pressure in $\frac{\text{dynes}}{\text{cm}^2}$ must be sustained by the walls of the cylinder? If the end of the cylinder has an area of 300 cm^2, what is the total force exerted on it?

4. The pump plunger A, of a hydraulic press, Fig. 71, has an area of 5 cm^2 and the ram B an area of 1000 cm^2; what is the mechanical advantage? How many kilograms-force must be applied to the plunger in order that the pressure head of B shall exert a total force of 9×10^4 Kg-force?

5. When a man presses down on the lever of the press of problem 4 at a distance of 60 cm from the fulcrum, what is the mechanical advantage of the lever if the plunger is 10 cm from the fulcrum? What force must the man exert in order that the force on the plunger may be 450 Kg-force? What is the total mechanical advantage of the press when the lever is used?

6. When the standpipe in Fig. 74 contains water to the height of 30 m, what pressure in $\frac{\text{gm-force}}{\text{cm}^2}$ does the water exert at the bottom? If one of the steel plates on the side there is 3 m long and 1.8 m high, what total force must it withstand?

7. Find the pressures in $\frac{\text{gm-force}}{\text{cm}^2}$ on the sides of the standpipe at depths of 1 m, 2 m, 5 m, 10 m, 20 m, 25 m, and, using any convenient scale, plot a graph showing the relations of pressure to depth of the water. Does this graph suggest to you anything about the relative thicknesses that the steel plates must have at these various depths in order not to burst?

8. Fig. 79 represents an experiment devised by Pascal to verify the conclusions stated in Art. 104. The bottom is held on to each of the three vessels in turn by the pull of the cord only. If the distance from the pointer to the bottom is 10 cm, and the area of the bottom is 50 cm^2, how many gm-force on the other balance pan are required to hold the bottom on when the cylindrical vessel is filled up to the pointer? How many for the wide-topped vessel? How many for the narrow-topped vessel? Would the result be the same if a vessel having any other shape were used, provided the other conditions remained the same? Explain how this apparatus may be used to verify statements 1, 3, and 4 of Art. 104.

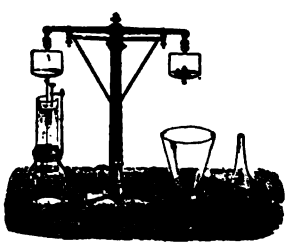


FIG. 79

9. Taking 1 $\frac{\text{Kg-force}}{\text{cm}^2}$ as the pressure of the atmosphere and 36 cm as the diameter of the Guericke hemispheres, Plate V, calculate the force with which they were held together, assuming that he got a perfect vacuum inside them. N. B.—The surface to be used is the area of greatest cross-section, not the surface of the sphere. (Why?) The latter would give the crushing force; calculate it.

10. A balloon contains 300 m^3 of illuminating gas, which weighs 0.75 $\frac{Kg}{m^3}$. One m^3 of air weighs 1.3 Kg; what weight, including its own, will the balloon support?

11. A canoe weighs 75 lb., 1 ft^3 of water weighs 62.4 lb. How many ft^3 of water must the boat displace when it is carrying two persons, weighing together 240 lb.?

12. The weight of a steamer is 6000 tons, and its gross displacement is 10,000 tons; what load can it carry?

13. Fig. 80 represents a syphon. Suppose the atmospheric pressure is 10^3 $\frac{\text{gm-force}}{\text{cm}^2}$ and that $EA = 10$ cm, $DB = 20$ cm. With how many $\frac{\text{gm-force}}{\text{cm}^2}$ does the water press down at A? At B? What is the resultant pressure in the direction ACB? In the direction BCA? What will the water do? Would the syphon work on a mountain top?. In a vacuum? Over how great a height can water be raised by it when the barometer stands at 75 cm?

14. Fig. 81, the cylinder C is hollow and has a capacity of 100 cm^3. P exactly fits it. P and C are balanced, as shown, but without any water in the vessel. The water is then placed in the vessel under P; will it remain submerged? If water is poured into C until equilibrium is restored, how many gm will be required? How many cm^3?

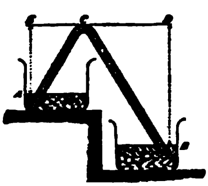


FIG. 80

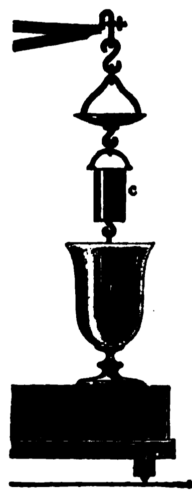


FIG. 81

15. Given: the mass of a piece of glass = 50 gm, the weight of the glass in water = 30 gm, the weight of the glass in gasoline = 36.26 gm. Required: the volume of the glass, its density, the mass of gasoline that has the same volume as the glass, and the density of the gasoline.

16. A piece of wood having a mass of 37.5 gm is attached to a piece of lead whose mass is 166.5 gm. The weight of both, when submerged in water, is 139 gm. The lead alone weighs in water 151.5 gm. Find: (a) the volume of water displaced by both together; (b) the volume of the lead; (c) the volume of the wood; (d) the density of the wood; (e) the density of the lead.

17. If you hold your finger at the outlet valve of a bicycle pump when the piston is at the top of the cylinder, and if the piston is then pushed half-way down, what is the pressure on your finger in $\frac{\text{Kg-force}}{\text{cm}^2}$ if the pressure of the atmosphere is 1 $\frac{Kg}{cm^2}$? What are the pressures when

the piston is $\frac{3}{4}$, $\frac{9}{10}$, $\frac{1}{3}$ of the way down? If you pump up your tire until the pressure in it is doubled, by how much is the density of the air in it changed?

18. Appended are some of the data obtained by Boyle in his experiment (Art. 114): Choosing a convenient scale, plot the lengths of the air columns as abscissas, and the corresponding pressures as ordinates. The graph will then represent the relation $PV = \text{const.}$ What does the graph show about the pressure when the volume becomes very large? What about the volume when the pressure becomes very large?

Lengths	Pressures
12	29.1
10	35.3
9	39.3
8	44.2
7	50 3
6	58.8
5	70.7
4	87.9
3.2	107.8

SUGGESTIONS TO STUDENTS

1. By means of a rubber tube connect a bubble pipe with the gas fixture, let the gas blow soap bubbles for you, and see what they will do. Can you explain their behavior?

2. Visit the water works and find out all you can about the pumping engines and the pumps. How is the pressure regulated by means of air chambers connected with the inlet and outlet pipes? Find out whether there is a standpipe connected with the works, and if so, what its use is.

3. Can you explain how your bicycle pump works? Find a compressed air tool at work (a riveter on a steel framed building or bridge, or a cutting tool at a marble works), and learn what you can about how they work.

4. When a sail-boat is tipped from the position in which the deck is horizontal, what moment of force does most of the tipping? What moment tends to restore it? Investigate this interesting and important problem of the stability of a boat and find out what you can about it.

5. Consult a book on physiology in which the action of the heart is described. Can you understand wherein it resembles a force pump? In what essential detail does its action differ from that of an ordinary pump?

6. If you are interested in the air ship shown in Fig. 77, read Santos Dumont's book on *My Air Ships* (N. Y. Century Co., 1904).

CHAPTER VII

HEAT

116. Heat and Work. In the preceding chapters we have studied the operation of the locomotive and of other machines and learned of motions and mechanical efficiencies. How is it with steam engines and gas engines? Every one is familiar with the fact that every engine consumes fuel in some form, and that without the fuel it will not move at all. Hence, all engines must in some way derive the energy with which they do work from the fuel that they burn, i.e., an engine is simply a device for converting heat energy into mechanical work.

The questions that arise in connection with the conversion of heat into mechanical work are many and interesting. Thus, how can we measure heat? How do bodies change when they are heated or cooled? What of the process of converting water into steam? Is heat absorbed in this operation? Does steam act as a gas and obey Boyle's law, or does it act differently? How is the heat transferred from the fire to the water in the boiler and from the water into steam? Is there any definite relation between heat and mechanical work? Is the efficiency of a steam engine high or low, and how is it determined? On what factors does the efficiency of such an engine chiefly depend?

117. Heat Sensations Unreliable. Perhaps the most familiar fact about heat is that some things feel hot to our touch while others feel cold. Yet our ability to judge how hot a body is depends on a number of varying circumstances, and at best is limited. For example, if we take three basins of water, one hot, one lukewarm and one cold, and place the right hand in the hot water and the left in the cold, and then transfer both hands to the lukewarm water, this latter will seem cold to the right hand and hot to the left. Hence, we can not rely on our sense of touch for accurate informa-

tion concerning differences of temperature. What, then, may we use?

118. Galileo's Thermometer. The first to give any scientific answer to this question was Galileo. He blew a bulb on the end of a glass tube of small bore and after slightly warming the bulb placed the end of the tube in a vessel of colored water (Fig. 82). When he warmed the bulb with his hand the liquid in the tube moved downward, showing that the air in the bulb expanded. Also conversely, when he cooled the bulb the liquid in the tube moved upward, showing that the air in the bulb contracted. He thus showed that air expands when it is warmed and contracts when it is cooled; and he suggested the use of this property of air for detecting small differences of temperature.

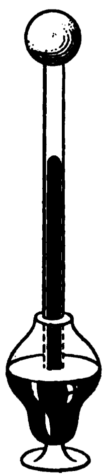


FIG. 82
GALILEO'S THERMOMETER

It is probably not necessary to state that Galileo's suggestion has been universally adopted for scientific work, although the instrument which he devised is practically useless as a thermometer, because the liquid in it is exposed to the pressure of the atmosphere. Since, as we learned in the last chapter, this pressure is always changing, the small column of liquid moves when the atmospheric pressure changes as well as when the temperature changes. Therefore, we can not be sure that a given position of the liquid indicates the same temperature at different times. Fortunately, allowance can be made for the error thus introduced, provided the barometer is observed at the same time with the thermometer, so that the pressure on the air in the bulb is known. But even so, how may we determine the amount of the change in temperature corresponding to a given motion of the liquid in the tube?

119. The Temperature Scale. We can not answer this question until we have adopted a scale of temperature, and defined the units in terms of which we shall measure differences of temperature. In order to establish such a scale it is necessary to have some fixed

temperature which may be used as the zero from which to count, and also to have some unit difference of temperature. It has been found convenient to adopt the temperature at which ice melts as the ZERO TEMPERATURE; hence, in scientific work this is called a temperature of zero degrees.

In order to determine a unit difference of temperature, we must select some other fixed temperature, and then define the interval between the zero and this other temperature as a certain number of degrees. The second fixed temperature that has been adopted by scientists is that of water boiling at normal barometer pressure, i.e., 76 cm. The interval between the temperature of melting ice and that of boiling water has been divided into a hundred equal temperature intervals called degrees. Another temperature scale, called Fahrenheit's, is in common use, but the one just defined is generally used in scientific work. Since, in defining this scale, the fundamental temperature interval is divided into one hundred equal parts called degrees, it is called the CENTIGRADE SCALE. Temperature degrees in this scale are denoted by the symbol °C. For example, 40°C. means forty degrees of the Centigrade scale. As the temperature does not involve either gm, cm, or sec, it has no symbol in the terms of these units.

Having defined our temperature units, we are now in a position to put a scale on our thermometer. This is done by placing the instrument in melting ice, marking the position of the drop of liquid in the tube, then placing the instrument in the steam over boiling water, and marking the position of the drop of liquid when it has become stationary. The interval on the tube between these two marks is now to be divided into one hundred parts representing equal temperature intervals.

120. Change of Volume at Constant Pressure. Let us now ask what the relation is between the volume of the air in the bulb and an increase in temperature of 1°. This relation has been determined experimentally with great accuracy, and it has been found that when a given mass of gas is heated from 0° to 1° its volume increases $\frac{1}{273}$, when heated from 0° to 2° its volume increases $\frac{2}{273}$, of its volume at 0°, etc., i.e., *for every change of* 1° *in temperature,*

the corresponding change in volume is $\frac{1}{273}$ *of the volume at* 0°. This ratio has been found to be the same for all gases and for all changes of 1° in temperature. It is called the COEFFICIENT OF EXPANSION OF GASES. Since the measurements by which these facts were first established were made by Charles and Gay Lussac, this relation is known as the LAW OF CHARLES AND GAY LUSSAC.

If we let V represent the volume of the gas at any temperature t, and V_0 its volume at 0° C., then, since the final volume (V) is equal to the volume at 0° (V_0), plus the increase in volume $\left(\frac{t}{273} V_0\right)$, this law is expressed analytically as follows:

$$V = V_0\left(1 + \frac{t}{273}\right).$$

On factoring out $\frac{1}{273}$, this equation becomes $V = \frac{V_0}{273}(273 + t)$.

Since $\frac{V_0}{273}$ is a constant for any mass of gas, we see that at constant pressure the volume of a given mass of gas is proportional to $(273 + t)$.

It is to be noted that this equation accounts for changes of volume due to changes of temperature only; and hence, *in stating this equation it is assumed that the pressure of the gas remains constant.*

121. Change of Pressure at Constant Volume. We have just found how the volume changes when a gas is heated at constant pressure; let us now try to find out how the pressure varies when the gas is heated while its volume is kept constant.

In order to do this we must add to the thermometer of Galileo a device for governing and measuring the pressure in the bulb This device usually consists of a rubber tube K, Fig. 83, which is fastened at one end R to the glass tube of the thermometer and at the other R' to another similar piece of glass tubing. This rubber tube is then filled with mercury until the mercury appears above both its ends. The instrument as thus arranged is called an AIR THERMOMETER. It will be readily seen that by raising or

lowering the free end R' of the rubber tube we can increase or diminish the pressure of the air in the glass bulb.

If the glass bulb is now heated, the air within it will expand and depress the mercury at the end R of the rubber tube. To compress the air to its original volume, the other end of that tube must be elevated until the mercury in the thermometer returns to its former level d. Thus, the increase in pressure produced by the rise in temperature is balanced by the pressure due to the weight of the mercury in column h; and the total pressure may be found by adding that of the column h to that of the atmosphere as read from the barometer (*cf.* Art. 115). What is the relation between the increase in temperature and that in pressure when the volume of gas is kept constant?

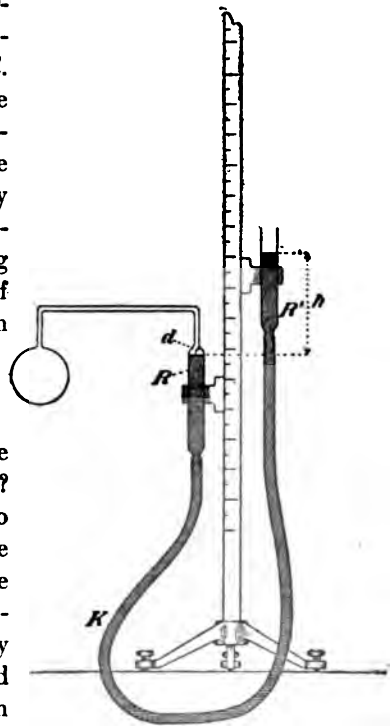


FIG. 83. AIR THERMOMETER

We can find the answer to this question by measuring the changes in temperature and the corresponding changes in pressure. This has been carefully done, and the relation is found to be similar to that between change of volume and change of temperature at constant pressure. It is stated as follows: *when a given mass of gas is heated at constant volume, the pressure increases* $\frac{1}{273}$ *of the pressure at* 0° *for every change of* 1° *in temperature.* This is true also for all gases and for all their changes of temperature.

If we let P represent the pressure of a given mass of gas at any temperature t, and P_0 its pressure at 0° C., then, since the final

pressure (P) is equal to the pressure at 0° (P_0), plus the increase in pressure $\left(\frac{t}{273} P_0\right)$, the result is expressed analytically as follows:

$$P = P_0\left(1 + \frac{t}{273}\right).$$

Or, factoring out the $\frac{1}{273}$, $P = \frac{P_0}{273}(273 + t)$.

We can therefore find the value of t with the air thermometer by substituting the observed values of P and P_0 in this equation, and solving it for t. Thus $t = \frac{273P}{P_0} - 273$.

Since $\frac{P_0}{273}$ is a constant for a given mass of gas, we see that at constant volume the pressure of a given mass of gas is proportional to $(273 + t)$. It will aid the memory to note that the equations of this article, and those of Art. 120 are similar in form.

The air thermometer is not a simple instrument to handle, therefore temperatures are generally measured by the ordinary mercury thermometer, but it must not be forgotten that *the air thermometer is the standard to which the mercury thermometers are all referred.*

122. When Pressure, Volume, and Temperature Change. Now, it has been shown in Art. 120 that, at constant pressure, $273 + t$ is proportional to the volume V. It now appears that at constant volume this quantity is also proportional to the pressure P. Therefore it follows that $273 + t$ *is proportional to the product of V and P;* i.e., $PV = \text{Constant} \times (273 + t)$. The numerical value of this constant depends on the density of the gas and the units in which the quantities are expressed. For solving most gas problems we may express this relation in a more convenient form. Thus if V is the volume of a mass of any gas at $t°$ C. and pressure P, and V' its volume at temperature $t'°$ and pressure P', then

$$\frac{VP}{V'P'} = \frac{273 + t}{273 + t'}. \qquad (9)$$

This equation expresses the relations of pressure, volume, and temperature for gases. It means that the product of the volume and

pressure of a given mass of gas is directly proportional to 273 + *its temperature.*

123. Absolute Temperature. Since the quantity PV is not proportional to the temperature as measured on the Centigrade scale, but is proportional to 273 plus that temperature, it is convenient for work of this kind to conceive that the zero temperature is placed at $-273°$. This new zero of temperature is called the ABSOLUTE ZERO, and temperatures measured from it are called absolute temperatures.

Therefore $273 + t$ *represents the absolute temperature.*

124. Expansion of Solids and Liquids. Thus far we have studied the changes in gases caused by changes in their temperatures. Do solids and liquids expand when they are heated, and contract when they are cooled? Everybody knows that they do. Just as in the case of gases, the volume of every solid and liquid is changed by a certain fraction of itself for every degree that the temperature changes. This fraction is called the coefficient of cubical expansion. Unlike gases, however, each liquid and solid has its own coefficient of expansion which is characteristic of it.

If V represent the volume of any solid or liquid at a temperature t, V_0 its volume at 0° C., and c its COEFFICIENT OF EXPANSION, then $V = V_0 (1 + ct)$ (*cf.* Art. 120).

Similarly for solids when the change in length only is important, the fractional change in length for 1° C. is called the COEFFICIENT OF LINEAR EXPANSION. If a represent this fraction, L the length of the solid at any temperature t, and L_0 the length at 0° C., then $L = L_0 (1 + at)$.

These coefficients are needed for the solution of many important problems which arise in everyday life because of the expansion and contraction due to changes in temperature. For example, when railroads are constructed in winter, spaces must be left between the ends of the rails to allow for the expansion in summer. Long span bridges must have their ends placed on rollers; and provision must be made in a hot water heating system for the expansion of the water and of the pipes.

125. Measurement of Heat. We have defined units in which we may measure temperature, but does the determination of the temperatures give us any information about the amount of heat? May not a small body and a large body have the same temperature but contain very different amounts of heat? Hence, in order to answer the question as to the quantity of heat absorbed by an engine or by any other device or body, we must make a further definition as to the units in which this heat is to be measured. *The unit universally adopted by physicists for quantities of heat is the quantity of heat absorbed by one gram of water when heated from* 15° *to* 16° *C.* This unit is called the GRAM CALORIE. Thus, if one liter (1000 cm) of water is heated until its temperature has risen through 100°, the quantity of heat thus imparted to it is $1000 \times 100 = 100{,}000$ gram calories.

126. Specific Heat. But does it require equal amounts of heat to increase the temperature of all substances by 1°? Evidently not, for it is well known that it requires more heat to raise the temperature of a gram of water through one degree than to raise the same mass of any other substance through the same temperature interval. In order to compare the heat capacities of different substances, it is, therefore, convenient to express their ability to absorb heat in terms of the heat-absorbing power of water. We may call this heat-absorbing power SPECIFIC HEAT, and define it as the ratio

$$\frac{\text{heat absorbed by 1 gm of the given substance in warming } 1^\circ}{\text{heat absorbed by 1 gm of water in warming } 1^\circ},$$

i.e., *the specific heat of any substance is the number of calories required to warm* 1 *gram of it through* 1° *C.*

The numerical value of the specific heat of any substance may be determined by experiment in a number of different ways. One of the simplest of these is illustrated by the following example: 100 gm of aluminum clippings at 98° C. are stirred into 200 gm of water at 2° C.; and the mixture comes to the temperature of 11.5°. If h represent the specific heat of aluminum, the heat given up by it in cooling to 11.5° is

$$h \times 100 \text{ gm} \times (98^\circ - 11.5^\circ).$$
$$\text{Sp. Ht.} \times \text{mass} \quad \times \text{change of temperature.}$$

The heat absorbed by the water in warming to 11.5° is

$$1 \times 200 \text{ gm} \times (11.5^\circ - 2^\circ).$$
$$\text{Sp. Ht.} \times \text{mass} \quad \times \text{change of temperature.}$$

The heat absorbed by the water must be equal to that given out by the aluminum; so that we may form the equation,

$$h \times 100 \times (98 - 11.5) = 1 \times 200 \times (11.5 - 2), \text{ or } h = 0.225.$$

In performing the experiment care must be taken to let as little heat as possible enter or escape, since it is assumed by the equation that none does so; and proper allowance must, in general, be made for the heat absorbed or emitted by the vessel that holds the water. Vessels used for this purpose are called CALORIMETERS, and are usually made of metal, and surrounded by a box designed so as to prevent heat from entering or escaping.

127. Steam. One other important phenomenon connected with an engine must be understood before we can determine its efficiency. This is the phenomenon of making steam. Heat is absorbed in this process; for a boiling kettle apparently ceases to emit steam shortly after being removed from the fire; and the greater the surface exposed to the air, the faster the water cools. But does water at any temperature ever cease emitting steam? If not, why do we have to heat it so hot in order to make it produce steam for use in the engine? Does water boil always at the same temperature, and what is the nature of the phenomenon we call boiling? In order to find answers to these questions, consider first a glass fruit jar of water exposed to the air. If this jar is placed in a warm room, what will happen to the water? Suppose that the cover is sealed on to the jar, what will then happen to the water? Will it evaporate at all? If so, how much? If not, why not? Suppose that we place the open jar under the receiver of an air pump and exhaust the air, will the water then evaporate; and, if so, to what extent? Will it evaporate in the same room faster if it is hot than if it is cold?

128. Evaporation. It is well known that water evaporates when left in open dishes, and that in the same room the evaporation

is faster the hotter the water; hence, we will all agree that water passes into aqueous vapor or steam at all ordinary temperatures. We may also grant that the reason why water evaporates from an open dish but does not disappear from a closed jar, is that the room is so much larger than the jar, and hence is capable of holding very much more water vapor than the space in the jar can hold. But how much water vapor will a given volume hold, and is this amount the same at all temperatures?

Elaborate experiments were necessary to determine these points, and the results show that water will always evaporate until its vapor exerts a certain pressure on the walls of the vessel containing it, and that this pressure can not, at a given temperature, exceed a certain definite value. When the pressure reaches this maximum value the space in the vessel is said to be SATURATED, and hence this maximum pressure is called the pressure of the saturated vapor at the given temperature. This pressure is independent of the other contents of the vessel, and depends only on the temperature of the water and its vapor. The case is similar for other liquids. Hence, we see that *evaporation takes place wherever there is an exposed surface of liquid, and continues until the vapor attains this maximum pressure of saturation.*

Further, as long as any liquid remains in the closed space we can not increase the pressure of this saturated vapor, provided the temperature remains constant. Nor can we alter that pressure by changing the size of the space: for if we increase the space, more vapor is formed; if we decrease it, some vapor is condensed into liquid; and the pressure exerted by the vapor remains constant. Hence, we may say in general *that the pressure exerted at a given temperature by saturated vapor in contact with its liquid is always the same.* Of course, the pressure exerted by the saturated vapors of different liquids at a given temperature are not necessarily the same.

A simple experiment will make these matters clear. In the barometer tubes b, b', b'', Fig. 84, the three mercury columns at first stand at the same height, depending on the amount of the atmospheric pressure at the time. Now, with a medicine dropper,

insert a few drops of water under the end of the tube b', tap the tube gently, if necessary, till the water rises to the top of the mercury column, and observe the result. The water evaporates till the water vapor exerts its pressure of saturation corresponding to the temperature of the experiment. Manifestly, this pressure is measured by the depression ct of the mercury column. Similarly, a few drops of ether, inserted in b'', partially evaporate, and exert the pressure of saturated ether vapor corresponding to this temperature. This pressure is measured by the depression cs of the mercury column in b'', and is seen to be greater than that of water vapor. If now a few drops of ether are introduced into the tube b', the depression of the mercury there is seen to be equal to $ct + cs$: i.e., the pressure of the ether vapor is the same as before, and so also is that of the water vapor. The total pressure, due to both, is simply the sum of the pressures which each would exert separately. So we see that the pressure of each is the same as it would be if the other were not present in the space.

FIG. 84

Now incline the tube b''. The mercury compresses the ether vapor in it. The vapor now occupies a smaller volume, so some of it must have been condensed into liquid alcohol, but the mercury still remains at the level s as before. The pressure cs of the ether vapor is therefore unchanged, although its volume has been much diminished. If the tube be held erect and lifted a little way (but not out of the mercury in the vessel), the mercury is seen to remain at the same level s. This time the volume has been increased, but the pressure remains constant, showing that some more ether must have evaporated to fill the space and exert its pressure of saturation.

By warming or cooling the vapor-filled space in either b' or b'', it may easily be observed that the pressure of saturation is greater when the temperature is higher, and less when the temperature is lower. The relations between temperature and pressure of

saturated vapor, as determined for water and for alcohol, are shown in the curves (Fig. 85). The abscissas represent temperatures, and the ordinates pressures in cm of mercury. The numbers on the vertical scale, when multiplied by 10, represent cm of pressure; those on the horizontal scale, when multiplied by 10, represent degrees C. What pressure does the line *Ap* represent?

129. Boiling Point. From the curve *W*, tell what pressure is exerted by saturated water vapor at a temperature of 20°, of 50°, of 80°, of 100°. Do you note any relation between the pressure corresponding to 100° and the normal barometer pressure, 76 cm? What pressure does alcohol vapor exert at 20° (curve *A*), at 50°, at 78°? Do you note any relation between the pressure corresponding to 78° and the normal barometer pressure? Since at 76 cm pressure, water boils at 100° C. and alcohol at 78° C., may we then define boiling point as the temperature at which the pressure of saturated vapor is equal to the surrounding pressure? Suppose the atmospheric pressure to be 42 cm, as on the top of Mt. Blanc, at what temperature would water boil there? At what temperature would alcohol boil there?

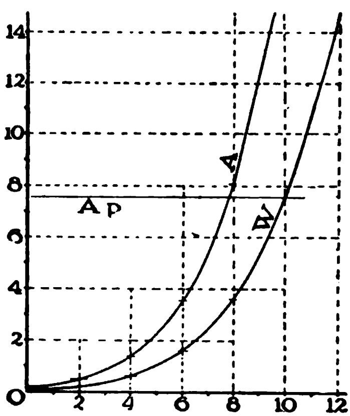


FIG. 85. RELATION OF PRESSURE OF SATURATED VAPOR TO TEMPERATURE FOR WATER AND ALCOHOL

Those who have answered correctly the questions just asked will understand that *the boiling point of any liquid may be defined as the temperature at which the pressure of its saturated vapor is equal to the surrounding pressure.* Thus we see that a liquid may boil at almost any temperature, since a reduction of the external pressure lowers the boiling point and an increase in that pressure raises it. For water, this change in boiling point corresponding to a change of 1 cm in the barometric pressure is 0.37° C. For example, when the barometric pressure falls from 76 cm to 74 cm

a correct thermometer, placed in the steam of boiling water, will read $100° - 2 \times .37 = 99°.26$ C.

That the boiling point is the temperature at which the pressure of the saturated vapor is equal to the surrounding pressure may be readily appreciated from the common sense point of view. For it is plain that ebullition (i.e., boiling) is different from evaporation in that the steam escapes in bubbles from the midst of the liquid instead of from the surface only. Now if the surrounding pressure were greater than the pressure of the steam in these bubbles, the bubbles would be unable to expand and float to the surface. On the other hand, if the external pressure were less than that of the steam composing the bubbles, the water would flash into steam instantaneously, as it sometimes does with explosive violence when a defective boiler gives way.

130. Superheated Vapor. Let us now suppose that we have a very small quantity of water in a closed fruit jar at a given temperature. As has just been stated, the vapor will soon become saturated and will exert the pressure that corresponds to this temperature. If, now, we increase the temperature, more of the water will evaporate, and the pressure of the saturated vapor will increase to that corresponding to this higher temperature. Let us now suppose that at this higher temperature all of the water has been evaporated; what will be the effect of a further increase in temperature? The pressure will increase, of course, but will it increase as fast as it would if more liquid were present so that more vapor would be formed? Clearly not. Therefore, when a saturated vapor not in contact with its liquid is heated in a closed vessel, its pressure at the higher temperature is less than that which it would exert if it remained in contact with its liquid so that it continued to be saturated. A vapor not in contact with its liquid and at a temperature higher than that corresponding to saturation, is said to be SUPERHEATED.

131. A Gas is a Superheated Vapor. Let us now consider how the pressure of a superheated vapor varies with the temperature. So long as the vapor is superheated, none of it will condense,

and there will be no liquid in the space, so that no more vapor can be formed therein and none of the liquid can exist. Therefore the changes in pressure produced by changes of temperature will not be complicated by evaporation and condensation. Hence we may surmise that such a vapor will behave very much like a gas. Experiment has shown that superheated vapors, when they are not too near the saturation point, do act in accordance with the gas laws of Gay-Lussac and of Boyle. We therefore conclude that *a superheated vapor is really a gas*. The converse of this conclusion, i.e., that *a gas is a superheated vapor*, has been verified by the reduction to liquids of the so-called permanent gases, hydrogen, oxygen, and nitrogen.

132. Critical Temperature. We have seen that water can exist as a liquid at all ordinary temperatures. Therefore at all such temperatures superheated water vapor can be condensed to a saturated vapor or to a liquid by the application of pressure alone. On the other hand, the so-called permanent gases do not exist at ordinary temperatures as liquids, and we find that we can not, by any amount of pressure, condense them to liquids without also cooling them. The temperature to which a gas must be cooled before it can be converted into a liquid, is different for different gases, and is called the CRITICAL TEMPERATURE. The critical temperature of water is 365° C.; that of air, −140° C. Other critical temperatures are: alcohol, 243° C.; ether, 194° C.; ammonia, 130° C.; carbon dioxide, 31° C.; oxygen, −119° C.; hydrogen, −242° C.

From what has just been said we learn that *we can not condense a gas or a superheated vapor into a liquid by applying pressure only, if the temperature is above the critical value for that gas.* This important fact is of far-reaching moment in the economy of nature, as a study of the preceding figures will show. We note that for water this critical temperature is high, so that at all ordinary temperatures water exists as a liquid: the same is true of alcohol. On the other hand, the critical temperature of air is very low. Hence, at all ordinary temperatures air is a superheated vapor or gas, and can not be liquefied by pressure alone: the same is true of hy-

drogen and oxygen. Hence, we see why such low temperatures are required for liquefaction of these gases. We can readily understand how fortunate it is for beings organized as we are that these substances are so endowed; for if the critical temperature of water were low, while that of air were high, we would know water at ordinary temperatures only as a gas, and air under like conditions largely as a liquid. The entire economy of nature would thus be overturned; for what could we do with liquid air to breathe and gaseous water to drink?

FIG. 86. WATER CHANGES TO INVISIBLE VAPOR WHICH CONDENSES INTO CLOUDS

133. Humidity. The relations we have just been studying act favorably to the maintenance of life on the earth in other important ways. Thus, the fact that at ordinary temperatures water and its vapor exist together shows us how it is possible for the water of the ocean to evaporate and be carried in the form of vapor over the land, to be deposited there as rain. We see also why there is always considerable water vapor in the air. This humidity of the air is an important factor in climate. Every one knows how oppressive a hot day is if the humidity is high; i.e., if the water vapor in the air exerts a pressure nearly equal to that of saturation at the temperature of the air. Under these circumstances it is plain that very little water can evaporate; and therefore we are not cooled by evaporation from our bodies.

FIG. 87. RAIN CLOUDS DEPOSIT THE WATER ON THE LAND

134. The Formation of Dew. The formation of dew is a familiar phenomenon. Drops of water appear on the outside of a pitcher of ice water on a warm day, because the temperature of the pitcher is below that at which the water vapor in the air would be saturated; i.e., below the DEW POINT. Thus, *by dew point is meant the temperature to which the air must be cooled in order to bring the water vapor in it to saturation, so that condensation begins.* Since the amount of water vapor in the air is of such great importance to climate, its determination is an important part of the work of the Weather Bureau. What is termed RELATIVE HUMIDITY is the ratio of the actual pressure of the water vapor in the air to the pressure of saturated vapor at the same temperature.

135. Latent Heat. Another important fact about the conversion of water into steam or into ice remains to be considered. If we place a thermometer in a vessel of water and heat it gradually, the thermometer indicates a gradually increasing temperature; but *when the water reaches the boiling point, although we continue to heat it, the temperature remains constant until the change of state is completed.* Thus, when water is boiling under any given pressure we can not raise the temperature of the water beyond the boiling point that corresponds to that pressure. But what becomes of the heat that is added after the boiling point is reached? It is used in converting the water into steam; so that energy is required to do this work. Is the quantity of heat thus required large? Experiment shows that it is large, for it is found that 536 *gm cal are required to convert* 1 *gm of water at* 100° *into* 1 *gm of steam at the same temperature.* Since this amount of heat seems to disappear in the process, it is called the LATENT HEAT OF STEAM. Also, conversely, when steam condenses into water, it gives up its latent heat, every gram of steam returning its entire 536 gm cal.

A similar phenomenon accompanies melting and freezing, but the amount of heat required is not so great; thus it requires 80 gm cal of heat to convert one gm of ice at 0° C. to one gm of water at the same temperature. Conversely, when water is frozen it gives up this same quantity of heat per gm. Every substance absorbs a definite amount of heat per gm while melting or evapo-

rating and gives up this energy while solidifying or condensing, the amount thus transformed being different for different substances.

136. Latent Heat is a Form of Energy. From the foregoing discussion it must be quite clear that latent heat is a form of energy, for heat energy is expended in doing the work of converting the liquid into the vapor form, and is given up again as heat when the vapor is condensed into the liquid form.

We can now understand why it is that when a substance is vaporizing or condensing, or when it is liquefying or solidifying under a constant pressure, its temperature remains constant until the transformation is completed. For when heat energy is doing the work of changing the state or internal condition of the substance, it can not at the same time be employed in raising the temperature. Conversely, when a mass of vapor is liquefying, each gm of vapor that condenses gives up its latent heat, so that the temperature of the liquid can not fall so long as any vapor remains to be condensed and supply it with heat. The case is the same when liquids solidify.

137. Water and Climate. We can understand also why water is so important in regulating atmospheric temperatures, because its specific heat, and its latent heat of vaporization and of solidification are so great. When water is warmed, or changed from ice to water, or from water to vapor, it absorbs large quantities of heat, and so prevents the atmosphere's heating as rapidly as otherwise it would. Conversely, when it is cooled, or changed from water to ice or from vapor to water, it gives out large quantities of heat, so that the temperature of the atmosphere does not fall as rapidly as otherwise it would. Since water is evaporated in great quantities from the oceans and since some of it is then carried with the winds over the land to be there condensed, it serves the earth very much as a steam heating system serves our offices and dwellings.

SUMMARY

1. Zero temperature is that of melting ice.

2. Unit temperature interval is the Centigrade degree. This is the $\frac{1}{100}$ part of the interval between the temperatures of melting ice and water boiling at 76 cm barometer pressure.

3. When the pressure remains constant, a given mass of gas expands $\frac{1}{273}$ of its volume at 0° C., for every increase in temperature of 1° C. (Gay-Lussac's Law.)

4. Absolute temperature is equal to 273° + Centigrade temperature.

5. When its volume remains constant, the pressure exerted by a gas is proportional to its absolute temperature.

6. Unit quantity of heat is the gram calorie, i.e., the quantity of heat involved in changing the temperature of 1 gm of water 1° C. Its symbol is gm cal.

7. Specific heat of a substance equals the number of calories absorbed by one gram of it in warming 1° C.

8. The total number of calories absorbed or given off by any body during any change of temperature = specific heat × mass × change of temperature.

9. Every saturated vapor exerts a pressure that depends on its temperature only.

10. A gas is a superheated vapor.

11. Gases can not be condensed into liquids by pressure, however great, at temperatures above the critical temperature.

12. Water vapor is an important constituent of the earth's atmosphere.

13. The dew point is the temperature at which the water vapor in the atmosphere becomes saturated.

14. The relative humidity of the atmosphere is the ratio of the pressure of saturated vapor at the dew point, to its pressure at the temperature of the atmosphere.

15. Heat is absorbed during the processes of melting and evaporation, and given out during the converse processes of solidification and condensation. (Latent heat.)

16. 80 gm cal of heat become latent heat when one gm of ice melts; and 536 gm cal, when one gm of water evaporates at 100° C.

17. When a body changes state, the number of calories absorbed or given off by it is equal to the corresponding latent heat, multiplied by the number of grams mass.

18. Latent heat is a form of energy.

QUESTIONS

1. How far can we rely on our sense of touch for information concerning temperature? Give some examples.

2. What elements are necessary in determining a temperature scale, and how is the Centigrade temperature scale defined?

3. Why is Galileo's air thermometer inaccurate? What device is employed to keep either the volume or the pressure of the air constant?

4. What other instrument must be used in connection with an air thermometer in determining temperature? Why?

5. How much does a gas expand when heated 1° C.? What is Gay-Lussac's law?

6. What do we mean by absolute temperature?

7. Is there any relation between the pressure of a gas at constant volume and its absolute temperature? What is the relation?

8. How do we define the unit quantity of heat?

9. How do we compare heat quantities? What is specific heat?

10. Does water vaporize at all temperatures?

11. If we have water vapor in contact with its liquid in a closed vessel, is there any limit to the pressure it can exert at a given temperature?

12. When is a vapor said to be saturated?

13. Does the pressure that a saturated vapor in a closed vessel exerts depend on the volume of the vapor, on the pressure of the air or other substances in the vessel, or on anything but the temperature?

14. Is a vapor in contact with its liquid in a closed vessel always saturated?

15. When is a vapor superheated?

16. Compare the properties of a superheated vapor with those of a gas.

17. When can we condense a superheated vapor to saturation by compression alone? When is cooling also necessary?

18. What do we mean by critical temperature? Is it the same for all substances? Why is it fortunate for animal and vegetable life that the critical temperature of water is high while that of air is low?

19. In what way does the large heat-absorbing power of water act favorably on the climate of places near large bodies of water?

20. Under what conditions does dew settle on an object?

21. What is meant by relative humidity of the atmosphere?

22. If only a small quantity of heat were to become latent heat when water passes into water vapor, what sort of climate would the earth have?

PROBLEMS

1. The freezing temperature of water is marked 32° on the Fahrenheit scale, the boiling temperature 212°, and each degree on this scale corresponds to a temperature interval of $\frac{5}{9}$ of a degree on the Centigrade scale. (*a*) The normal temperature of the human body is 98.4° Fahrenheit. How many Fahrenheit degrees is this above the freezing point of water? To what reading on the Centigrade scale does it correspond? To what on the absolute scale? (*b*) When the temperature of a school-room is 70° Fahrenheit, what will a Centigrade thermometer read? (*c*) Show that the following rule is correct: To find the Centigrade reading that corresponds to any Fahrenheit reading, subtract 32 from it, and multiply by $\frac{5}{9}$. Make up a rule for changing Centigrade readings to Fahrenheit. (*d*) Mercury freezes at −38.8° C. What is the freezing point of mercury on the Fahrenheit scale?

2. A student in the chemical laboratory collects 156 cm^3 of oxygen gas at 20° C. and 78 cm barometric pressure. What would its volume be at 0° C. and 76 cm pressure? Its density under the latter conditions is 0.00143 $\frac{gm}{cm^3}$. What is its mass?

3. How many cm^3 of air are there in a schoolroom whose dimensions are $10 \times 15 \times 5$ m? The density of air at 0° C. and 76 cm pressure being 0.00129 $\frac{gm}{cm^3}$, what is its mass? Its specific heat being 0.237, how many calories of heat are required to raise its temperature from 0° to 20° C.?

4. Suppose the air in the room, problem 3, must be completely changed every 15 minutes, how many calories are required each hour? If this heat is to be taken from hot water, which enters the radiators at 85° and leaves them at 76°, how many gm of water must be delivered from the boiler each hour for this room? One gm of coal when burned gives up 7500 gm cal. How many Kg of coal are required per hour if 50 per cent of the heat of the coal gets to the radiators?

5. When 100 gm of lead shot at 99° are mixed with 25 gm of water at 5°, to what temperature will both come, if the specific heat of lead is 0.033?

6. The specific heat of steam is 0.48, of ice, 0.505. Steam at 105° is mixed with 10 gm ice at −10°, and the temperature is found to be 40°. How many calories were required to warm the ice to 0°? To melt it? To warm the resulting water to 40°? How much heat did each gm of the steam give up in cooling to 100°? In condensing to water? How many calories did each gram of this condensed steam give up in cooling to 40°? Let m represent the whole mass of the steam, and write the expression for the whole quantity of heat given up by the steam in changing from steam at 105° to water at 40°. Supposing no heat entered or left the vessel in which they were mixed, how must this

amount have compared with that absorbed by the ice in warming to 0°, melting to water, and coming to 40°? Find the value of m, the mass of steam used.

7. If 100 gm water at 50° are mixed with 200 gm ice at 0°, will all the ice be melted? If not, how much? If so, what will be the resulting temperature? Answer the same questions if the mass of the water was 500 gm, and its temperature 80°.

8. An iron girder bridge is 30 m long when its temperature is —10° C. Taking the mean coefficient of expansion of iron as 0.000012, what is the length of the bridge when its temperature is 37° C.?

9. A bottle contains 2500 cm of alcohol at 20° C.; what will be the volume of this alcohol at 0° C.? The coefficient of cubical expansion of alcohol is 0.0011.

SUGGESTIONS TO STUDENTS

1. Repeat the experiment described in Art. 117. Have you noticed a similar phenomenon when going from a cold room to one of medium temperature and comparing your sensations with those of your schoolmates who have come from a room that is overheated?

2. How is water purified by distillation? Petroleum consists of a number of different components, each having its own boiling point; suggest a method for separating these components.

3. Examine the pendulum of a regulator clock at a watchmaker's, and see if you can find out how the downward expansion of the pendulum rod is compensated by the upward expansion of another metal.

4. Lay a thin strip of brass on a similar strip of iron, rivet the two together, throw the combination into a fire, and see what it will do. Examine the balance wheel of your watch; does your observation on the brass-iron strip help you to explain how the watch is compensated?

5. Examine the device (thermostat) by which the temperature of an incubator is kept constant. Make a diagram of it and report.

6. How does a wheelwright put an iron tire on a wheel so that it will be tight?

CHAPTER VIII

TRANSFER OF HEAT

138. Conduction and Convection. In the preceding chapter, when heating and cooling were mentioned it was assumed that these terms would be understood. It may be well before we leave the subject to give fixed form to our ideas concerning these processes. When we are heating water in a tea-kettle, we notice that before it can reach the water, the heat must first pass through the copper bottom of the kettle. Further, those portions of the water that are nearest the fire must become heated first. How do they transfer this heat to other portions of the water? Why does covering boiler or a steam pipe with an asbestos coating prevent waste of heat?

In considering these questions, the first point to note is that when we wish to impart heat to a substance we bring it near or in contact with something hotter; and conversely, when we wish to cool it we place it near something colder. This almost instinctive practice is based on the universally accepted concept that heat in some way passes from the hotter body to the cooler, and not the reverse. But is this strictly true? Do we never find cases in which heat passes from a cooler to a hotter body? Before answering these questions we must distinguish among several different kinds of heat transference.

One form of heat transference is illustrated in the passage of heat through the bottom of a kettle or along a solid rod when one end is heated in a flame. In this case the particles of the kettle or the rod do not change their relative positions, but merely pass the heat along from particle to particle. This form of heat transfer is called CONDUCTION, and it is the process by which heat moves from one part of a solid to another. In conduction, heat always flows from portions at higher temperatures to others at lower temperatures.

In the case of liquids and gases, however, the process of heat transfer is somewhat different. All fluids are very poor conductors of heat, but when a small portion of the fluid becomes warmed by conduction from a heated body, it expands, and hence becomes less dense than the surrounding portions of the fluid. It is therefore pushed upward by those heavier surrounding portions, which creep in below; and as it goes it carries its heat with it. Thus currents are set up in the fluid, the cooler portions settling downward and pushing the warmer portions upward.

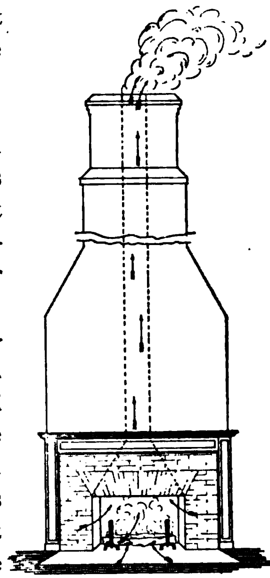


FIG. 88. THE HEAVY FLUID DISPLACES THE LIGHTER

The name of this process is CONVECTION. It is the process by which heat usually spreads through liquids and gases, and it continues as long as there is any difference in temperature between the different parts of the fluid. This process is illustrated in Fig. 88, as it takes place in a chimney. The smoke shows how the hot gases flow away at the top, and the arrows show how the cold air pushes its way in below, forming a so-called "draft" along the floor and up the chimney.

139. Applications of Conduction and Convection. The knowledge of the conducting powers of different substances is very useful in daily life. The asbestos coverings on locomotive boilers and steam pipes are poor conductors of heat, and so prevent its escape. So does wool, whether in its natural state on a sheep's back, or as cloth in our garments. Air conducts very little heat; hence the air spaces between the walls of refrigerators. Down comforters are very warm for their weight, because the air, entangled with the down, keeps the heat from escaping. Water is a very bad conductor of heat; hence it must be heated from below, in order that convection currents may start in it. The circulation of air

in a hot-air heating system, and of hot water in a hot-water system, is in many cases secured entirely by convection. The drafts up a lamp or factory chimney are not essentially different from tropical whirlwinds.

140. Radiation. In both of the processes just described the substance heated must be in contact with the hotter body. Now there must be some other process of heat transfer; for we all know that a hot stove will heat objects in its vicinity, though not in contact with them, to a temperature higher than that of the surrounding air, and that the life-giving energy of the sun somehow succeeds in reaching the earth without any apparent contact between the two. Hence, for describing this process of heat transfer, we are compelled to imagine another mechanism which is called RADIATION.

141. Diffusion. We can most easily gain some conception as to how radiation takes place, if we pause for a moment to consider some other phenomena which will enable us to form an idea as to what heat is. When any gas having an easily recognizable odor, such as illuminating gas, is liberated in a room, in a very short time the odor can be detected in every part of the room. This familiar fact justifies us in concluding that the gas has spread throughout the entire space. How may this have happened? Evidently the gas must consist of numerous particles, and these particles must have moved from the place where they were liberated into all other portions of the room. Hence we are led to think that the particles of this substance must have been in motion before they were liberated, and that to liberate them it was merely necessary to open a stopcock, or take out a cork, so as to provide an opening in the confining walls, through which the gas particles might escape. The simplest idea that we can form of their motion is that each particle is highly elastic and that it continues to move in a straight line until it collides with some other particle or with the sides of the vessel; when it immediately rebounds and starts off again in a new straight path.

142. Evaporation. Let us now consider whether this hypothesis will help us to a better understanding of the phenomena of evaporation, of which we learned in the preceding chapter. We there found that water evaporates at all temperatures, i.e., the water particles fly away from the liquid surface and diffuse themselves into the surrounding air. If we assume that the water particles are in violent motion while within the body of the liquid, we can form a mental picture of how the particles that are at the surface might be more free to fly out into the air than to fly back into the liquid. Further, since a vapor always occupies so much more space than does the liquid from which it has been formed, we must conclude that the particles of water vapor are farther apart than are those of liquid water, and therefore we can understand why they have far greater freedom of motion.

143. Diffusion of Solids. Again, we are familiar with what often happens when we put a solid into a liquid. For example, a lump of sugar, placed in a cup of coffee, disappears after a time, even without stirring; and, if left long enough, it will sweeten all the coffee in the cup. Here again we are led to conclude that the sugar and the liquid consist each of a large number of particles which are already in motion, and that when the solid is put into the liquid, the moving particles of each spread themselves amongst those of the other. When the particles of two or more substances thus spontaneously mix together, they are said to diffuse into one another, and the process is called DIFFUSION.

144. Gaseous Pressure. With the aid of our hypothesis we can now get a very satisfactory conception of the manner in which a body of gas exerts a pressure on the walls of a vessel in which it is confined. For it is easy to see that if the millions of tiny gaseous particles are flying with great velocities in all directions, the sides of the vessel will be struck at every instant by a multitude of these particles, and that each of these when it strikes and rebounds will give the wall a push or impulse. The magnitude of this impulse will depend on the mass and velocity of the particle (*cf.* Art. 39). The sides of the vessel will thus be bombarded

at every instant by a multitude of the little particles, which come so thick and so fast that the sum of their impulses can not be distinguished from a continuous pressure.

145. Effect of Heating. We have learned, (Art. 121) that when a gas is heated in a closed space at constant volume, the pressure that it exerts is increased; but we have just seen that the pressure depends both on the masses and on the velocities of the gaseous particles. Hence, since heating the gas does not change the mass of the flying particles, it must increase their velocities. Also, since at a given instant each particle has a certain mass m, and a certain velocity v, it has a certain amount of kinetic energy $\frac{mv^2}{2}$; and since, as we have just found, heating increases the velocity v, we are forced to conclude that what the heat energy does when it raises the temperature is simply to increase the kinetic energy of the little particles. Thus we are led to infer that *heat is nothing more nor less than the kinetic energy of these moving particles.*

146. The Kinetic Hypothesis. The hypothesis at which we have now arrived includes the following ideas:

1. *Every substance consists of a great number of very small particles, each of a definite mass. These particles are called* MOLECULES.

2. *These molecules are constantly in rapid motion.*

3. *Heat is the kinetic energy of these moving molecules.*

4. *The temperature of a body depends on the average kinetic energy of the individual molecules of the mass, while the total quantity of heat possessed by it depends on the sum of the kinetic energies of all its molecules.*

This kinetic hypothesis has helped us to a better understanding of diffusion, evaporation, and gaseous pressure; let us now return to radiation and see if it will assist us there.

147. Radiation. We have learned that radiation consists in the transfer of energy from one body to another when the two are not in contact, as, for instance, from a stove to your hand. Now,

if the particles of the stove are in rapid motion, and if heating the hand consists in making its particles move more rapidly, by what possible mechanism may the rapidly moving particles of hot iron communicate some of their motion to the particles of your hand across the intervening space? Does anything like this happen when a stone is thrown into a pool? Does not the

FIG. 89. ENERGY MAY BE TRANSMITTED BY WAVES

motion of the stone produce waves, as shown in the photograph (Fig. 89), which move in gradually widening circles until they reach the borders of the pool? What happens to the pebbles there when the waves reach them, and what becomes of the waves themselves? Do they not set the pebbles in motion, thus passing along to the pebbles the energy that they received from the stone? May we not, then, imagine that just as water waves spread out in rings from the spot where a stone falls, so heating waves spread out in all directions from the vibrating molecules of hot bodies; and just as water waves break and give up their energy to pebbles on the shore of the pond, so the heating waves strike and give up their energy to particles on the boundaries of their realm?

148. The Ether. In the case of the pebbles that are set in motion when a stone is thrown into a pool, it is evident that water is the medium through which the energy travels, and that without some such medium, no energy can be thus transferred. Now if heat is transferred by waves, what is the medium in which

these waves are propagated? That it is not air, nor ordinary matter of any kind, must be manifest to any one who will but hold his hand near an electric glow lamp; for the air has been pumped out of the bulb, yet the filament radiates both heat and light through this vacuum. So also we are warmed by the energy that comes to us from the sun, although there are good reasons for believing that the greater part of the space between does not contain any sensible amount of ordinary matter. Hence the adoption of the wave hypothesis for radiation makes it necessary to assume that there exists in space a medium which is not ordinary matter, but which is capable of transmitting such waves. This medium is called the ETHER. When we come to study electricity and light, we shall meet with it again, and are likely to become more deeply impressed with the utility of the ether-wave hypothesis.

149. Prevost's Theory of Exchanges. Suppose that the fire goes out so that the temperature of the stove falls to that of the hand; then you can no longer warm it at the stove. Has the stove then ceased to send out heat waves? If now we bring a piece of ice near the stove, will not some of it melt? Yet when the stove has further cooled to the temperature of the ice, this latter will no longer be melted by the heat from the stove. Has the stove then ceased to send out heat waves? Is it not simpler to suppose that all bodies are radiating heat waves at all temperatures, and that whether a body grows hotter or colder depends on whether, in a given time, it absorbs more than it radiates, or radiates more than it absorbs?

We may state this theory as follows: *All bodies are radiating and absorbing heat energy at all times. If, in a given time, the amount of energy that a body absorbs is greater than that which it radiates, the temperature of the body rises; while if the amount of energy that it absorbs is less than that which it radiates, its temperature falls.* This theory was first propounded by Prevost, and hence it is known as Prevost's Theory of Exchanges.

150. Absorption. Radiant heat, though not absorbed by the ether, is always absorbed to a greater or less degree by ordi-

nary matter. Elaborate experiments have been made to determine how much of the energy of radiant heat is absorbed by various substances; and it has been found that of all the gaseous substances, water vapor has the largest absorbing power. Thus the experiments of John Tyndall showed that a layer of air saturated with water vapor at ordinary temperatures, and four feet thick, absorbs 20% of the radiant heat energy that falls upon it. Of course solids and liquids absorb more of the radiant heat than this.

151. Absorbing Power of Water Vapor. The fact that water vapor is a powerful absorber of radiant heat furnishes us with another admirable example of nature's adaptation of means to an end; for what would be the condition of the earth's surface

FIG. 90. GLACIER AND SNOW-FIELD ON A HIGH MOUNTAIN

if the water vapor did not absorb a large portion of the sun's energy? We should be exposed to a blistering heat in the daytime, and a freezing temperature at night. That this is actually the case on high mountains, where the earth is not so thickly blanketed with water vapor, is well known to every one. This is largely due to the fact that water vapor is transparent to light, i.e., it absorbs very little of the sun's energy that reaches it in that form; while on the other hand it absorbs a very large proportion of the radiant heat waves. When the light energy from the sun has passed through the vapor-laden atmosphere, it is absorbed by the bodies on the surface of the earth, and is there converted into heat. At night, when these bodies are sending out this energy as radiant heat, and are radiating more energy than they are receiving, the water vapor in the atmosphere absorbs

this radiant heat, and prevents it from escaping into space. Thus, acting like a trap to catch and hold the sunbeams, the water vapor accumulates heat energy in the day-time and throughout the summer, and holds it over for the nights and the winter.

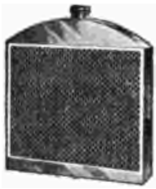


FIG. 91. AUTOMOBILE COOLER

The glass of a hothouse acts in a similar way: it lets the light in; but when the light has been absorbed and converted into heat, the glass will not let it escape by radiation. So it remains and keeps the plants warm. For this reason vegetables can be grown under glass very early in the spring-time without the aid of artificial heat.

152. Radiation and Absorption. A close relation is found to exist between radiation and absorption. Substances that send out large amounts of radiant heat and light when red hot, are found also to absorb large quantities when cold. Thus, a substance like lampblack is dark when you look at it because it absorbs nearly all the light that falls on it; it is also found to be a powerful absorber of radiant heat. We all know well that this substance is a splendid radiator, for carbon is used in the manufacture of electric lights, and is present in large quantities in oil and gas flames. Furthermore, coal and wood-charcoal, which are also forms of carbon, when burning become powerful radiators of both heat and light.

FIG. 92. DETAIL FIG. 91 SHOWING AIR CELLS AND WATER-PIPES WITH LARGE RADIATING SURFACE

Since radiation and absorption take place only at the surfaces of bodies, it follows other things being equal, that the greater the surface the greater is the radiation or absorption. Hence radiators for heating houses and

for cooling automobile engines are better if black and rough than if bright and polished, because then they radiate more outward and reflect less inward for a given surface, and because when they are rough they have a larger surface.

153. Heat and Light. When any substance is heated, as in a blacksmith's forge, it becomes red-hot, at a temperature of 520°C., i.e., it begins to give out red light; and if it is further heated, it not only gives out more heat, but the light that it emits becomes first yellow and then white. Since bodies at high temperatures give out both radiant heat and light, and since both radiant heat and light are converted into heat when they are absorbed, it appears that they must be closely related phenomena. It is, therefore, reasonable to suppose that if the radiant heat is a wave motion, light is also a wave motion. If this conjecture is correct, how does light differ from radiant heat?


FIG. 93. THE HOT IRON SENDS OUT WAVES OF VARIOUS LENGTHS

We may imagine that, just as on a calm day we have the long, steady roll of the ocean, so from bodies at low temperatures we have long heat waves; and on the other hand, just as, during a storm, we have not only that same long roll of the ocean much intensified, but also a multitude of shorter waves added to it, so from red-hot bodies we have not only the long radiant heat waves intensified, but also shorter waves which give us the sensation of light. We shall have occasion to test this assumption in a number of ways when we come to take up the detailed study of light in the last five chapters of this book.

SUMMARY

1. Heat transference is of three kinds, conduction, convection, and radiation.

2. The facts of diffusion, evaporation, and gaseous pressure lead to the hypothesis that heat is molecular kinetic energy.

3. The facts of radiation suggest the hypothesis that radiant

FIG. 94. THE BIG ROLLER CARRIES SMALLER WAVES ON ITS BACK

heat and light are forms of wave motion in a medium called the ether.

4. All substances are radiating heat waves at all temperatures.

5. A body does not change its temperature when it receives in a given time as much radiant energy as it itself sends out.

6. The amount of radiation depends: 1, on the radiating substance; 2, on the difference in temperature between the radiating body and the surrounding space; 3, on the amount of surface exposed.

QUESTIONS

1. How do we describe the process by which heat travels along an iron rod when one end of it is held in a flame? What name do we give to the process?

2. What can you say of the conductivities of different substances, solid, liquid, and gaseous?

3. What is the name of the process of heat transfer in fluids? Describe the process, and show how it differs from conduction.

4. When bodies are not in contact, how do we conceive that heat travels from one to the other? What is the name of this process?

5. When two bodies not in contact have the same temperature, what must be their action with regard to radiation?

6. Why do we suppose that all bodies are sending out radiant heat waves at all temperatures?

7. Do all bodies at the same temperature have the same radiating power? What ones radiate most?

8. What relation has the radiating power of a substance to its absorbing power?

9. What are some of the conditions that are favorable to rapid radiation?

10. Mention some practical applications of conduction, convection, and radiation.

PROBLEMS

1. Why are wooden handles put on short fire pokers, but not on long ones?

2. At what level should the cool water return pipe and the cold water supply pipe enter the hot water reservoir of a water heating system operated by convection? From what level should the hot water main leave? Should the heating pipe in the stove or furnace be horizontal or inclined? Which should be at the higher end, the cold water supply pipe or the hot water delivery pipe? Give reasons.

3. How are both evaporation and diffusion illustrated by liquid perfumes? Do solid perfumes act in the same way?

4. Do clothes dry while frozen, when they are hung out on the line in freezing weather? Does snow ever disappear from the ground without melting? Do crystals ever form on the sides of a bottle of gum camphor? Do solids never pass from the solid to the vapor state without first passing into liquid?

5. Do you become warmer in summer if you wear a black suit than if you wear a white one of the same material? Why? Why is loosely woven, thin material more comfortable to wear in summer than thick, compact material?

6. What is the use of tarred paper in the walls of houses? In what way do double windows save heat?

7. By what process or processes is heat distributed when a room is heated by hot water radiators? By steam radiators? By indirect radiators? By hot air registers? By stoves? By grates? Can you get hot air to come out of a register into a room that is tightly closed?

8. What advantage arises in the hot water heating system from the high specific heat of water? In the steam system, from the high latent heat of steam?

9. Why does a room become so very hot in summer when the sun shines into it through a closed window?

10. Sketch an arrangement for cooling a house by means of a furnace which is to consume ice instead of coal, and send cold air instead of hot air out of the registers. Sketch a similar scheme for reversing

the action of a hot water heating system and an ordinary stove, specifying in each case the location of the cooler in the house.

11. Why is the word "draft" an inappropriate one to use in connection with convection?

SUGGESTIONS TO STUDENTS

1. With a thermometer, take the temperature of carpet, oil-cloth, metals, and wood in a cold room. Feel each of the substances with the hand. Explain why the sensations do not agree with the indications of the thermometer. Make the same experiment with several substances after warming them in an oven.

2. Stir some scrapings of blotting paper into a glass of water, add a lump of ice, and make a diagram of the convection currents that are shown by the scrapings. Make the same experiment, using a hot clay marble instead of the ice.

3. Examine a refrigerator. Do you find any provision for securing a circulation of air in it by convection? If so give diagram and brief description.

4. Investigate a hot air heating system, and hand in a diagram with a brief explanation of how the air circulates, and how it heats the rooms. Do the same for a hot water system and a steam system Visit the stores where such heaters are on sale, and ask for information illustrated circulars, etc. Investigate particularly the heating system in your own home, and give in a brief paper the results of your experience as to the best methods of managing it, especially the regulation of the cold air supply.

5. Read the chapter on Heat in *Experimental Science*, by Geo. M. Hopkins (Munn & Co., N. Y.). Many interesting and easy experiments are described in it.

6. Tyndall's *Heat as a Mode of Motion* (Appleton, N. Y.) contains the descriptions of many interesting experiments in conduction, convection, radiation, and absorption. Much of the *theory* of this book is not up-to-date, but the *facts*, told in Tyndall's charming and masterly style, are well worth reading. Some of the experiments may easily be repeated.

CHAPTER IX

HEAT AND WORK

154. The Mechanical Equivalent of Heat. Let us now pass finally to the consideration of the relations between heat and mechanical work. Nearly everybody has polished a cent by rubbing it on a carpet, or seen sparks fly from a grindstone when a tool is being ground, and therefore knows that heat can be produced by mechanical work. No definite relation between work and heat was recognized in the early days of science. When Sir Humphry Davy, in 1810, showed that he could melt two pieces of ice simply by rubbing them together, the idea that heat could be produced by mechanical work began to prevail.

Since heat can be converted into work, the question at once arises, how much work can one gram calorie of heat do? The first to solve this problem experimentally was James Prescott Joule (1818–1889), who attained the result in a very interesting and instructive way. His apparatus is pictured in Fig. 95, and consisted of a calorimeter filled with water, in which paddles were made to rotate by the weight of a falling mass. Thus the energy of the falling mass is expended in heating the water by friction. The amount of the work done is measured by the product of the weight of the mass and the distance through which it falls. The number of calories of heat generated is the numerical product of the specific heat of the water, its mass, and the change of temperature (*cf.* Art. 126). More recently Rowland, at Johns Hopkins University, made a most careful and accurate determination of this constant. The method employed by him was not different in principle from Joule's. The result of his experiments has been adopted as the most probable value of the number of ergs that is equal to one gram calorie. It is

$$1 \text{ gm cal} = 4.19 \times 10^7 \text{ ergs.} \qquad (9)$$

This ratio is known as JOULE'S EQUIVALENT, OR THE MECHANICAL EQUIVALENT OF HEAT.

155. A Gas is Heated When it is Compressed. Who has not noticed that a bicycle pump becomes heated when it is being used to pump up tires? Since a great deal more heat is thus developed than can be attributed to friction, we are forced to conclude that the greater part of it is produced by the work done in compressing the air. This is perhaps the most direct and simple case of the transformation of energy into heat.

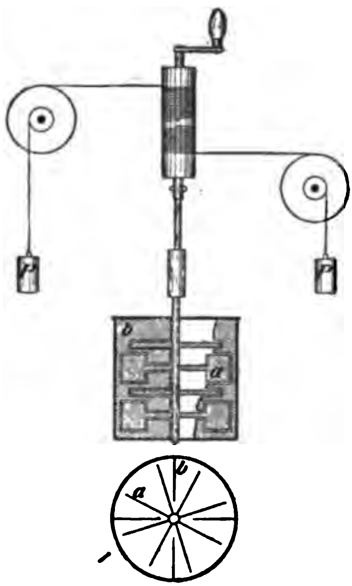


FIG. 95. JOULE'S APPARATUS
a. Revolving Blades.
b. Stationary Blades.

With the help of the kinetic hypothesis, which we adopted in the preceding chapter, we may easily form a mental picture of the way in which the heat is added to the gas when it is compressed. For if the piston of the pump is moving downward, while millions of the little molecules are flying upward against it, each little molecule will rebound with an increased velocity, just as a base ball rebounds from a moving bat. But this increased velocity implies an increase in the kinetic energy of the molecules; and the total kinetic energy that has been thus added to all the molecules—i.e., the added heat—is the equivalent of the work done by the advancing piston in compressing the gas.

156. A Gas Cools When it Expands. That a gas cools when it expands also follows at once from our hypothesis. For just as a base ball strikes against the hands of the catcher and gives up its kinetic energy in putting them into motion, so the little molecules, when they bombard the piston so as to put it into motion, must give up some of their kinetic energy in doing this work. The total kinetic energy thus lost by the molecules—i.e., the lost heat—is the equivalent of the work done by the gas on the piston.

It will make no difference whether the piston is present or not; for if a compressed gas is allowed to expand directly into the air, it must do work in pushing aside the air; and it must lose heat in order to do this work. Doubtless many of you have made an experiment that verifies this prediction; for you may have held your hand near the valve of your bicycle tire when the compressed air was escaping from it, and observed that the jet of air seemed very cold. You may also have noticed that when a bottle of ginger ale or pop is opened, a cloud of condensed vapor appears near its mouth. The gas escaping from the bottle becomes so cold from doing the work of pushing away the air, that it cools this air below the dew point.

157. Liquid Air. This principle is the one that is used in the liquefaction of gases, notably of air. The air is first cooled and compressed as far as possible, and is then allowed to escape through a small opening into the room. In this escape the atmosphere must be pushed back by the expanding air; and heat must, therefore, be supplied to do this work. The only heat immediately available is that still remaining in the compressed and cooled air. So much heat is taken from it that part of it is condensed into a liquid.

158. Cooling by Evaporation. In Chapter VII we learned that a large amount of heat must be supplied in order to change a liquid into a vapor. We are now in a position to appreciate that this heat is employed in doing the work of changing the liquid into the vapor. We can also understand that if this heat is not supplied from some external source, it may be obtained from the liquid itself, in which case the temperature of the liquid falls. When we recall the fact that the amount of heat required to vaporize liquids is usually very large—in the case of water for example, more than 500 gm calories per gm—we can see why this principle may be advantageously employed in cooling processes. By the rapid evaporation of liquid air, temperatures lower than $-182°$ C. may be obtained.

159. Manufactured Ice. Ice is now made in all large cities, and in the process the principle just discussed is applied. Ammonia or carbon dioxide gas is condensed into a liquid by means of a powerful force pump driven by a steam engine or an electric motor. The liquefied gas is then allowed to escape through a valve into a system of pipes from which the air has been pumped. These pipes pass back and forth in a large tank which is filled with salt water, so that the heat required for evaporating the liquid carbon dioxide is taken from this brine. This is then cooled until its temperature is several degrees below 0° C.; but it does not freeze, because the freezing point of salt water is lower than this. The pure water that is to be frozen is placed in large iron molds, which are submerged in the cold salt water, and are kept there until the water in them is frozen.

160. Cold Storage. Cold storage rooms are generally operated in connection with artificial ice plants. These rooms have thick walls made of nonconducting materials, and around them on the inside are rows of pipes. The brine from the freezing tanks is pumped through these pipes on its way back to the cooling tank, and thus serves to reduce the temperature of the room to the point desired. Immense quantities of eggs, butter, fruit, and other perishable foods are thus preserved in cold storage for use in the winter months.

161. The Steam Engine. We are now prepared to consider the way in which heat is converted into mechanical work by steam engines. We select the locomotive as a typical case of these engines, because it is complete in itself. Fig. 96 shows the construction of a modern locomotive. The heat is derived from a fire, which is built in the fire-box *Fb* at the rear end of the boiler. In order that this heat may pass easily into the water in the boiler, the smoke and hot gases produced by the fire are sent through a large number of tubes *Ft*, which pass through the boiler from the fire-box to the forward end. The water in the boiler surrounds these tubes and the fire-box, so that it is in a position to absorb a large part of the heat of the fire. When the engine

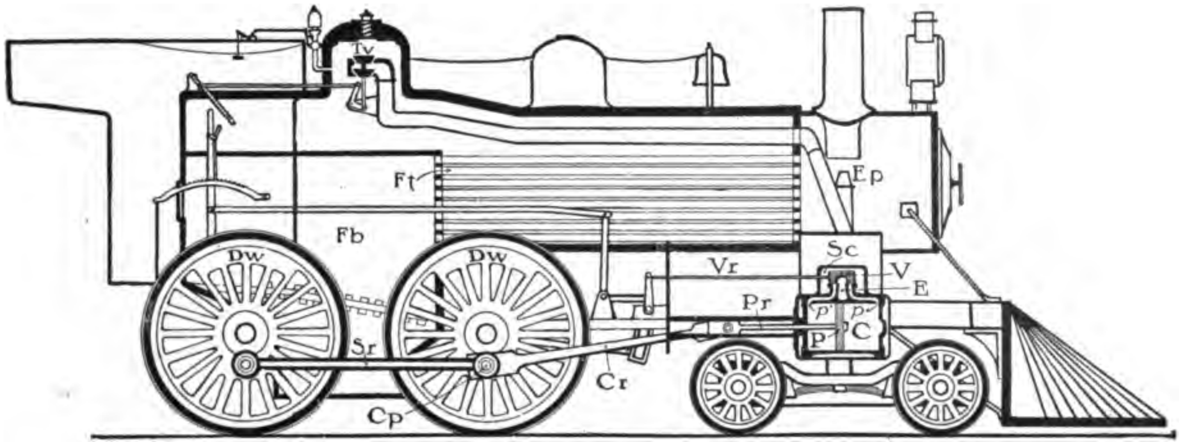


FIG. 96. DIAGRAM OF A LOCOMOTIVE ENGINE
Names of the parts are as follows:

Fb, Fire box.	V, Slide valve.	E, Exhaust.	Cr, Connecting rod.
Ft, Fire tubes.	pp, Steam ports.	Ep, Exhaust pipe.	Cp, Crank pin.
Tv, Throttle valve.	C, Cylinder.	Vr, Valve rod.	Sr, Side rod.
Sc, Steam chest.	P, Piston.	Pr, Piston rod.	Dw, Driving wheels.

has "steam up," the upper part of the boiler is filled with steam at high pressure.

When the engineer wishes to start the engine, he pulls on a lever called the throttle; and this lever opens the throttle valve *Tv*, which is placed at the highest point in the boiler. When this valve is opened, the steam rushes into the large supply pipe, which conducts it to the steam chest *Sc*, whence it passes to the cylinder *C*. Having reached the cylinder through the port *p*, the steam pushes against the piston *P* and moves it backward. This motion of the piston is transmitted by means of the piston rod *Pr* and the connecting rod *Cr* to the driving wheels *Dw*, which are thus made to turn.

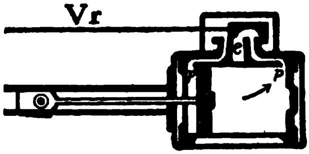


FIG. 97. THE PISTON STARTS FORWARD

When the piston has reached the rear end of the cylinder, the slide-valve automatically opens the port *p′*, and also connects the port *p* with the exhaust *e*, Fig. 97. The high pressure steam in the steam chest then rushes into the rear end of the cylinder and pushes the piston forward, driving the steam in the forward end out through the port *p* and the exhaust *e* into the exhaust pipe *Ep*, whence it escapes with a puff up the stack. The valve then automatically connects *p* with the steam chest and *p′* with the exhaust and the piston is again pushed backward, as in Fig. 98; and so on. Thus the steam, by expanding in the cylinder, moves the piston; and this motion is used in doing the mechanical work of turning the drivers.

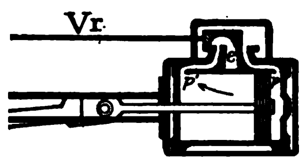


FIG. 98. THE PISTON STARTS BACKWARD

162. Work Done by the Steam. The most important question about a steam engine is: What is its efficiency? In order to answer this question, we have to determine how much work is done by the steam, and how much energy is supplied to it (*cf.* Art. 37). How can we do this? How shall we measure the work done by the steam? How determine the amount of heat energy supplied?

The work done by the steam may be determined with the help of equation (5), Art. 34, $W = fl$. For if the total force of the steam against the piston is f, and if this force acts through a distance l equal to the length of the stroke, the work done at each stroke of the piston is simply the product of these two quantities. But the total force of the steam on the piston is the steam pressure, i.e., the force per cm^2, multiplied by the area in cm^2 of the piston. Therefore the work done by the steam is W = steam pressure $\times$ piston area $\times$ length of stroke.

This result may be interpreted in another way. For piston area $\times$ length of stroke is the volume of the cylinder; therefore, the work done in one stroke is W = steam pressure $\times$ volume of cylinder. Hence we can determine the work done by the steam when we know these two factors.

In actual practice it is an easy matter to measure the volume of the cylinder; but the pressure is not constant throughout the stroke. When the steam enters the cylinder, it has the pressure that exists in the boiler; and, if the port were left open during the entire stroke, the steam would be exerting that same pressure when the exhaust was opened. It would then expand suddenly into the air, and thus waste a large amount of its energy. Therefore it is customary to arrange the valve V, Fig. 96, so that it closes the port p when the piston has made about one-quarter of its stroke. The steam then pushes the piston the remaining three-quarters of the stroke by its own expansion. But while the steam is expanding and doing this work, its pressure is decreasing; therefore when the exhaust is opened, the steam pressure is found to have fallen nearly to that of the atmosphere.

163. The Pressure-Volume Graph. Since the pressure of the steam changes during the stroke, we must find an average value to use in calculating the work. This is generally done by measuring the pressure at different parts of the stroke by means of an ingenious pressure gauge attached to the cylinder (*cf.* problem 8, page 187). The pressures thus found are plotted as ordinates with the corresponding volumes as abscissas, and the average pressure obtained by measurements on the graph. Fig.

99 is such a graph taken on a locomotive cylinder when it was pulling a train. The point p_1 represents the pressure and v_1 the volume when the steam had just entered the cylinder. As the piston moved, the volume v_1 increased, while the pressure remained constant and equal to that in the boiler until the point p_2 was reached. At this point the port was closed and the steam, being thus "cut off" from the boiler, was left to expand. During the remainder of the stroke,

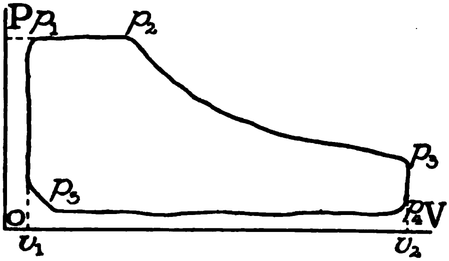


FIG. 99. THE PRESSURE-VOLUME GRAPH

while the steam was expanding and the volume increasing, the pressure was falling as shown by the curve p_2 p_3. At p_3, the piston having now reached the end of its stroke, the exhaust was opened, so that the pressure fell suddenly to that of the atmosphere, represented by p_4. As the piston returned, the volume of the cylinder decreased; but because the exhaust was open the pressure there remained constantly equal to atmospheric pressure up to the point p_5. At this point the valve shifted so as to close the exhaust and let in the live steam (*cf.* Art. 161); and consequently the pressure jumped quickly to p_1, that of the boiler, and the entire process was repeated.

164. Back Pressure. In determining the average pressure from a graph like this, one thing must be carefully noted. When the piston returns, it has to push the exhaust steam out of the cylinder against the pressure of the atmosphere. The work of doing this is numerically equal to atmospheric pressure × volume of cylinder. This amount of useless work must be subtracted from that done by the steam, in order to find the amount of available mechanical work done by it. Therefore, when an engine exhausts into the atmosphere, as the locomotive does, the average pressure by which its work is determined is the difference between the total average pressure and that of the atmosphere.

165. Lower Pressure at Exhaust. It has probably occurred to many of you that we might increase the amount of work done if we could allow the steam to exhaust into a vacuum instead of into the air; for then no work would have to be wasted in pushing the used up steam out of the cylinder. This may be partially accomplished by allowing the exhaust steam to escape into a coil of pipes which contain no air, and which are surrounded by cold water. When the exhaust steam enters these pipes, it is condensed into water, and so the pressure there is that of the saturated vapor at the temperature of the cold water that surrounds the pipes (*cf.* Art. 128 and Fig. 85). Such a device is called a CONDENSER. The water condensed from the steam has to be pumped out of the condenser against atmospheric pressure; but since its volume is so much less than that of the steam, the useless work done against the atmospheric pressure is much less. Condensers can not be used on a locomotive, because they are too bulky, and because they require a large amount of cold water to keep them cool. They are useful, however, on steamboats, where space is more plentiful, and where a large supply of cool water is always at hand.

166. Increased Boiler Pressure. Another way to increase the amount of work done by the steam is to increase the boiler pressure by increasing the temperature of the steam; for the work = average pressure × volume of cylinder, and this process increases the average pressure. There is, however, a practical limit to increasing the efficiency in this way, because the pressure must never be allowed to approach that at which the boiler would burst (*cf.* Arts. 128, 129).

167. Heat Energy Consumed. Having now found how the work done by the steam may be measured, let us consider how the quantity of heat energy supplied to the engine is determined. This is done practically by first noting the amount of fuel consumed by the engine in a given time, and then finding out how many heat units are liberated when this amount of fuel is burned. For example, a locomotive of the type shown in Plate I con-

sumes 1500 gm of coal for every horse-power that it furnishes for one hour. The number of gm cal liberated by each gm of coal when it is burned is found to be 7800. Hence the amount of heat liberated by 1500 gm of coal is $1500 \times 7800 = 117 \times 10^5$ gm cal. Since the mechanical equivalent of 1 gm cal is 4.19×10^7 ergs [*cf.* equation (9), Art. 154], this amount of heat is equal to $117 \times 10^5 \times 4.19 \times 10^7 = 49 \times 10^{13}$ ergs. This is the energy supplied to the engine for each horse-power hour.

Since (*cf.* Art. 43) 1 horse-power $= 746 \times 10^7 \dfrac{\text{ergs}}{\text{sec}}$, and since 1 hour $= 3600$ sec, 1 horse-power hour $= 3600 \times 746 \times 10^7 = 268 \times 10^{11}$ ergs. This is the work done. The EFFICIENCY is then, $\dfrac{\text{work done}}{\text{energy supplied}} = \dfrac{268 \times 10^{11}}{49 \times 10^{13}} = \dfrac{268}{4900} = 0.055 = 5.5\%$.

168. Efficiency and Temperature. We have just found that a locomotive, when considered from the point of view of efficiency, is a very inefficient machine. Yet even stationary and marine engines seldom have efficiencies greater than 17%. This fact leads us to ask whether there is any theoretical reason for this. Are the conditions under which every heat engine must work such that its efficiency is necessarily small? Or may we hope some day to make heat engines of high efficiency?

We can find answers to these questions by carefully tracing the heat through an engine. We note that heat is absorbed by the water when it passes into steam. The steam then carries this heat with it when it goes into the cylinder, where part of the heat is used up in doing the work of moving the piston. The rest of the heat is carried with the exhaust steam into the air or condenser, and is then no longer available for doing work in the engine. The essential things in this process are: 1. Heat energy is imparted to the steam at a high temperature (boiler temperature); 2. The temperature of the steam must fall when work is done; 3. Heat is given up to the condenser at a low temperature.

The only heat available for doing work is that given up by the steam in cooling from the temperature of the boiler to that of the

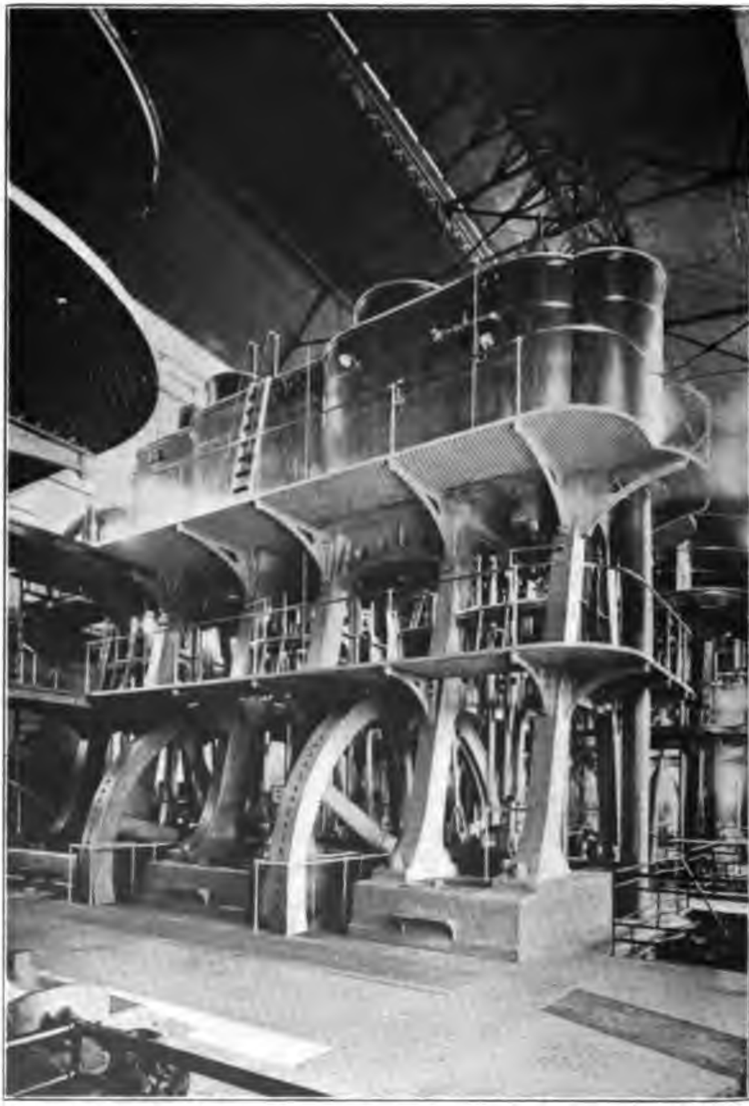


PLATE VI. A TRIPLE EXPANSION PUMPING ENGINE

condenser. Hence the heat energy available for work depends on this difference in temperature. An engine that could convert all of this available heat energy into work would be a perfect engine.

The total heat energy in the steam when it leaves the boiler depends in a similar way on how many degrees the boiler temperature is above the absolute zero. We may then say that the total amount of heat energy of the steam depends on its absolute temperature, i.e., on $273 + t$ (*cf.* Art. 123).

Finally, since the efficiency of an engine is defined as $\frac{\text{heat converted into work}}{\text{total heat supplied}}$, and since in a perfect engine the work depends on the difference in temperature $(t - t')$ between the boiler and the condenser; and since the total heat energy supplied depends on $273° + t$, we may conclude that the efficiency of a perfect engine might be expressed by the ratio $\frac{t - t'}{273 + t}$.

169. Comparison of Efficiencies. Plate VI is a picture of one of the large pumping engines that supply the city of St. Louis with water. It is a triple expansion engine. The steam from the boiler enters the smaller cylinder at the left of the picture and there expands and cools somewhat. It then passes into the second cylinder, in which it expands and cools some more; and from this it exhausts into the third and largest cylinder, at the right of the picture. From this it passes to the condenser. This arrangement, by which the steam is allowed to do part of its expanding in each of the three cylinders, has many practical advantages, such as distributing the strains among three cranks instead of concentrating them in one; greater compactness, since a single cylinder that would do the work of the three would have to be of enormous size; greater economy in steam, since the fall in temperature in each cylinder is only one-third of the total fall, so that the steam does not condense so readily into water in the cylinder.

In this engine the steam has a pressure of 10 atmospheres (760 cm of mercury) when it enters the first cylinder, and it ex-

hausts into a condenser in which the pressure is 53 cm of mercury. On consulting a table in which the relations between temperature and pressure of saturated water vapor are given, we find that the temperature corresponding to the boiler pressure is 180° C., and that corresponding to the condenser pressure is 80° C. If the engine were perfect, its efficiency would then be $\frac{t-t'}{273+t}=\frac{180-80}{273+180}=\frac{100}{453}=22\%$.

It was found that 500 gm of coal was consumed at the boiler for every horse-power hour furnished by the engine. Since the locomotive just considered (Art. 167) consumed 1500 gm of coal per horse-power hour, the efficiency of this engine is just three times that of the locomotive, or $5.5 \times 3 = 16.5\%$. The real engine is thus seen to be about three-quarters perfect.

Sometimes boiler pressures as high as 15 atmospheres are used. The corresponding temperature at the boiler is found from the table to be about 200° C. If the pressure in the condenser is reduced to 3 cm of mercury, the corresponding temperature would be 30° C. So the efficiency of a perfect engine working between these temperatures would be $\frac{t-t'}{273+t}=\frac{200-30}{273+200}=\frac{170}{473}=36\%$ nearly. Since boiler pressures greater than 15 atmospheres are not very safe, and since it is very expensive to reduce the condenser temperature below 30° C., 36% represents the practical limit for the efficiency of a perfect steam engine. No steam engine has yet been made with an efficiency as high as this.

170. The Gas Engine. The efficiency of a steam engine is thus seen to be low because the range of temperature $(t-t')$ through which we can use the steam is comparatively small. In the gas engine, Fig. 100, the conditions are more favorable. In this machine the fuel is burned in the cylinder where the work is done. The temperature of the gas in the cylinder may, therefore, be very high. A mixture of gas and air is introduced into the cylinder and there exploded. The pressure developed by this explosion pushes the piston outward. It is then pushed back

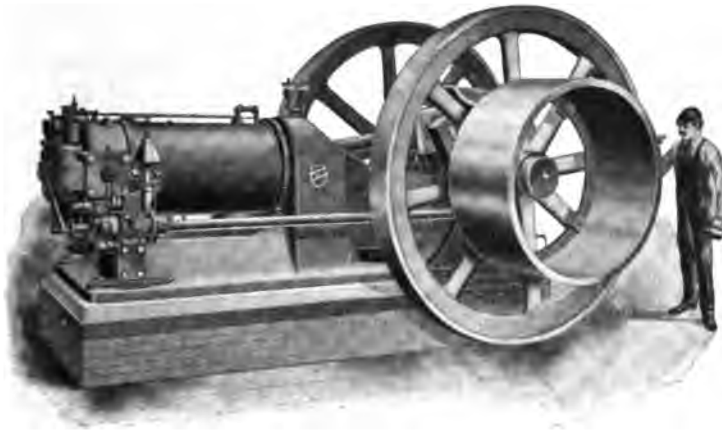


FIG. 100. A GAS ENGINE

by the atmospheric pressure: another explosion pushes it outward again, and so on.

A good gas engine consumes about 16 cubic feet of gas from the city mains for every horse-power hour that it supplies. The heat of combustion of illuminating gas in New York has been found to be 18×10^4 gm cal per cubic foot. Hence the heat supplied by 16 feet of gas is $16 \times 18 \times 10^4 = 288 \times 10^4$ gm cal. This heat energy is equal to $288 \times 10^4 \times 4.19 \times 10^7 = 1200 \times 10^{11}$ ergs. This is the heat energy supplied to the engine.

FIG. 101. THE OLD-FASHIONED WATER-WHEEL

In Art. 167 the number of ergs in a horse-power hour was found to be

268×10^{11} ergs. Therefore the efficiency of this gas engine is $\frac{268 \times 10^{11}}{1200 \times 10^{11}} = 22\%$.

171. Turbines. In the engines thus far considered the rotary motion of the drivers or of the flywheel was produced by a translatory motion of the piston to and fro. Engines of this type are therefore called reciprocating engines. In all such engines,

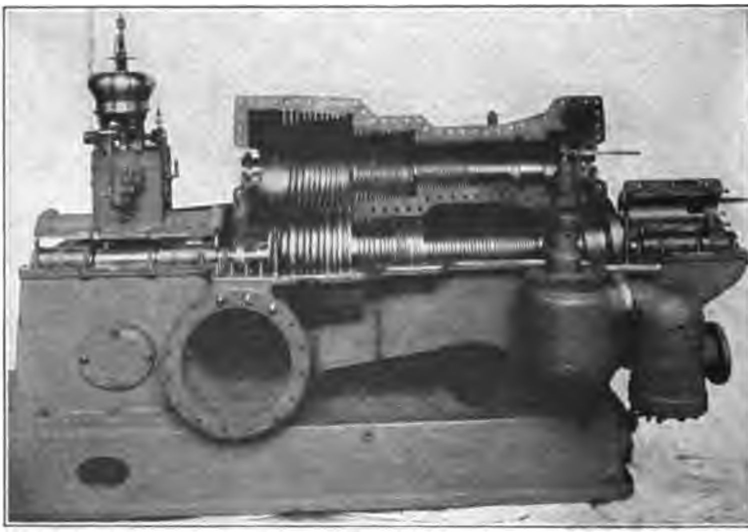


FIG. 102. A STEAM TURBINE

useless work has to be done in starting and stopping the piston at each stroke; and this action always produces a jarring which is harmful both to the engine and to the building or boat in which it is placed.

The old-fashioned water-wheel, Fig. 101, and the modern water turbines, such as are used so extensively at Niagara, illustrate another method of converting kinetic energy of translation into kinetic energy of rotation. The moving water is projected against the paddles or blades of the wheel, and thus keeps it steadily turning. Many attempts have been made to construct

an engine in which a wheel would be set into rotation by blowing steam against blades or paddles on it. It is only within the last few years that engineers have learned how to apply this principle so as to make a steam turbine equal in efficiency to the best reciprocating engines.

Fig. 102 is a picture of one of these modern steam turbines. The cover has been removed so that we can see how it is made. Instead of a few large blades, like the water wheel, it has many rows of small blades fastened to a steel cylinder called a rotor. These movable blades pass between rows of stationary blades fastened to the case of the machine. High pressure steam enters the turbine at the smaller end of the case, and, in elbowing its way amongst the forest of blades, sets the rotor into rapid rotation. The arrangement of the fixed and moving blades in this turbine is shown in Fig. 103. The dotted line *cemo* indicates the path of the steam.

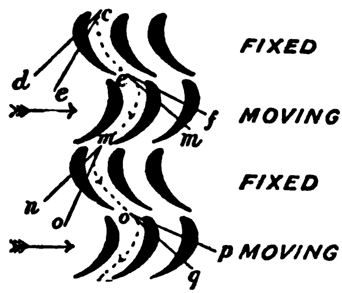


FIG. 103. BLADES IN THE STEAM TURBINE.

Turbines have now been so far perfected that their efficiencies are greater than those of reciprocating engines. On account of their freedom from jarring, their high efficiency, and their compactness, they are now coming into general use. It is interesting to note that the principle of the steam turbine was known to Hero of Alexandria (B.C. 120). The technical difficulties involved in the practical construction of a steam turbine of high efficiency have delayed its perfection for 2000 years.

SUMMARY

1. The mechanical equivalent of 1 gm cal is 4.19×10^7 ergs.
2. A gas is heated when it is compressed and cools when it does work in expanding.
3. When a liquid evaporates, heat is absorbed.
4. The work done by the steam in a steam engine is measured

by the product of the average pressure and the volume of the cylinder.

5. The efficiency of an engine may be increased (*a*) by raising the boiler temperature; (*b*) by using a condenser.

6. The efficiency of a perfect engine is equal to the difference in temperature between the boiler and the condenser divided by the absolute temperature of the boiler.

QUESTIONS

1. A tea-kettle of liquid air boils furiously when placed on a cake of ice. Does this case differ essentially from that of a kettle of water on a hot plate of iron?

2. Describe the experiment of Joule and Rowland. What relation was established by them?

3. With the aid of diagrams made from memory, explain the action of the steam in the cylinders of a steam engine, and the manner in which this action is controlled by the slide valve.

4. Why is it possible to "cut off" the entrance of the steam to the cylinder before the completion of the stroke, and still get work out of the steam that has entered?

5. What is the use of the condenser of a steam engine?

6. From the expression for efficiency in Art. 168, can you tell why the cylinder of a gas or gasoline engine is arranged so as to be cooled with water or air?

7. Why should the rotor and case of the steam turbine, Fig. 102, be larger at the end where the steam leaves than it is at the end where it enters?

8. We can convert a given quantity of mechanical energy into heat. Can we convert a given quantity of heat entirely into mechanical work? Why?

PROBLEMS

1. When a warm, moisture-laden wind strikes the sides of a mountain range, it is forced up the inclined plane, and rises to where the atmospheric pressure is less. What effect has this change of pressure on its volume? Since it is expanding against some atmospheric pressure, what effect does this expansion have on its temperature? What effect may this change of temperature have on the invisible water vapor that it contains? If the air then blows over the mountains, will it be likely to deposit much rain on the other side?

2. How does Pascal's principle operate in the cylinder of an engine? Does equation (9), Art. 122, apply there?

3. Niagara Falls are about 53 m high. If all the energy of the falling water were transformed into heat, how much would each gram heat itself by falling?

4. The cylinders of a locomotive are 60 cm long and 50 cm in diameter. What is the volume of each? Take $\pi = 3.14$. What volume of steam is used per stroke? The average effective pressure of the steam is found to be 3.5×10^6 dynes. How many ergs of work are done per stroke? If 3.3 strokes are made per sec, what is the power in $\frac{\text{ergs}}{\text{sec}}$? What is the horse-power? Remember that there are two cylinders, and that a complete stroke is twice the length of the cylinder.

5. In June, 1892, in a test of the Empire State Express, the following data were recorded when the train was going at a speed of 60 $\frac{\text{miles}}{\text{hour}}$. Find the horse-power developed. Area of each piston, 283.5 inches^2; pressure, 53.7 $\frac{\text{pounds-force}}{\text{inch}^2}$; length of cylinder, 2 ft.; revolutions per minute of drivers, i.e., strokes of piston, 260.

6. Fig. 104 is a pressure-volume graph. When the volume is increasing from p_1 to p_2, does the pressure change? If the number of cm in the length of the ordinate $v_1\,p_1$ be multiplied by the number of cm in the length of $v_2 - v_1$, what area does this product represent in the figure? Does this area also represent the work done by the gas in expanding from volume v_1 to volume v_2? Might we get the numerical value of this work by multiplying this area of the rectangle $v_1\,p_1\,p_2\,v_2$ by the number of dynes pressure and the number of cm^3 volume respectively, that 1 cm represents on the diagram? If the pressure changed to a different value p_3, for a change of volume $v_3 - v_1$, might we get in the same way the work done during this new period of expansion?

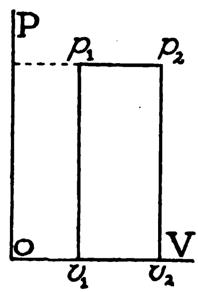


FIG. 104

7. In the graph, Fig. 99, draw at equal distances ten vertical lines from the atmospheric line $p_5\,p_4$ to the broken line $p_1\,p_2\,p_3$, measure these lines, and take their average. Will this average represent approximately the mean effective pressure of the steam against the piston? If we multiply this average by the length $v_2 - v_1$, what area will it represent? Will this area represent approximately the work done by the piston in traveling from one end of the cylinder to the other? Why?

8. Fig. 105 represents the pressure gauge mentioned in Art. 163. The lower end of the tube at the left is coupled to one end of the engine cylinder, and opens into it; so the steam can enter and push up the little piston (seen inside the tube). This piston is held down by the spiral spring near the top of the tube, but when the piston is pushed

up against this spring, it lifts the lever. The lever carries a pencil which it moves vertically along the little drum at the right of the diagram. A card is wrapped around this drum and held by the spring clips. A cord, which is wrapped around the drum, runs over a pulley at the right of the drum and thence to a pin on the piston rod of the engine. As the piston moves back, the cord is unwound, and rotates the drum. When the piston moves forward, a spring turns the drum back again and winds up the cord. Can you apply your knowledge of the composition of motions so as to tell how the pencil draws on the card a pressure-volume graph like Fig. 99? This pressure gauge is known as the steam indicator. It was invented by James Watt. The graph is called by engineers an indicator diagram and is much used in determining the work done by a steam engine. The horse-power is obtained by dividing the work done per sec by 550 (*cf.* Art. 43), and is called the indicated horse-power.

FIG. 105

9. What is the efficiency of an engine that consumes 1000 gm of coal per horse-power hour?

10. An engine with an efficiency of 15 per cent does work at the rate of $746 \times 10^8 \frac{\text{ergs}}{\text{sec}}$. How many gm of coal must it burn per hour (*cf.* Art. 167)?

11. In a water turbine, moving water turns the wheel and the wheel turns machinery. Mention a case where a similar wheel is turned by an engine and made to produce motion by pushing against the water. A steam turbine and a windmill are used to propel machinery. Mention a familiar case of a similar wheel that is turned by a motor or engine and made to put air into motion.

SUGGESTIONS TO STUDENTS

1. Hammer a nail rapidly for several minutes on a piece of iron, pick it up and see what happens.

2. If you have a toy engine, bring it in and explain how it works.

3. Visit a roundhouse or engine shop and make a report of what you see that interests you. Visit an ice and cold storage plant and report.

4. Find out what you can about Count Rumford and his studies of the conversion of work into heat.

5. You will find interesting information about heat in the following books: Mach, *Heat,* Open Court Publishing Company, Chicago; D. E. Jones, *Heat, Light and Sound,* Macmillan, New York; *The Twentieth Century Locomotive,* Sinclair Company, New York. A good description of the indicator diagram will be found in Pullen's *Mechanics* pp. 253–260. You will find much interesting information concerning new forms of engines in the "Scientific American" and its Supplement, in "The Engineering News," "The Technical World," Cassell's and Cassier's magazines, and in "Railway and Locomotive Engineering."

6. Read the *Life of James Watt,* by Andrew Carnegie; the *Life of Robert Fulton,* by R. H. Thurston. An interesting account of the development of the steam engine is given in *A History of the Growth of the Steam Engine,* by R. H. Thurston. Who was Count Rumford? Read *Memoir of Sir Benjamin Thompson, Count Rumford,* by G. E. Ellis (American Academy of Arts and Sciences, Boston).

7. The following books may interest you: Balfour Stewart, *The Conservation of Energy* (Appleton, N. Y.); Ray S. Baker, *The Boys' Book of Inventions,* and the *Second Boys' Book of Inventions* (Doubleday-Page, N. Y.); R. H. Thurston, *Heat as a Form of Energy* (Houghton-Mifflin, Boston); T. O'Connor Sloane, *Liquid Air and the Liquefaction of Gases* (N. W. Henley, N. Y.).

8. A great deal of useful information about all kinds of engines and fuels is given in Wm. Kent, *Mechanical Engineer's Pocket Book* (Wiley N. Y., 6th Edition, 1903).

CHAPTER X

ELECTRICITY

172. The Transmission of Power. In the preceding chapters, we have seen how the energy of wind, of water, and of steam may be utilized in windmills, water-wheels, and steam engines for doing mechanical work. When the energy of these contrivances is transmitted by means of belts, cables, gear wheels, or shafting to the machinery which it is to operate, this machinery and the source of its energy must be near together. The efficiency of such a system of transmission is not so great as could be desired, and it diminishes rapidly as the distance between is increased. Furthermore, the practical difficulties in the way of these methods of transmitting power become prohibitive at comparatively short distances. When the work to be done is that of transportation, the engine is a locomotive, and in doing its work it goes over any distance desired; but it must carry with it a heavy load of fuel and water, and it is not nearly so efficient for light loads and frequent stops, as for heavy loads and without stops. Is there no form of energy that may be transmitted cheaply and conveniently over considerable distances, and used at such times, at such places, and in such amounts as may suit the convenience of the user?

Most readers know in a general way that there are two methods by which energy may be thus distributed. One is by converting the coal into fuel-gas, and sending it through pipes to the various places where it is to be used for light, for heat, and for operating gas engines for power. The other is by converting the energy of the water-wheel or the steam engine into that of an electric current, and distributing it by means of copper wires to electric lamps for light, and to electric motors for power.

173. Electric Generators. Plate VII is a photograph of one of the large dynamo-electric machines of a power plant, whence electrical energy is distributed. It is direct-connected with a

Plate VII. Dynamo in a Large Power Plant

big compound steam engine. The engine transforms heat energy into mechanical energy; and the dynamo transforms the mechanical energy into electrical energy. This electrical energy is sent out along a system of wires to the places where it is to be used.

These powerful machines suggest many interesting questions for study, some of which we shall try to answer in the next four chapters. How do these machines work? What are some of the elementary facts of electricity and of magnetism? What are the relations between electricity and magnets? What are some of the useful inventions by means of which the discoveries in electrical science are applied so as to multiply both our means of doing business and our facilities for enjoying life? Who were some of the great discoverers that sowed the seeds from which this harvest has sprung?

174. Early Knowledge of Electricity and Magnetism. Before history began, man feared the lightning and thunder, and had some primitive explanation for it. Amber, when rubbed with wool, attracts light bodies. Doubtless the fact was known long before Thales of Miletus recorded it, about 600 B.C. The early Greeks also knew that lodestone, or magnetic ore of iron, attracts pieces of iron; and some truth and many extravagant fables were written of it by Pliny and others. The magnet is supposed to have received its name from Magnesia, in Asia Minor, where deposits of lodestone were found. The word "electricity" is derived from "electron," the Greek name for amber.

The Greeks never used the scientific method of study, and hence they learned nothing of electricity beyond a few simple facts, which in themselves are useless. They contented themselves with supposing that amber possessed a soul, which gave it its strange powers.

175. Gilbert. The first man who ever studied electricity and magnetism to any purpose was William Gilbert of Colchester (1540–1603), a contemporary of Shakespeare and Bacon. Queen Elizabeth appointed him her physician, and gave him a salary, in order that he might be free to pursue his studies. He collected and recorded all that was then known about the subject; and as

a result of his experimental studies he discovered many new facts. These he published in 1600 in a book, *De Magnete*, which is still of good scientific repute.

176. Electrification. Gilbert found that other substances besides amber, such as glass, sulphur, and the resins, would, when rubbed, attract light bodies. When in this condition, they are said to be electrified or CHARGED WITH ELECTRICITY. He found also that he could not charge metals by rubbing them; therefore he called the former *electrics* and the latter *non-electrics.*

177. Conductors and Insulators. Gilbert's lack of success with his "non-electrics" was due to a very important electrical property, of which he failed to learn, but which was discovered a century later by another Englishman, Stephen Gray. This property, now so well known, is called CONDUCTION. An electric charge will travel over or through some bodies very easily, somewhat as heat does; and it can thus be transmitted along them from one place to another. Accordingly, these substances are called GOOD CONDUCTORS of electricity. Those substances which do not conduct well are called POOR CONDUCTORS OF INSULATORS. As a result of experiments, substances may be arranged as below, in the order of their electrical conductivities.

CONDUCTORS
Silver
Copper
Iron
Mercury
Carbon
Solutions of Salts
Pure Water
Resins
Hard Rubber
Porcelain
Glass
INSULATORS

It is worthy of note that for most substances the order of their electrical conductivities is the same as that of their heat conductivities. There is no dividing line between conductors and insulators; for every substance has some conducting power. The difference is merely one of degree.

We ought now to know how to succeed where Gilbert failed. To ELECTRIFY A PIECE OF METAL, we have only to fasten it to a handle made of one of the substances near the foot of the list; and we shall find that we can charge it as highly as we can any

of the others. The human body and the earth are fairly good conductors; and if it were not for the insulating handle of glass or rubber, the charge would escape through the body of the experimenter, and spread itself over the earth. Consequently, there would be so little energy left at any one place that no perceptible work would be done by it.

178. Repulsion. That an electric charge may cause repulsion as well as attraction was first noticed by Guericke, who devised the first electrical machine as well as the first air pump. When two pith balls suspended by threads from an insulating support are approached by an electrified glass rod, the rod attracts the balls to itself. If the balls touch the rod, some of the charge from the rod is communicated to them, and they are repelled. The balls now repel each other; they also repel the rod, as we should expect from the third law of motion (*cf.* Art. 40). This may easily be shown by suspending the rod at its middle by a paper sling, attached to a silk thread. In the repellent movement, the rod, of course, has a smaller acceleration than have the balls, because its mass is greater.

179. Discharge. If now the pith balls are touched by the hand, or by any other conductor connecting them with the earth, their charges will be dissipated. When this has occurred, they are said to be DISCHARGED.

180. Two Kinds of Electrification. If the balls are both charged from any other electrified body, they will repel each other just as they did when charged from the glass. We find, however, that their electrification is not always of the same sort; for if we electrify them by contact with glass that has been rubbed with silk, and then present to them a stick of sealing wax that has been rubbed with flannel, we observe that though there is *repulsion* between the balls and the glass, there is *attraction* between the balls and the wax. Thoroughly discharge two pairs of pith balls, A and B, electrify the pair A from the glass, and the pair B from the wax. Those of the pair A repel each other;

those of the pair *B* repel each other. But those of the pair *A* attract those of the pair *B*, and are attracted by them. It was thus found that there are two kinds of electric charges, and it became necessary to name them. That kind of charge which is developed on glass by rubbing it with silk, is called a *vitreous* or + (POSITIVE) CHARGE; and that kind which is developed on sealing wax by rubbing it with flannel, is called a *resinous* or − (NEGATIVE) CHARGE. These names are, of course, purely arbitrary, and are adopted solely for convenience.

181. Dufay's First Law. The results of experiments like those just described are included in the following general statement, which we may call the law of electrostatic attractions and repulsions, or the first law of electrostatics. It is also known by the name of its discoverer, Dufay.

Like charges repel each other; unlike charges attract each other.

182. To Determine the Kind of Charge. Since a charged body always attracts an unelectrified body, as well as one having a charge of opposite sign, *attraction is not a satisfactory test* of the kind of charge that a body has; but if repulsion occurs between two bodies we may be sure that they have like charges. If, then, we wish to determine the sign of an unknown charge, we may impart some of this charge to a pith ball, and place the ball first near a glass rod rubbed with silk, then near a stick of sealing wax rubbed with flannel. If the ball is repelled by the glass rod, the unknown charge is of the positive kind; and if it is repelled by the wax, the unknown charge is of the negative kind.

183. Electroscope. A suspended and insulated pith ball thus serves as an electroscope, by means of which we may detect a charge, and determine its sign; but for many experiments a more sensitive instrument is needed. The one shown in Fig. 106 answers well. Two strips of gold or aluminum leaf take the place of the pith balls; and the flask serves for an insulating support, as well as to protect the light and fragile leaves from any disturbing currents of air. A very slight charge communicated

to the metallic ball or plate at the top is conducted to the leaves, and causes them to repel each other. Also, the greater the charge, the greater the divergence of the leaves. When the ball or plate is touched by the hand, the leaves collapse, showing that the electroscope is discharged.

To test a charge by means of the GOLD LEAF ELECTROSCOPE, give the leaves a known charge sufficient to cause a moderate divergence. If now a charge of the same sign is approached, the divergence of the leaves is seen to increase; but if a charge of the opposite sign is approached, their divergence is seen to diminish. Therefore, if we have given the electroscope, say, a negative charge, and if we then bring near it the unknown charge, an observed increase in the divergence of the leaves will prove the unknown charge to be negative, and a decrease in their divergence will prove this charge to be positive. The PROOF PLANE P, Fig. 106, is a disc of metal with an insulating handle. It is used to carry a small charge from a charged body to the electroscope in order to test the body's electrical condition.

FIG. 106 THE ELECTROSCOPE

184. Both Substances Charged. We may now ask, Is it likely that the substance with which we rub the glass or the wax suffers no change of condition? Ought we not to suspect that it also receives a charge? And will the charge, if it has one, be of the same kind as that of the substance rubbed, or of the opposite kind? For answer, rub the glass and silk together. When tested with the electroscope, the silk will prove to be negatively electrified. Rub the wax and flannel together; and the flannel, when tested, will prove to be positively electrified. In this experiment, the silk and the flannel must be tied to insulating handles of glass or rubber, as they themselves are not sufficiently

good insulators to retain their charges when held in the hand. In this way it has been shown that whenever two dissimilar substances are rubbed together, one gets a positive charge and the other a negative charge.

185. The Two Charges are Equal. But what about the relative amounts of the electric charges of the glass and the silk, or the flannel and the wool? Are they equal or unequal? We may call them equal if they produce equal effects, or if one exactly neutralizes the effect of the other. Let us make an experiment which, though rather crude, is nevertheless convincing. Two brass discs, *A* and *B*, Fig. 107, are fastened to insulating handles. The one on the right, in the picture, is faced with a disc of flannel or fur, of exactly its own size, and neatly cemented on with sealing wax.

Fig. 107. The Charges are Equal

First thoroughly discharge both discs and the electroscope. Fit the discs accurately together, face to face, and twist one of them half way around and back again, so as to rub them together. Hold *A* two or three centimeters from the electroscope, and note the amount of divergence of the leaves. Withdraw *A*, and put *B* as nearly as possible in the same place. The leaves are seen to diverge; and the divergence is the same in amount as before. Now, without having allowed the discs *A* and *B* to touch anything, fit them accurately together again; and while they are thus held, bring them near the electroscope. Observe that while they are together there is no effect on the leaves of the electroscope, i.e., the two opposite charges exactly neutralize each other's effects. Thus we know that they are equal in amount.

These matters have been thoroughly and accurately tested

by many experiments, all of which go to prove the following general statement:

Any two dissimilar substances when brought into intimate contact and then separated, acquire equal electrostatic charges of opposite sign.

186. Franklin's Theory. One of the most noted discoverers in the field of electrostatics in the eighteenth century was our own Benjamin Franklin (1706–1790). Franklin proposed a theory, which in his own time was very widely accepted. He supposed that all unexcited bodies have an indefinite supply of electricity, which is of one kind only, and that charges are generated by one body getting some of this electricity from some other body, so that the former has an excess of electricity, while the latter has an equal deficiency. The body having the excess was said to have a positive charge, and the other an equal negative charge. It was Franklin who proposed the use of the positive and negative signs as suggested by this theory.

This theory is very useful as a working hypothesis. With some slight modifications as to ideas and terms, it is still competent to describe all electrostatic phenomena, including some very remarkable ones recently discovered.

187. Electrostatic Polarization. Let us now see how we can use this theory as a working hypothesis for the description of phenomena and the discovery of new facts. In experimenting with the electroscope, we can hardly have failed to notice a remarkable fact which requires explanation, namely, that the leaves diverge widely whenever an electrified body approaches the instrument. This divergence occurs although the electrified body does not touch the electroscope, nor even approach it near enough for a spark to pass. Using our hypothesis, we may say, in explanation, that the + electrification of the glass rod, when brought near the uncharged electroscope, causes a disturbance of the neutral electrification of the electroscope; it repels positive (+) electrification into the end farthest away, and an equal negative (−) charge remains at the nearer end. The leaves, there-

fore, being both positively charged, repel each other. In this way, any neutral body may be given a + charge at one end, and a − charge at the other, both of these charges being equal in amount to the charge that causes this redistribution. A body in this condition is said to be ELECTROSTATICALLY POLARIZED, Fig. 108, a. That a conductor is really in the condition just described, when under the influence of a near-by charge, may easily be shown by taking small portions of its charges on a very small proof plane (*P*, Fig. 106), and testing them with the electroscope. We can thus prove that the end nearest the influencing charge has a charge of opposite sign, that the other end has a charge of the same sign as that of the influencing body, and that the middle region is neutral.

188. Grounding the Repelled Charge. If the influencing charge be removed from the neighborhood without our having touched the electroscope, the latter returns to the neutral condition, as is evidenced by the collapsing of the leaves. We may say, then, that the opposite charges at its two ends have united and neutralized each other. But if we touch the plate

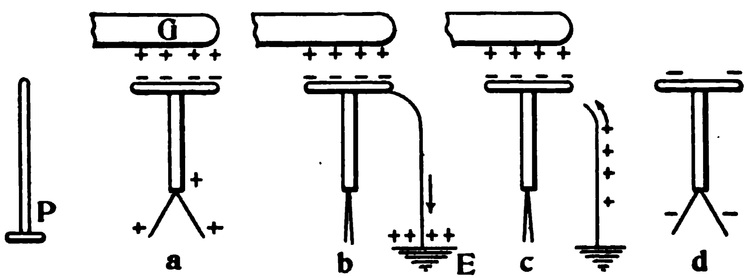


FIG. 108. CHARGING BY INDUCTION
a. Polarized. b. Grounded. c. Bound charge. d. Charged.

with the hand while the influencing charge is held near, the theory leads us to expect that the repelled + charge, in trying to get as far as possible from the glass rod, will go to the earth through the body, while the − charge will remain on the plate, bound by its attraction for the + charge on the glass rod. That the repelled charge does go from the leaves to the earth is indicated by

the fact that as soon as we touch the electroscope plate so as to connect it with the earth *E*, Fig. 108, the leaves collapse. Allowing a charge thus to pass to the earth along a conductor is often called GROUNDING THE CHARGE, Fig. 108, b.

189. Charging by Influence. The theory now suggests another step. When we have polarized the electroscope and grounded the repelled + charge, suppose that we remove the earth connection before we remove the influencing charge. Will not the electroscope then be left with a — charge? It would seem so, because breaking the earth connection would certainly prevent the return of the + charge, while the – charge would be held by its attraction for the influencing charge, Fig. 108, c. When this bound charge is released by removing the influencing charge, it will move freely over the electroscope and reveal its presence by causing a divergence of the leaves, Fig. 108, d. That this excess of electrification is really present and is of the negative sort, may easily be shown by the fact that the leaves collapse when the positively charged glass rod is brought near, and increase their divergence when approached by a negative charge. This is the most convenient method of charging the electroscope. Any conductor whatever may be charged in this way. This kind of charge is called an INDUCED CHARGE, and the process by which a conductor *B* is electrified by another body *A* without loss by *A* of any part of its own charge, is called CHARGING BY INFLUENCE. Let us review the steps of the process. They are:

1. Bring the influencing charge *A* near the neutral conductor *B*; *B* becomes polarized. 2. Touch *B* with the hand, or with any earth-connected conductor; the repelled charge, equal in quantity to *A*, and the same in kind, is grounded. 3. Break the earth connection; the grounded charge can not get back, and the bound charge, equal to *A* and opposite in kind, remains. 4. Remove *A*; the bound induced charge is freed, and spreads over the surface of *B*. That the induced charge is equal to the influencing charge may be proved, if both bodies, *A* and *B*, are good conductors, by allowing *A* to touch *B*, when the two charges will unite and neutralize each other. That the two charges are

opposite in kind may be proved by testing them with the electroscope.

190. A Static Charge Resides on the Outside of the Conductor. When a charge is communicated to a conductor, any two portions of it will repel each other; and therefore every portion of the charge will get as far away from every other portion as it can. Thus we should expect to find a charge distributed over the surface of the conductor, and not at all on the inside. That this is true, may be proved by using a very small proof plane (Art. 183), with which to take samples of the electrification from the different parts of the conductor. Thus we may electrify an insulated, hollow brass globe, or even a common tin cup placed on a cake of resin. With the proof plane, we may then test all places on the outside of the hollow conductor, carrying to the electroscope the sample charges, which will reveal their presence by the divergence of the leaves. But try as we may, provided the proof plane is not allowed to come too near the edges of the opening in the hollow conductor, we can not succeed in getting any charge from the inside.

191. Coulomb's Law. The magnitude of the force between two charged bodies was first determined in 1777 by Coulomb, an eminent French engineer and physicist. He measured this force by balancing it against the torsion (twisting force) of a fine wire. He found that when the distance between two charges was made twice as great, the force was reduced to $\frac{1}{4}$ its former value; and when the distance was made four times as great, the force was reduced to $\frac{1}{16}$. Coulomb's experiments also proved that if either charge was increased in amount, the force increased in the same proportion. The facts established by such experiments may be summarized in the following general statement, which we may call Coulomb's law of electrostatic force, or the third law of electrostatics.

The force between two electrostatic charges varies directly as the product of their quantities, and inversely as the square of the distance between their centers. This law is strictly true only when

the charges are situated on spherical conductors, which are very small in proportion to the distance between them. This statement furnishes us with an appropriate unit in which to measure a charge, for we may define unit charge *as that charge which, placed in air at a distance of* 1 *cm from an equal charge of like sign, repels it with a force of* 1 *dyne.*

192. The Leyden Experiment. In the year 1745 a discovery was announced from Germany, and a few months later from Leyden, in Holland, which caused experimenters to redouble their activity. While experiments were being made in electrifying water in a bottle held in one of his hands, Musschenbroek, a renowned science teacher of Leyden, touched the wire by which the charge was passing into the water, with his other hand. He received a muscular shock of extraordinary power. The news of the experiment spread rapidly, astonishing all Europe, and it was repeated everywhere, both in Europe and in America, with dramatic effect. At the French court, birds and small animals were killed by electricity; and 180 soldiers in line were given a shock simultaneously. A line of Carthusian monks 900 feet long was formed, and the dignified ecclesiastics were made to jump up all together, by the discharge of the "Leyden bottle." All this partook rather more of the spectacular than of the scientific; but it served a good purpose, first in arousing general interest in scientific experiments, and second in providing physicists with a new combination to investigate, and a means of collecting greater charges than had previously been at their disposal.

193. Condensers. The essential parts of the Leyden apparatus are: 1. Two conductors of large surface; 2, a thin layer of some good insulating substance between the conductors. The two conductors are called the coatings, and the insulating layer is called a DIELECTRIC. Such an arrangement of two conductors and a dielectric is called a CONDENSER. It soon took the familiar form of a glass jar, coated inside and out with tin foil, reaching to within 5 or 10 cm of the mouth. The mouth is closed with an insulating stopper, through which passes a brass rod, terminated

above by a brass ball, and below by a chain which touches the inner coating. This is called a LEYDEN JAR.

194. To Give a Condenser a Large Charge, one coating must be connected with one of the knobs of an electric machine, and the other coating with the other knob or with the earth. As the machine is operated, and the electrical energy given out, this energy is stored in the condenser. To *discharge the condenser*, the two coatings must be joined by a conductor. When the conductor has almost completed the circuit, the air between the knob of the condenser and the end of the discharger breaks down, or is punctured, and a spark passes, accompanied by a loud snapping noise. This is known as a DISRUPTIVE DISCHARGE. Such sparks are shorter, thicker, hotter, noisier, and in every way more energetic than those which the machine can give without the condenser.

195. How the Condenser Operates. Since each of the coatings of the condenser is connected with one of the poles of the electric machine, one coating becomes charged positively and the other negatively. These two charges are not able to neutralize each other's effects, because of the dielectric between them. They, however, strongly attract each other, and hold each other bound on the opposite surfaces of the dielectric. Thus the charges cling to the two sides of the dielectric. This fact was discovered by Franklin, and may be easily proved by means of a Leyden jar, whose inner and outer coatings can be removed. Such a jar is shown in Fig. 109. After the jar has been charged, the two coatings are removed and discharged. When they are replaced, the jar will be found to have retained its charge, for a bright spark may be obtained from it. Hence the charge remained bound on the two sides of the

FIG. 109. THE CHARGE IS IN THE DIELECTRIC

dielectric, even after the outer conducting coatings had been removed.

196. The Dielectric is in a Strained Condition. It has been found useful to conceive that when a condenser is charged, the dielectric between the two charges is in a strained condition. This idea was first suggested by Faraday. The discharge of the jar, then, consists merely in the release of the strain. Faraday also enlarged our conceptions of condensers by showing that whenever a conductor is charged, an equal, opposite charge is induced on some neighboring conductor, or on the walls of the room. The dielectric between these two charges is in a state of strain. Hence every charge, from that of a little pith ball to that of a thunder cloud, may be regarded as the charge of a condenser; for there are always two equally and oppositely charged conductors separated by a dielectric.

197. A Disruptive Discharge is Oscillatory. Since we have seen that an electric charge always implies electric strains in the medium between the two oppositely charged bodies, we might suspect that a discharge consists in the release of this strain. We may get a rough mechanical picture of the state of things by considering two plums, imbedded in a mass of elastic gelatin. If the two plums are separated by stretching the gelatin, they tend to come together again and resume the positions which they had before the gelatin between them was strained. If we release them by allowing them to slip back gradually, they will cease to move when they reach the position of no strain. If, instead of releasing them gradually, we release them suddenly, they first fly beyond their positions of equilibrium, and then fly back nearly to their starting points. So they continue to swing, or oscillate, back and forth, but through smaller and smaller distances, until they come to rest finally in their normal positions. Now, if an electric discharge is the releasing of a strain in an elastic medium, as we have conjectured, we can see from the consideration of the crude gelatin model that when the spark passes, an oscillation of some sort must take place. These electric displacements,

first in one direction and then in the opposite direction, may be conceived to be electric charges of opposite sign; and a disruptive discharge would then consist of a rapid surging movement, or alternating current, between the two conductors. If Faraday's elastic displacement theory is anywhere near the truth, and if we have reasoned correctly from it, we ought to conclude that the spark from an electrostatic machine, or from a condenser, is oscillatory, and it ought to be possible to prove it by actual experiment. It has been shown mathematically that the oscillations which make up the discharge are exceedingly rapid. Neverthe-

FIG. 110. THE SPARK OSCILLATES

less, it has been possible to photograph them and determine their period, by means of a rapidly vibrating or rotating mirror. The periods of oscillation of the sparks from Leyden jars depend mainly on the sizes of the jars, and range from one thousandth to one ten-millionth of a second. Fig. 110 is a photograph of such an oscillatory discharge. The mirror was turned to the left while the charge surged alternately up and down between two brass balls. The mirror, as it passed, threw an image of each surging on to a photographic plate and since the mirror turned a little each time, the successive images were thrown on different parts of the plate.

198. Lightning. During his extended experiments with condensers and their powerful effects, Franklin began to be impressed with the many resemblances between condenser discharges and lightning. Conjectures of this kind had been advanced from time to time by European physicists, from the days of Gray, but in the mind of Franklin it had grown into a conviction. In 1749 he stated his conviction, gave his reasons for it, and proposed an

experiment with a pointed aërial wire by means of which the electricity might be conducted quietly from the clouds to the earth. His proposition was received with skepticism or indifference, except at the French court, where one experimenter tried it and was successful.

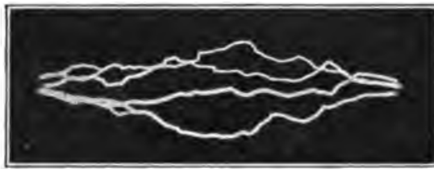


FIG. 111. THE ELECTRIC SPARK IN THE LABORATORY

Franklin, however, did not regard this experiment as conclusive, because the wire did not reach into the clouds; and he therefore thought that the charge might have been received in some other way. Accordingly, he devised an experiment which, for boldness of conception and dramatic interest, as well as for its conclusive character, stands unsurpassed in the history of electrical research. He made a kite from a large silk handkerchief, and tipped it with a pointed wire. He awaited a thunder storm, and went out with his son to fly the kite. After sending the kite up into the overhanging rain cloud, he held it by a strip of silk attached to the hemp twine, and waited calmly for the appearance of the sparks, which he hoped to receive from a metal door-key attached to the string. "No man," says a modern writer, "ever so calmly, so philosophically, staked his life upon his faith." He believed that the pointed wire would bring, not a disruptive discharge, which he knew would kill him if it came, but a quiet flow down the kite-string. At first there was no result; but after

THE ELECTRIC SPARK IN NATURE

it had begun to rain, and the string had become wet, the hempen fibers began to bristle up, and sparks came in plenty from the key. A Leyden jar was charged, and the charge was thoroughly tested for all the properties of electricity. Thus were the lightnings and the thunders of Nature's great laboratory identified with the sparks and the snappings of the machine in the philosopher's laboratory. Thus were the valuable researches of "the many-sided Franklin" crowned by a great discovery. Its publication produced a profound sensation in Europe as well as in America. The honors which his researches brought him were well deserved, for he was the greatest experimental philosopher of his time.

In closing this chapter, it may be remarked that the BENEFIT TO MANKIND resulting from the study of electrostatics, up to the point that we have now reached, was almost wholly intellectual. Electrostatic phenomena are almost entirely devoid of practical applications; but the intellectual progress which resulted from the experimental study of these phenomena prepared the way for discoveries of the phenomena of electricity in motion, and for the vast throng of important inventions which depend upon them. Some foolishly "practical" person once asked Dr. Franklin what use might be made of the facts proved by some of his experiments. The great philosopher replied pithily by asking him, What is the use of a baby? The discovery of the oscillatory character of the condenser discharge illustrates the force of Franklin's answer to this much asked question, What is the use of a scientific discovery? In the course of time, Maxwell deduced from theory that the ether (*cf.* Art. 148) might be the medium in which electrostatic strains and oscillations take place; and that if so, the discharge of a condenser must start waves in the ether. Hertz went to work to start such waves, and detect them. He succeeded. Then followed the discoveries of more sensitive detectors by Branley and Lodge, and the great work of the inventors in wireless telegraphy, which is now going on. Like a baby, a scientific discovery may be small and uninteresting to many; but no one can tell how important it may become.

SUMMARY

1. Mechanical energy may be converted into electrical energy. This electrical energy may be economically transferred to distant places, and there reconverted into mechanical energy, heat or light.

2. In order to understand how this is done, one must know the elementary facts and laws of electrical phenomena.

3. Amber and sealing wax, when rubbed with wool or fur, attract light bodies, and exhibit other remarkable properties. They are then said to be charged with electricity.

4. Electrification travels along some substances easily, but along others with great difficulty. The former are called conductors, the latter, insulators.

5. Both conductors and insulators are essential to the transference of electrical energy.

6. There are two kinds of electrification, positive and negative.

7. Like charges of electricity repel each other; unlike, attract.

8. Any two dissimilar substances, when brought together and then separated, become equally and oppositely charged.

9. The theory of Franklin affords a convenient language for the description of electrostatic phenomena.

10. A neutral body may be electrostatically polarized, and charged by influence. The induced charge is equal to the inducing charge, and opposite in kind.

11. A static charge does not exist anywhere inside a closed conductor; it is always found on the outside.

12. The electrostatic unit quantity of electricity is that quantity which, placed in air at a distance of 1 cm from an equal quantity of like sign, repels it with a force of 1 dyne.

13. The repulsive or attractive force between two charges is directly proportional to the product of their quantities, and inversely proportional to the square of the distance between them.

14. A condenser consists of two conducting plates with a thin layer of dielectric between.

15. The charge of a conductor is in the dielectric, not in the conductor.

16. Franklin proved that lightning is a disruptive electrical discharge from cloud to cloud, or from cloud to earth.

17. The dielectric between two charged conductors is in a condition of strain.

18. Every disruptive discharge is a condenser discharge.

19. A disruptive discharge is oscillatory and starts ether waves.

QUESTIONS

1. What great advantage has electrical energy over other forms?

2. What is the most obvious property of a body that is charged with electricity?

3. What are conductors and insulators? Why are both necessary to electrical transmission?

4. Why do we say that there are two kinds of electrification? How are the two kinds named? How may the presence and the kind of charge be determined with the aid of the electroscope?

5. With the aid of Franklin's theory, explain how a conductor may be polarized, and charged by influence. How do the induced and inducing charges compare with each other in kind and amount?

6. Define the electrostatic unit of quantity.

7. What are the essential parts of a condenser? How is it charged and discharged? Describe the manner in which the charge is accumulated, accounting for the large capacity, i.e., ability to hold a large charge.

8. What is a dielectric? Does the charge of a condenser belong to it or to the coatings? How may this fact be shown?

9. What is the physical condition of the dielectric between two charged conductors?

10. Describe a disruptive discharge. Explain why every charged body must be regarded as one of the coatings of a condenser.

11. What is the peculiarity of a disruptive discharge?

12. How may the heat, light, and noise of a disruptive discharge be accounted for?

PROBLEMS

1. Two unlike electrostatic charges of Q and Q' units, respectively, are distant d cm from each other. Will they attract or repel each other? Call their force in dynes f, and write the expression for its amount.

2. A certain charge is placed at a distance of 10 cm from a + charge of 250 units, and the two charges are found to repel each other with a force of 10 dynes. What was the amount and sign of the unknown charge?

3. In Fig. 108, a, suppose the charge of the glass rod is 10 units, and its distance from the plate of the electroscope is 2 cm. How many — units are attracted to the plate? How many + units are repelled to the leaves? Suppose this + charge to be 10 cm from the repelling charge. With how many dynes force is it repelled? What is the amount and direction of the resultant force between the rod and the electroscope? Do your answers suggest a reason why a charged body attracts an uncharged body?

4. Does the self-repulsive property of a charge suggest why it is found that when a body with sharp points is electrified, a large proportion of its charge is collected at the points? It is also found in such cases that the charge escapes rapidly from the highly charged points, with streams of air that flow away from each of the points, sometimes with force enough to blow out a candle. From your knowledge of electric attraction and repulsion and of electrifying by contact, explain how such streams of air are maintained until the body is discharged.

5. Does question 4 suggest the reason why bodies intended to hold electrostatic charges are usually made round and smooth? Does it suggest why an electrostatic machine usually refuses to "spark" when a pointed wire is attached to it, or grounded and brought near it? Does it show grounds for Franklin's belief that the charge from the pointed wire on his kite would not harm him?

6. It is found that *the electrostatic capacity* of a condenser, i.e., its ability to accumulate a large charge under given conditions, is directly proportional to the area of its coatings, inversely proportional to the thickness of the dielectric between the coatings, and also depends on the material of the dielectric. If a charge is of the nature of a strain in the dielectric, can you see a reason for each of these three relations?

7. The dielectric of a condenser is often found to have a residual charge, i.e., a second discharge is obtained from it when a little time has elapsed after the first. Does this fact indicate that the charged dielectric is in some such condition electrically as an elastic body is mechanically when it is compressed, stretched, or twisted?

8. Can you show that an electrostatic charge has potential energy, like a strained spring? What work is done to store this energy?

SUGGESTIONS TO STUDENTS

1. Read Tyndall's *Elementary Lessons in Electricity* (Appletons, New York) for a fascinating account of electrostatic phenomena, with many practical hints for those who wish to make experiments inexpensively for themselves. Read also Hopkins's *Experimental Science*, pp. 359–391.

2. Find out what you can about the early experimenters in the field of electricity, especially Gilbert, Franklin, and Faraday. The following books contain much that may interest you, and that you can find easily if you will consult their indexes: Benjamin's *The Intellectual Rise of Electricity* (Longmans, New York); Benjamin's *The Age of Electricity* (Scribners, New York); Cajori's *History of Physics* (Macmillan, New York); Arabella B. Buckley's *A Short History of Science.*

If you have access to a good cyclopedia, consult it often for further information about the men and the subjects mentioned in these pages. The *Encyclopedia Britannica* is especially strong on the side of science.

3. If you will repeat for yourself the experiments mentioned in this chapter, and as many of those made by your teacher as you can, you will find that by thus becoming somewhat of an independent experimenter, you will not only increase your ability to understand the lessons of this course, but you will also get a great amount of pleasure out of it, and acquire a kind of skill that may be of great service elsewhere.

CHAPTER XI

MAGNETISM

199. Lodestone and the Compass. Having learned in the last chapter some of the fundamental facts concerning electric charges and their properties, we will now take the next step toward finding out how the dynamo operates; and seek to discover what a magnet is, what an electric current is, and what relations exist between electricity and magnetism.

It is interesting to note in the first place that the attraction of a magnet for iron has been known from time immemorial; for magnetic iron exists in nature in the form of the mineral called magnetite, or lodestone. This mineral was known to the Egyptians and Greeks long before the Christian era, for in their writings we find them speculating about its attraction for iron.

Besides the attraction of lodestone for iron, little was known of magnetism, and no use was made of it, until the introduction of the compass in Europe by the Arabs, about the year 1200 A.D. A COMPASS, as is well known, consists of a small strip of steel, which is first magnetized by rubbing it from end to end on a lodestone, or any strong magnet, and is then suspended on a pivot so that it is free to turn in a horizontal plane. Such a magnetic needle always tends to set its length in a north and south direction, and therefore it has been used by all civilized people for determining the north and south line. Since every magnet, when freely suspended, like the compass needle, settles in a definite position, which is nearly north-south, the magnet is said to have *polarity*. The end which points toward the north is called its NORTH-SEEKING POLE, and the other end, its SOUTH-SEEKING POLE.

The reason for this property of the magnet was not known until the time of Gilbert. He conceived that *the earth is itself a great magnet, having its magnetic poles near the geographical*

poles. To demonstrate that his theory was sound, he made a sphere of a piece of lodestone, and showed that a small magnetic needle, when near the surface of this miniature model of the earth, points in directions similar to those in which it points at corresponding places on the surface of the earth (Fig. 112).

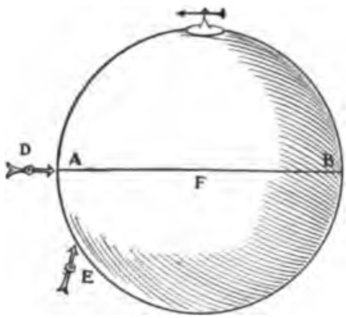


FIG. 112. GILBERT'S MODEL OF THE EARTH

200. Magnetic Curves. This study of the direction in which a small, freely-suspended magnet comes to rest in the neighborhood of a large one, is of great interest and importance. The experiment may be performed as Gilbert performed it, by placing a small suspended magnet at various points near the large magnet, and marking at each point the direction in which it comes to rest.

The same result may be accomplished more quickly by covering the magnet with a sheet of cardboard, and sifting fine iron filings over it. When the cardboard is lightly tapped, each bit of iron acts as a small compass needle, placing itself with its axis in the same direction as would a compass needle. Figure 113 is a photographic reproduction of the result. It will be noted that the iron filings trace well defined curves. These curves indicate the direction of the magnetic force at every point about the magnet. Many of these curves appear to begin at points on the magnet near its end. If the card were large enough to show them all entire, they would all appear to end at points on the magnet and near its other end. There are two points, near the ends of the magnet, toward which the lines of force appear to converge. They are called the POLES. We also note that the

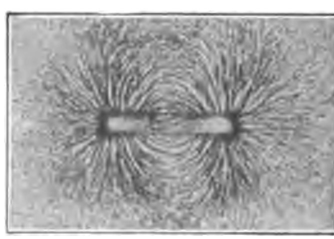


FIG. 113. MAGNETIC FIELD OF A BAR MAGNET

curves do not intersect each other. It is customary to think of the lines represented by the filings, as passing through the magnet and forming closed curves.

201. Magnetic Field. Now, the behavior of the iron filings tells us that the space about the magnet is permeated with magnetic forces, whose directions are indicated by the curves. This space around the magnet is called a MAGNETIC FIELD. The lines traced by the filings are called lines of magnetic force, because they show the direction in which a free north-seeking pole would move at any point in the field. This direction that a north-seeking pole would take is often shown by an arrow point.

Let us now consider the field of force produced by two magnets. To do this, we place two magnets under the card, and let the iron filings trace the directions of the lines of force as before. Fig. 114 shows the result when the adjacent poles of the magnets are of opposite kinds. It will be noted that the lines of force about each magnet are distorted, and that some of the magnetic lines appear to pass through both magnets. Since experiment shows that unlike magnetic poles always attract each other, we are led to conclude that the magnetic force is a kind of tension along the lines of force, as if these lines were elastic and were trying to shorten themselves.

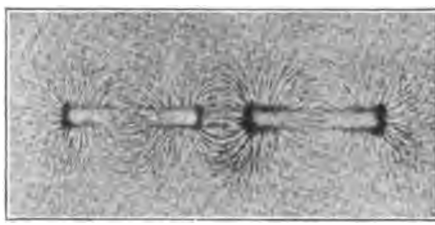


FIG. 114. UNLIKE POLES

If we investigate the shapes of the lines when like poles are placed near together, we obtain the curves shown in Fig. 115. In this case it will be noted that, though the field of each magnet is distorted by that of the other, yet none of the lines of either magnet enter the other. Since experiment shows that like magnetic poles always repel each other, and since, in the space between the two magnets, the lines proceed parallel to each

other and in the same direction, we are led to think of the lines of force as repelling each other in a direction at right angles to their lengths.

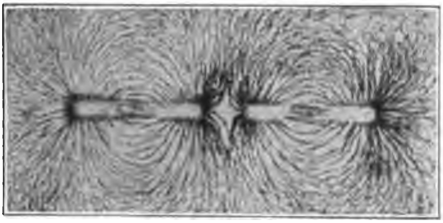


FIG. 115. LIKE POLES

We shall therefore adopt the hypothesis that *a magnet is, in some way, able to produce a strain in the medium about it, that this strain has a definite direction at every point, and consists of a tension in the direction of the lines of force, and a repulsion at right angles to that direction.*

202. Permeability. The diagrams with the iron filings enable us to form a picture of the attraction between a magnet and a piece of iron. For when we place a piece of soft iron between two unlike magnetic poles, the shape of the field of force is that shown in Fig. 116. It will be noted that the field is distorted, and the lines of force are concentrated by the iron. Some of the lines of force that pass from one into the other go through the iron; and so we have attraction. The iron thus acts like a magnet, and so, in fact, it is. Its magnetism is said to be *induced.* So we see that *iron, when placed in a magnetic field, becomes itself magnetic by induction.*

Further, the fact that the lines are gathered in and led through the iron shows that the magnetic force acts more easily through iron than through the air. *This property of gathering in and conducting the lines of force is called permeability.* It is of great importance in connection with all kinds of apparatus in which magnetic force is used. By placing iron in the gaps between magnetic poles, the

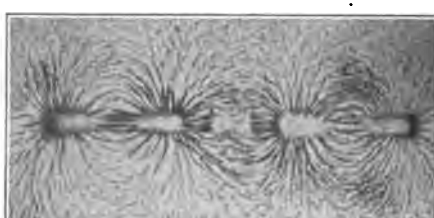


FIG. 116. PERMEABILITY

lines are kept from leaking out; and the force, instead of being dissipated, is concentrated within a small space. The more nearly we can approach to having a CLOSED MAGNETIC CIRCUIT of iron, the more efficient the apparatus will be. This is clearly shown by the fact that a horse-shoe magnet is much more powerful if it is fitted with a soft iron bar, or armature, as shown in Fig. 117.

203. The Earth's Magnetism. The study of the direction of the lines of magnetic force about the earth is of great importance to navigation, for it has been found that the direction of the earth's magnetic lines do not coincide with the true north and south direction, and that the deviation is different in different places. This fact was discovered by Columbus on his memorable voyage in 1492, and when it became known to his sailors, their fear drove them almost to mutiny. As the voyage progressed, it was found that the needle did not always point in the same direction, but varied from the direction of the pole star by different amounts; i.e., *the magnetic meridian does not always coincide with the geographic meridian.*

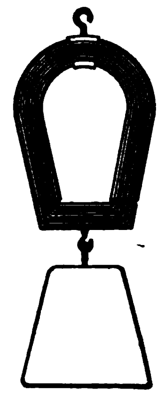


FIG. 117. CLOSED MAGNETIC CIRCUIT.

The departure of the needle from a geographic meridian at any point is its DECLINATION. The declination of the needle and the variation in its amount in different places are easily explained by assuming that the magnetic poles of the earth do not coincide with the geographic poles, and that the needle points toward the former, not toward the latter. At points east of the magnetic pole, the declination is toward the west, and vice versa. From a study of the declination, it has been found that the earth's north magnetic pole is situated in Boothia Felix, near Hudson Bay, in latitude about 70°.5 north, and longitude 97° west.

The position of the magnetic pole itself is not absolutely constant, and therefore we have a variation in the declination at different times in the same place.

The positions of the magnetic meridians, as well as other facts concerning the earth's magnetism, have been determined by different governments, at great expense, because the indications of the compass to mariners and surveyors would be very inaccurate without the corrections made necessary by the magnetic declinations.

204. The Unit Magnetic Pole. We may now inquire how we can compare the strengths of two magnets. In order to do this, we must adopt a magnetic unit. The definition of the unit pole adopted for scientific work is the following: *A unit magnetic pole is a pole of such strength that when it is placed at a distance of one centimeter from a like pole of equal strength, the two repel each other with the force of one dyne.*

205. Law of Magnetic Force. When the measurements are made in the units just mentioned, it is found that *the force with which two magnetic poles act on each other is equal, numerically, to the product of their strengths divided by the square of the distance between them.*

206. The Chief Characteristics of Magnets may be summed up as follows: 1. When freely suspended, a magnet takes a definite position, with its axis nearly north-south; 2, like poles repel each other, while unlike poles attract; 3, we conceive a magnet to be surrounded by a field of force, made up of lines of force whose directions are indicated by the curves traced with iron filings; 4, there is tension along the lines of force and repulsion at right angles to them; 5, a unit magnetic pole is one of such strength that when it is placed at a distance of 1 cm from a like pole, the two repel each other with a force of 1 dyne; 6, a piece of iron or steel, placed in a strong magnetic field, becomes a magnet by induction; 7, steel is harder to magnetize than soft iron, but it retains its magnetism better; 8, when a magnet is broken into pieces, each piece is found to be a magnet.

207. Electric Currents. At the beginning of our study of electricity we were led to ask how dynamos and motors work

but in order to find this out we had first to learn something of electricity in the static condition and some of the properties of magnetism. We shall not be prepared to understand the operation of electric machinery until we have learned a few facts about electricity in motion and the relation between electric currents and magnets.

In a general way we are all familiar with electric currents, for we know that they are used to operate the telegraph, the telephone, the electric light, the trolley cars, and even to ring our door bells. Electric currents were wholly unknown until the beginning of the nineteenth century.

The first knowledge of current electricity was derived from the researches of Alessandro Volta (1745–1828), who was professor of physics at the University of Pavia. Italian medical men had been much interested in investigating the effects of the electric shock on animal and human subjects; and Galvani, professor of anatomy at Bologna, was experimenting with the legs of a frog. He found that when he twisted together the ends of two wires made of different metals, and then touched a muscle and a nerve of a dead frog's leg with the free ends of the wires, the muscle would contract convulsively, and the legs would kick as if they had been brought to life. The greatest excitement followed this discovery, and the most extravagant hopes were entertained. It was thought that electricity possessed the principle of life, and would cure all ills.

Volta had already done much experimenting in electrostatic induction. He believed that the electricity which caused the frog's legs to kick was generated at the contact of the two dissimilar metals, and not in the frog's leg, as Galvani contended. By a series of very interesting experiments he proved that the charge could be obtained from two different metals immersed in a liquid and that it did not originate in the frog's leg.

208. The Voltaic Cell. This discovery was announced in the year 1800. While seeking still further to increase the electric output of his apparatus, Volta invented the simple cell which goes by his name, and which has been but slightly modified in

the best forms of modern commercial cells. This cell consists of a copper and a zinc plate, each terminated by a wire and placed face to face, but not in contact, in a jar of dilute sulphuric acid. By joining a large number of such cells IN SERIES, i.e., the copper of the first to the zinc of the second, the copper of the second to the zinc of the third, etc., effects of considerable power were obtained. His electroscope showed that the wire attached to the copper was positively charged, and that attached to the zinc negatively charged.

In 1800, only a few weeks after Volta had written of his researches to the Royal Society at London, two members of that society, Carlyle and Nicholson, were experimenting with Volta's apparatus, and discovered that if the terminals were placed in water containing a little sulphuric acid, the water would be decomposed into its constituent gases, oxygen and hydrogen, and would be so decomposed continuously. This decomposition of a chemical substance by electricity is called ELECTROLYSIS. We shall learn more of voltaic cells and electrolysis in Chapter XIII. It interests us just now, because in this way it was first found that the voltaic battery can produce, and transfer along a wire, a constant supply or CURRENT OF ELECTRICITY. Thus electricity was at once transformed from a subject of purely intellectual research into a powerful means of investigation in every branch of natural science, and a source of energy whose practical uses are so numerous and far-reaching that the boldest imaginings of that time are surpassed by the realities of the present.

209. Electromagnetism. Now, although the method of producing continuous currents had been discovered, no one had been able to prove that there was any relation between such currents and magnetism. That such a relation exists had been suspected, and it was discovered in the year 1816 by Hans Christian Oersted (1777–1851), professor in the university at Copenhagen. Oersted had given much study and thought to the voltaic battery and the possibilities of proving the long suspected connection between the electric and magnetic forces. In a moment of inspiration, while lecturing before his class, the idea occurred to him of joining

the wires from a battery above a suspended magnetic needle, the wire being parallel to the needle but not touching it. The needle instantly turned on its axis, and set itself at right angles to the wire. He reversed the current, and the needle turned in the opposite direction. He had shown that an electric current possesses magnetic properties, in that it can move a magnet. He interposed metals, glass, and other materials between the current and the magnet, but found that none of them prevented the action of the current on the magnet. Later it was learned that iron will screen the magnet from the effects of the current, though none but magnetic substances, such as iron, have this effect.

Oersted's great discovery was published in 1820, twenty years after Volta's. The new territory which it opened was immediately occupied, and other discoveries quickly followed. To the untrained and unthinking mind, Oersted's discovery might seem of little importance; but let us see what we may learn from it by the scientific method of inquiry.

210. The Current Has a Magnetic Field. From the fact that the needle always takes a definite position with reference to the direction of the electric current, we have a right to infer that the current has a magnetic field—that it is, in fact, a magnet.

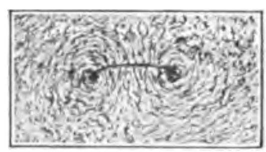


FIG. 118. THE CURRENT HAS A MAGNETIC FIELD

In order to test this inference, let us take a wire conveying a strong current, and dip it into a box of iron filings. The filings cling to it, and if carefully examined, will be seen to cling to one another so as to form a number of rings. They do not stand out radially from the wire as they do from the poles of a magnet, but are like a lot of curtain rings strung along the wire. Break the current, and they fall off. The field is instantly destroyed. In order more conveniently to investigate the form and extent of the field, let us pass the wire up through a small hole in a smooth board, and then down through another hole, close the circuit, sprinkle filings, and tap the board. The filings are seen to jump into

concentric rings around the two portions of the wire (Fig. 118). The lines of force, then, are circumferences of concentric circles, whose planes are all perpendicular to the direction of the current.

In which direction will these forces cause a north-seeking pole to move? If we place a pocket compass at various points around the wire, and mark by short arrows the directions of the needle at these points, using the arrow tips to denote the direction of the north-seeking pole of the needle, we shall see at once, from the two maps, that *when we look along the wire in the direction in which the current is going;* i.e., from copper to zinc, *the direction of every line of force is that in which the hands of a clock revolve.* Where the current is coming up through the board, the lines of force appear to circulate counter-clockwise, but that is only because we are facing the current so that it is moving toward us. If we were to get down under the board and look upward, the lines would appear to be clockwise as before.

211. Test for the Direction of a Current. This rule enables us to predict the direction which the needle will take when placed near a current, and conversely to tell the direction of the current, if unknown, by the direction which the needle takes. Thus, if we explore the field around the wire and find that there is a deflection of the north-seeking pole clockwise with reference to it, we know that there is a current traversing the wire, and that it is flowing away from us. Conversely, if the deflection appears counter-clockwise to us, then the current must be coming toward us.

212. How are Currents Related to Magnets? Notice again the field of the current-bearing loop. Look along the top of the loop in the direction of the current. The positions taken by a compass needle show that all the lines go out of the left-hand face of the loop, and enter the right-hand face, curving around on the outside of the loop. Referring back to the field of the bar magnet (Fig. 113), does not this suggest that the right-hand face of the loop is a north-seeking pole? Make another loop with a smaller diameter and with the lines closer together, the field smaller and stronger.

Can we not increase the strength of the field by making a coil of several loops, so as to get the lines of all the loops into the same space? On trying this, we find the needle and filings more strongly affected, and the map, Fig. 119, much more distinctly like that which a magnet ought to give us, if it were made short and very thick compared to its length.

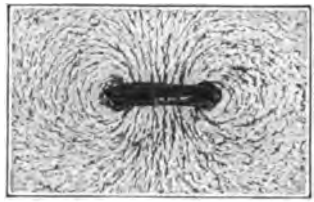


FIG. 119. MAGNETIC FIELD OF A COIL

Follow up this suggestion by making a new coil in the form of an elongated helix, and more like the bar magnet in shape (Fig. 120). The field of this helix looks almost exactly like that of the bar magnet. Each little loop has its own small circular lines, but inside and outside the helix they combine to form strong resultant lines which are closed curves exactly like those of the magnetized steel bar.

We now have a right to infer that our current-bearing coil, or helix is a veritable magnet. Will it do the things that a magnet does? We have seen that it strongly attracts iron, and that like the whole coil every little loop does so, though less strongly. But, if freely suspended, will it point north and south? If placed near another suspended magnet or another current-bearing loop or helix, will the poles mutually attract? Are the effects strongest at its poles? The requisite experiments show that in every case the answer is "yes," *a current-bearing conductor is a magnet.*

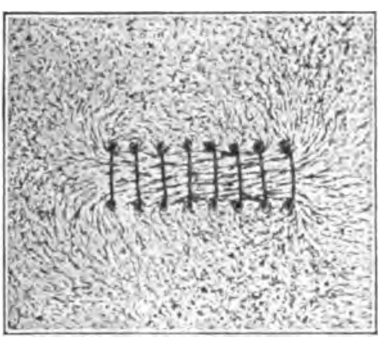


FIG. 120. MAGNETIC FIELD OF A HELIX

213. Permeability—Electromagnets. We found (Art. 202) that soft iron placed in the field of a bar magnet, gathers in the magnetic lines, i.e., concentrates the force. Ought we not, then, to be able to increase greatly the force of the coil or

the helix by putting soft iron into it? On trying the experiment, it is found that the iron core adds immensely to the magnetic strength of the coil or the helix. We have thus been led to construct an electromagnet (Figs. 121, 122). Such a combination has two great advantages over a steel magnet: 1. Since its strength is proportional to the strength of the current, and also to the number of turns of the wire around the coil, we can thus make a very strong magnet. 2. We can magnetize it and demagnetize it at will. We may now ask, can it not be made to do various kinds of mechanical work at some distance from the point at which the electric circuit is opened and closed?

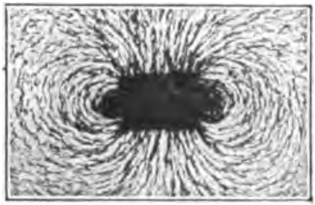


Fig. 121. An Iron Core Strengthens the Field

This question leads us into a field in which notable discoveries were made by an eminent American, Joseph Henry (1799–1878). Henry was a busy teacher in the Albany Academy, and afterwards professor at Princeton, and Secretary of the Smithsonian Institution at Washington. He was incessantly overworked, but in spite of that fact he made researches that brought him high honors among scientists all over the world.

214. The Electromagnet and the Telegraph. Popularly, Henry's name is scarcely known in connection with the electromagnetic telegraph. The credit for that invention has been given by the American public to Samuel F. B. Morse. But what Morse did was simply to combine principles and apparatus discovered by Henry and others, and make the public believe in the possibilities of the electromagnetic telegraph. Important as were the services of Morse, the honor of the

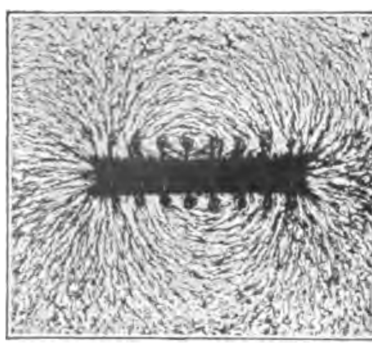


Fig. 122. Field of an Electromagnet

invention belongs to Henry, who, like his great English friend Faraday, was content to make fundamental scientific discoveries and leave their practical applications to others. Recognizing the possibilities offered by the electromagnet, physicists everywhere were trying to construct an apparatus for signaling at a distance by means of it, and had given it up, because the current became so weak after traversing a few hundred feet of wire, that the magnet would not move anything. A member of the Royal Society had tried, and claimed that he had demonstrated the impossibility of the scheme. Henry read this paper and to his mind it was a challenge; so with characteristic American audacity he set out to accomplish the impossible.

Henry discovered that if he insulated his wires by covering them with silk, and then wound many turns of fine wire on the core as thread is wound on a spool, this LONG COIL MAGNET would work at great distances from the battery, even though the current was very weak. Using the horse-shoe form of core, he pivoted in front of it a lever, carrying a little soft iron bar or armature. The lever terminated in a clapper, which would strike a bell when the armature was attracted. Placing this apparatus in circuit with a battery of many cells in series, he was able to make the clapper strike the bell whenever he closed the circuit, and fall back in obedience to the tension of a spring whenever he destroyed the magnetic field by breaking the circuit.

Thus, representing each of the letters of the alphabet by a combination of bell strokes, a message could be spelled out. In the instrument which Morse afterwards patented, the lever carried a pencil which it pressed against a moving roll of paper when the armature was attracted. By making short and long contacts, dots and dashes were made on the paper strip; and in the *Morse Alphabet* each letter was represented by a combination of dots, dashes, or spaces, thus:

a	g	t	m	l
. –	– – .	–	– –	—
H	e	n	r	y
. . . .	.	– .	. . .	. . . .

The original idea of the Morse recorder survives in the "ticker" of the stock exchanges and brokerage offices to-day, but the instrument now generally used for commercial telegraphy is the speedy and more familiar "sounder," Fig. 123. Each signal of the sounder is begun by the lever *L* clicking against a stop *P* when the armature *A* is pulled down on closing the electric circuit, which passes around the core of the magnet *M*. It is ended by the lever clicking against another stop *Q* when the circuit is opened, and the armature, released from the magnetic attraction, is pulled back by a spring *S*. Thus the differences between the dots and the dashes are represented by differences in the time intervals between the double clicks of the sounder.

FIG. 123. TELEGRAPH SOUNDER

215. The Relay. Neither a bell nor a sounder will work over lines many miles long, however, because the current, weakened by the resistance of the long circuit of wire, can not produce enough energy to do the heavy work. Henry overcame this difficulty by devising the RELAY. He found that a single battery cell with large plates would produce great effects when the circuit was of short, thick wire, offering little resistance to the current, and that powerful magnetic effects could be produced with such a battery by using a few turns of thick wire around the core.

Suppose we have such a battery and magnet at the receiving station connected in a circuit by short, thick wires. This magnet *SM*, Fig. 124, with a strong current from a LOCAL BATTERY *LB* can operate the sounder lever *SL*. Now let us place in the MAIN LINE CIRCUIT *L* a magnet *RM* wound with many turns of fine wire. The main current, though exceedingly weak, can furnish enough energy to this sensitive magnet, so that it can work a very

light armature *A*, Fig. 125, with its lever. Now, this light lever of itself can neither do any printing nor make any noise, but it may easily make or break an electrical contact between two little points *CC*, one of which projects from the lever and the other from a fixed metal post.

Suppose we cut the wires of the local circuit, and join one of the cut ends to the contact point *C* on the relay lever, and the other to the fixed point *C* against which it strikes, as in Fig. 124. All we have to do now to work the big sounder, by means of the key at the distant station, is to send our weak main line current *L* around the core of the sensitive relay magnet *RM*. The relay armature is instantly attracted and the contact point on the light armature lever strikes the fixed contact point *C*. This completes the local circuit, and lets the powerful local current go around the core of the sounder magnet. The sounder lever is drawn down and makes a loud click. Open the main line circuit; and the relay armature, in obedience to the tension of its spring *S*, Fig. 125, flies back. This separates the contact points *CC*, thus opening the local circuit; and the local current ceases to flow around the sounder magnet. Instantly, in obedience to the tension of its spring, the sounder lever flies back. Thus the powerful local current is released or throttled at will by the operator at the distant sending station.

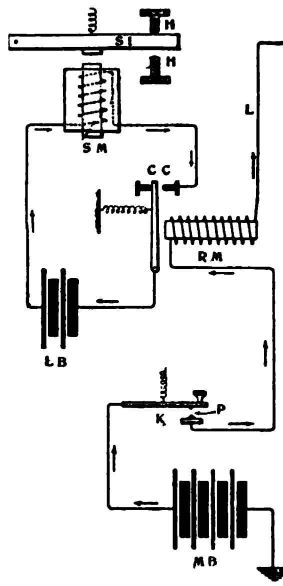


FIG. 124. TELEGRAPH DIAGRAM, ONE STATION

This was the principle of Henry's relay. It is something like using the weak current to pull a hair trigger, and discharge a big gun. We make a powerful source of energy do heavy work, but we control it by a weak current from a distant source. Long distance transmission of electric signals of any kind is commercially a practical impossibility without the relay.

216. The Telegraph Key. The opening and closing of the circuit is accomplished by a key, Fig. 126 and K, Fig. 124, worked by the thumb and first two fingers of the operator. It is simply a lever with a contact point P attached to its under side, and striking another contact point attached to the metal base, but insulated from it. In telegraphic practice there is a key, relay, sounder, and local battery at each station; and when one key is worked, it operates all the keys and their sounders simultaneously. Accordingly, each station has its own signal letter or call by which the attention of its operator may be attracted.

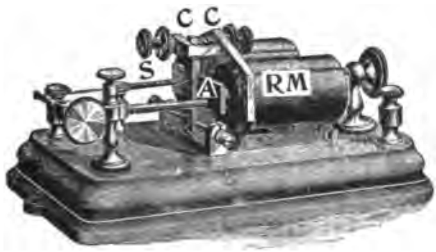


FIG. 125. TELEGRAPH RELAY

217. Grounding the Wires. Before practical telegraphy had progressed very far, the fact that the earth conducts electricity suggested the possibility of using the earth as part of a telegraphic circuit. This idea is realized in the following manner: One of the battery wires is joined to a metal plate or pipe G, Fig. 124, buried in damp ground, and the farther end of the line wire is grounded in the same manner. The current which has passed from the other pole of the battery along the line wire and through all the instruments, proceeds to the ground connection at the farther end, and completes its circuit through the earth. It is somewhat as if we had a pump on one side of a lake, which would lift water and force it through a pipe to a place on the other side of the lake, where it might do work in turning water motors, and then return to

FIG. 126. TELEGRAPH KEY

the lake when its energy was exhausted. Just as in the case of the water there is a *current* through the pipe, and a *drift* across the lake, so in the case of the grounded electric current, there is an electric current in the wire, and an electric drift back along the ground. As the ground resistance to the electric drift is very small, this arrangement not only saves copper, but also saves energy, as we shall see later on.

218. The Electric Call Bell. The electric call bells and buzzers in common use work very much like a telegraph sounder, except that the armature which carries the bell clapper, or the reed which makes the tone of the buzzer, is made to vibrate automatically, as long as the current is continuously supplied to it by keeping a push button depressed.

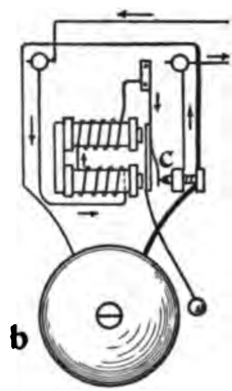


FIG. 127. THE ELECTRIC BELL

The construction and operation of the bell will be understood from the diagram, Fig. 127. The path of the current may be traced by the arrows. When the button is pressed, the circuit is closed, the armature is attracted by the electromagnet, and the clapper strikes the bell; but at the instant when this happens, the contact breaks between the contact-screw C and the armature, because the armature has been pulled away from the contact screw. The breaking of the electric circuit at this point destroys the magnetic field; and the armature, no longer attracted, is pulled back by the elastic spring on which it is mounted. Thus it touches the contact screw; and since the circuit is now again closed, the operation is repeated. Thus the armature vibrates automatically, receiving its periodic impulses from the periodic magnetic field.

219. Galvanometers. Shortly after Oersted's discovery, Ampere suspended a delicate magnetic needle inside a coil of wire, and used it for detecting the presence of a current and estimating

its intensity. Such an instrument is called a galvanoscope or a galvanometer. When the plane of the coil is placed in the magnetic meridian, the needle, directed by the earth's magnetism, remains in the plane of the coil—i.e., with its axis north-south—unless a current is passed around the coil. In this case, the magnetic field of the current exerts a moment of force on the needle, tending to pull it around so that it will lie along the lines of force of the coil—i.e., east-west; and the angle through which the needle is turned from the north-south direction depends upon the strength or intensity of the current.

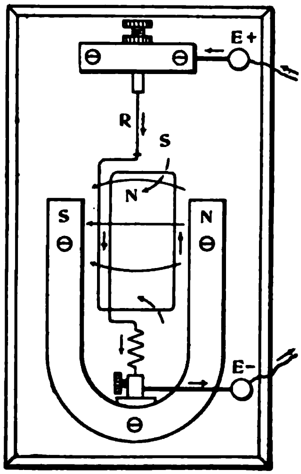


FIG. 128. D'ARSONVAL GALVANOMETER

Many different types of the galvanometer are now in use. The most useful are: (1) The *astatic*, a galvanometer which may be made so sensitive that it can be used for receiving signals sent through the submarine cables, (2) The *tangent galvanometer*, commonly used in measuring currents of considerable strength, and (3) The *D'Arsonval galvanometer*. This last form we shall pause to examine, both because it is the best for all-around use, and because it will assist us in making another important step in our inquiries.

In the other forms we have a fixed coil with a needle suspended at its center. In the D'Arsonval (a simple form is shown in Fig. 128) we have a large, fixed, U-shaped steel magnet, with a small coil suspended between its poles. The current to be tested enters at the binding post $E+$, passes into the coil through a thin metallic ribbon R by which the coil is suspended, and passes out below by a similar ribbon. When the current passes through the coil, the latter becomes a magnet and tends to set its lines of force in the same direction with those of the large fixed magnet. The magnetic lines of the magnet and of the coil

are represented by arrows. The deflections of the coil are read by a pointer moving over a circular scale, or by the displacement of a beam of light, reflected from a mirror attached to the coil. With all galvanometers the deflection of the suspended coil or magnetic needle is greater when the current is greater. The exact relation between the deflection and the strength of the current depends on the kind of galvanometer.

220. A Suggestive Experiment. Let us pass the current from a single voltaic cell through the coil of D'Arsonval's combination. The coil, Fig. 128, hangs with its plane parallel to that of the U-magnet. As soon as the current passes, one face of the coil becomes a north-seeking pole and the other a south-seeking pole, as is suggested by the short, curved arrows which represent two of its lines of force. The like poles of the magnet and coil repel each other and their unlike poles attract each other. So the coil turns through a right angle, and stops with its north-seeking face next the south-seeking pole of the magnet.

If we reverse the current in the coil, its magnetic poles will be reversed also; so that each pole of the coil will be facing a like pole of the magnet. Now, we know that like poles repel each other and unlike poles attract; therefore we may infer that the magnetic forces will cause the coil to face about, so that each of its poles will be adjacent to an unlike pole of the magnet. When we shift the battery wires, so as to reverse the current through the coil, we find that the coil turns through half a revolution, just as we inferred.

With the exercise of THE SCIENTIFIC IMAGINATION we may now arrive with a single bound at the principle of one of the greatest inventions of all time. Can we, by any device, modify this apparatus, so that when the current is again reversed, so as to be in its original direction, the coil will turn through the second half of the circle, instead of going back on its path? In other words, can we, by reversing the current at the end of a half turn, cause the coil to rotate continuously, instead of merely vibrating back and forth through a semicircle? If we can but do this, what boundless possibilities wait on the labors of inventors! Electric

motors, turning machinery miles away from the source of current —electric power, distributed by wires everywhere and converted into mechanical work in the factory, the street or even in the private home—power in just the amount wanted, available at the instant when it is wanted, and the expense stopped the instant the power is not needed, by merely turning a switch; these are among the achievements which the device suggested would make possible.

This was the dream that possessed the imaginations of scientists at the beginning of the second quarter of the nineteenth century. Before the middle of that century the necessary discoveries had all been made. The principles were in the hands of the inventors, and in another forty years thousands of motors were in successful operation.

221. The Electric Motor. In our electrical studies thus far we have learned some of the most important facts and principles that were known when Faraday and Henry were seeking to discover the principles of the electric motor. Suppose that the motor exists only in our imaginations as it then did in theirs, and returning to our magnet and little suspended coil, let us see what we can discover.

Let us send the current into the coil and observe what happens. The coil rotates through a right angle, and sets its faces opposite the poles of the magnet; but it does not stop at the instant when it reaches that position. On account of its inertia, it goes beyond that position. The magnetic force and the torsional elasticity (twisting force) of the suspending ribbon stop it and bring it back. After a few oscillations it settles into the position just mentioned, in which its lines of force coincide with those of the magnet.

If we can manage to reverse the current just as it passes this position, and if we can also free the coil from the torsion of the suspending ribbon, then instead of oscillating and settling in the definite position mentioned, it will go on around through half a revolution more. But on account of its inertia, it can not stop itself; and if we again reverse the current just at the right instant, it will continue to rotate through another half turn. Thus it appears

that if we can reverse the current just at the end of each half turn, and if we can also get rid of the twisting force of the suspension, we may produce continuous rotation.

By a little practice in timing the reversals of the current, we may easily make the coil of a galvanometer execute one or two complete revolutions, for in this case it stops only when the suspension ribbon gets twisted up. Evidently we must overcome this mechanical difficulty. This may be done by inventing a sliding contact.

222. Armature, Field Magnets, Collecting Rings, Brushes. We can arrange such a sliding contact by fastening the coil to a steel axis or shaft, whose ends turn freely in suitable bearings. It will be better also to mount the shaft horizontally rather than vertically (Fig. 129). We shall now call the coil an ARMATURE and the U-magnet the FIELD MAGNET. Then we may place near one end of the axis a pair of metal rings, and solder the ends of the coiled wire to the rings. These rings, $R+$ and $R-$, we shall call the COLLECTING RINGS. In order to let the current in at one of these collecting rings and out at the other, we attach the battery wires to two light metal springs, $B+$ and $B-$, which we will call the BRUSHES, and hold these brushes so that one of them shall rub lightly against each of the collecting rings. While making the apparatus, there is something very important which we must remember and provide against. The steel shaft, or axis, about which the coil rotates, is a conductor; and if our collecting rings were in metallic contact with it, the greater part of the current, coming to the first ring, would take the short cut along the steel shaft from this ring to the other, and thence back to the battery. Only a very small fraction of it would go around the turns of the coil. Therefore our collecting rings must be mounted on a sleeve

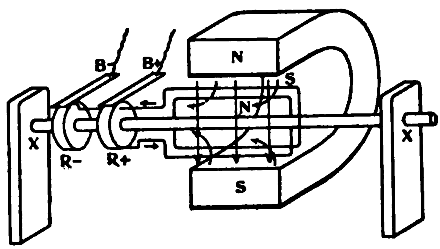


FIG. 129. THE COIL IS FREE TO ROTATE

of hard rubber which will *insulate them from the armature shaft, and from each other.*

Having ready this apparatus, we make the experiment—first touching the brush $B+$ to ring $R+$, and the brush, $B-$ to $R-$; then reversing the brushes, so that the brush $B+$ touches $R-$ and the brush $B-$ touches $R+$. If we are skillful enough in shifting the brushes at the right instant, we secure continuous rotation. We have, then, experimentally demonstrated the possibility of the electric motor; but our apparatus is as yet very crude and inefficient, even for a toy. What we need now is either an alternating current, or a device for making the coil itself reverse the current from the battery. We shall not here consider the first proposition, because when the invention had reached this point nobody had even dreamed of producing an alternating current; i.e., one that would flow first in one direction and then in the other.

223. The Commutator. Since the necessary motion of shifting is only relative, it makes no difference whether the brushes reverse their positions while the positions of the collectors remain fixed, or whether we attach the brushes to a fixed insulated support, and let the collectors reverse their positions. How can we do this?

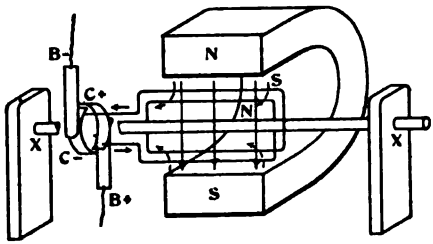


FIG. 130. THE MOTOR DIAGRAM

A little imagination and some careful thinking will help us to hit on a plan. Manifestly, when the coil turns, one-half of the ring is passing you during the first half of the rotation and the other half of the ring is passing you during the other half of the rotation. It is, then, easy to solve the problem by discarding one of the rings, splitting the other ring parallel to the shaft XX, and attaching the two ends of the coiled wire to the two separated halves of the ring.

We must now fix the brushes so that they touch diametrically opposite points of the split ring. This arrangement is shown in Fig. 130, the shaft being broken away and the split ring ($C+$, $C-$) being enlarged for the sake of clearness.

If the brushes are properly placed, it will be seen that at the instant when the armature comes into the position where its poles (i.e., its flat faces) are nearest the attracting poles of the field magnet, the half ring $C+$ will shift from the brush $B+$ to the brush $B-$, and the half ring $C-$ will shift from the brush $B-$ to the brush $B+$. The result is that the current through the armature coil is reversed, and so its poles are also reversed. Therefore the armature poles are repelled by the poles of the field magnet, and so the coil continues to rotate. Since a similar reversal of the current will occur at the completion of every half turn of the armature, the latter will continue to turn so long as the necessary power is supplied.

This split ring device we call a COMMUTATOR, because it reverses the current through the coil whenever it shifts the brush contacts.

The problem, then, of providing a sliding contact, and a device for automatically reversing the current through the armature, is satisfactorily solved in our imaginations. If now we have the completed apparatus before us, and send into the armature a current of sufficient power, we shall be rewarded for all our labors by seeing the armature spin merrily round and round. The solution of the problem is now a reality.

224. From Toy to Practical Machine. We must not stop at this point, however, for our motor is very weak, being barely able to run itself, to say nothing of driving machinery; and, furthermore, we already possess knowledge which we may apply to increase its efficiency. We know that the turning force is that of the two magnets, and we have learned some ways of making these magnets stronger.

1. We know of the permeability of soft iron (Art. 202), so the next step is obvious. It is to fill the coil with a soft iron core.

2. We know that an electromagnet will have a stronger field; so we can further increase the strength of the field, by substituting an electromagnet for the steel magnet as in Fig. 131.

FIG. 131. MOTOR FRAME, POLES, AND FIELD COILS

3. We may make our magnet shorter and thicker, with large pole-pieces, shaped so as to embrace the coil as closely and completely as possible without interfering with its rotation (Fig. 131). This fills the air-gaps as nearly as may be with soft iron, and gathers in the lines of force; so that more of them pass through the armature, instead of leaking away where they will not be utilized. By a little further modification in shape, we may make the field completely "iron-clad," as represented in Fig. 132. This is one of the modern forms and leaves little to be desired in the matter of packing the lines of force into the effective space, and protecting the machine from anything that might get into it.

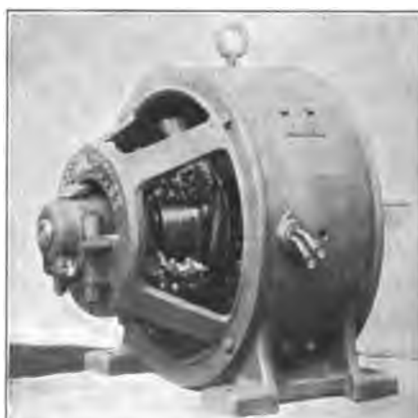


FIG. 132. A MODERN MOTOR

4. We may greatly increase the effectiveness of the armature by winding on the core another coil at right angles to the first, so that when one coil is turning into the least effective position, the other is turning into the most effective position. The number of

coils may be increased to four, six, eight, and so on (Fig. 133), till all the available space on the core is filled, but the commutator must be then re-divided so as to have one segment for each coil. Increasing the number of coils not only increases the magnitude of the force, but also makes it approximately uniform in intensity.

Fig. 133. Motor Armature, Commutator, and Shaft

225. Winding the Field Magnets. The current may be passed by the brushes through the armature coils and thence through the field coils (series winding, Fig. 134), or it may divide at the brushes, part going through the field coils and the remainder through the armature coils (shunt winding, Fig. 135), or both these methods of winding may be used together (compound winding, Fig. 136).

There are many other ways employed in the construction of modern motors, by which their efficiency is still further increased. Figs. 131 to 133 are photographs of one of the best modern types of direct current motor.

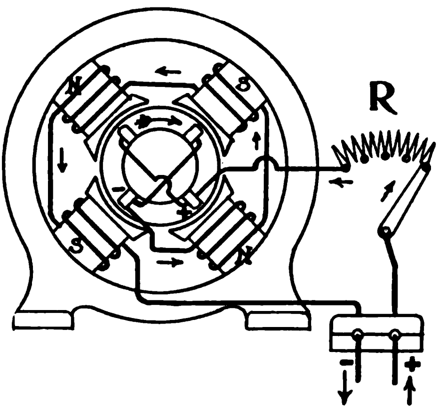


Fig. 134. Four Pole Motor, Series Wound

226. Ampere's Theory of Magnetism. A theory of magnetism which was proposed by Ampere is generally held at present, though it has received considerable extensions in order that it may describe some facts discovered since Ampere's time.

If we break up a magnetized knitting needle, we find that each part is a magnet, even to the smallest parts into which it may be

broken. There is no reason why we should believe that the smallest particles of iron into which a magnet can be divided, should fail to have their magnetic poles. Ampere, therefore, believed that the magnetic properties were permanently possessed by the molecules of the iron. The amended and extended hypothesis of Ampere is:

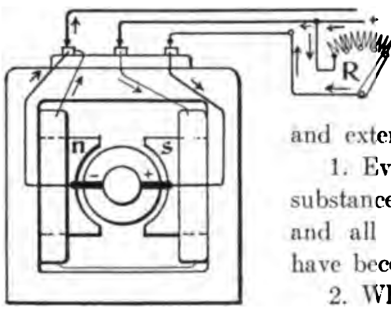


FIG. 135. TWO-POLE MOTOR, SHUNT WOUND

1. Every molecule of a magnetic substance is a magnet, possessing poles, and all the properties with which we have become familiar in large magnets.

2. When the magnetic substance is unmagnetized, the magnetic axes of the molecules are turned in all possible directions, so that their magnetic forces neutralize one another.

3. When the lines of force of an external magnetic field pass through the bar, the little molecular magnets tend to set themselves along the lines of the external field.

4. When the molecules are all as nearly parallel to the axis of the bar as is possible, the bar is said to be magnetically saturated.

5. The perfect magnetization of a molecule may be accounted for by supposing, either that an electric current is always circulating around it in a plane perpendicular to its axis, or that it has an electrostatic charge and is rapidly spinning. In view of the experiment described in the next article the latter idea seems to have the advantage.

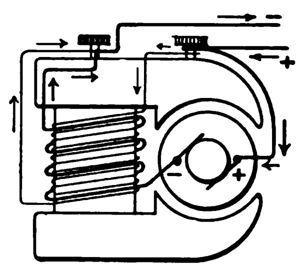


FIG. 136. TWO-POLE MOTOR, COMPOUND WOUND

227. Magnetic Field of Moving Charges. In order to find out whether a rapidly moving charged particle has a magnetic field, just as a current has, Rowland devised the following experiment: A metal disc was mounted so that it could be rotated about an axle perpendicular to its plane. When this disc is charged electrostatically,

and is also rotating rapidly, it is found that a compass needle near the disc is deflected just as it would be if a current of electricity were flowing in the path of the rotating charge. Thus we learn that *charged bodies in rapid motion produce electromagnetic effects, just as currents do;* and therefore we are led to conceive that a current may be simply a series of electrostatically charged particles darting rapidly along a wire.

The numerical relations obtained from this experiment are very suggestive. It was found that if a unit charge (Art. 191) moves with a velocity of $3 \times 10^{10} \frac{\text{cm}}{\text{sec}}$, the strength of the magnetic field produced is the same as that of a unit magnetic pole (Art. 204). We shall learn in Chapter XXII that both light and the electric waves, predicted by Maxwell and detected by Hertz (Art. 198), travel with this same velocity, $3 \times 10^{10} \frac{\text{cm}}{\text{sec}}$. These facts suggest the idea that there must be some intimate relation between electricity and light. In Chapter VIII we learned that heat and light are related phenomena. Therefore we may reasonably ask whether electricity, heat, and light may not be simply different manifestations of one and the same form of energy. This question will be considered further in the later chapters.

228. Energy of a Magnetic System. A magnet can not do work, unless mechanical work is done on it, or a current of electricity gives up some of its energy to it.

Thus, if a magnet has potential energy, so that it can attract a piece of iron to itself, work had to be done to store up this energy. Either the iron had first to be pulled away from the magnet, or the magnet had to be pulled away from some other magnet from which it got its magnetism.

If an electromagnet does work in moving a piece of iron, some of the current which energizes the magnet is used up in doing this work. Thus, if we pass a current through an electric lamp and a magnet coil, the lamp diminishes in brightness while the work is being done by the magnet. If we replace the lamp by a galvanometer, the deflection diminishes while the magnet is doing its work.

This is only another particular case of the general LAW OF THE

CONSERVATION OF ENERGY, which states that no energy can be created or destroyed.

SUMMARY

1. Lodestone is a compound of oxygen and iron, and is a natural magnet.

2. A magnet attracts iron, and when freely suspended, takes a definite position with its axis nearly north-south.

3. The magnetic properties of a magnet are strongest at its ends, which are called its poles.

4. Like magnetic poles repel each other, and unlike poles attract each other.

5. A magnet induces magnetic properties in every piece of iron or other magnetic substance which is brought into its neighborhood.

6. Every piece of a magnet is itself a magnet.

7. The poles of a magnet may be determined, if unknown, either by suspending it freely, or by presenting its poles to those of a magnet.

8. The space that surrounds a magnet is a field of magnetic force.

9. A vector drawn tangent to a line of force shows the direction in which a free north-seeking pole is urged at the point of tangency.

10. Lines of force are always closed curves which pass outside the magnet from its north-seeking to its south-seeking pole, and inside the magnet from its south-seeking to its north-seeking pole. These lines never intersect each other.

11. Magnetic forces act as if the lines of force were elastic threads which tend to shorten themselves lengthwise and repel each other sidewise.

12. If soft iron is placed in a magnetic field, the lines of force are gathered in and pass through it. This is due to permeability of the iron.

13. The earth is a great magnet, having one of its magnetic poles near Hudson Bay. This explains why a magnetic needle points nearly north-south.

14. The declination of the needle is explained by the fact that the earth's magnetic poles do not coincide with its geographical poles.

15. Variations in declination at a given place are caused by the small variations in the positions of the earth's magnetic poles.

16. A unit magnetic pole is one which, when distant 1 cm from an equal like pole, repels it with a force of 1 dyne.

17. The force between two magnetic poles, expressed in dynes, is equal to the product of their magnetic strengths divided by the square of the distance between them.

18. By chemical action a voltaic cell can supply a continuous current of electricity, which transfers energy along a conductor.

19. An electric current has a magnetic field, and can do mechanical work by moving a magnet.

20. Only magnetic substances act as screens to cut off magnetic force.

21. The magnetic lines of a current in a straight wire are circumferences of circles, whose planes are perpendicular to the direction of the current.

22. The direction of a line of force is always clockwise to an observer looking in the direction in which the current is going.

23. The magnetic field of a current-bearing helix is similar to that of a similarly shaped steel magnet.

24. This fact leads us to infer that a current-bearing helix will always behave like a magnet, and this conclusion has been fully verified by experiment.

25. Placing a soft iron core in a current-bearing helix increases the strength of its field. Such a combination is an electromagnet.

26. Electromagnets are used in telegraph sounders and call bells to send signals over short conducting lines, and in relays to send them over long lines.

27. Galvanoscopes are used to detect currents and determine their directions; galvanometers, to measure their intensities.

28. An electromagnet, mounted on an axis and placed between the poles of another magnet, may be made to rotate continuously. This is the principle of the electric motor.

29. The efficiency of an electric motor may be greatly increased

by arranging the soft iron parts so as to form a closed magnetic circuit, and also by increasing the number of coils in the armature.

30. According to Ampere's theory of magnetism, every molecule of a magnetic substance is supposed to be itself a magnet, because it carries an electric charge and constantly spins on its axis.

31. A unit electrostatic charge, moving with a velocity of $3\times10^{10}\frac{\text{cm}}{\text{sec}}$, is equivalent to a unit current and therefore produces a unit magnetic pole.

32. A magnetic system can do no work unless it has been supplied with energy, either by mechanical work, or by using up some of the energy of an electric current.

QUESTIONS

1. Describe a series of simple experiments for demonstrating the properties of magnets.

2. Explain how you can prove whether a substance is strongly magnetic or not. If it is a magnetic substance how can you prove that it is or that it is not a magnet?

3. What is a magnetic field? What is a line of magnetic force?

4. Describe the methods of mapping a magnetic field.

5. Sketch the appearance of the magnetic field of a bar magnet; of two like poles repelling each other and of two unlike poles attracting each other.

6. Sketch the effect on the field of introducing soft iron into the magnetic circuit. What name is given to this property? What are the effects on the properties of the field?

7. What is the magnetic meridian of a place?

8. What great principle was established by Oersted's discovery?

9. Describe experiments by means of which the magnetic field of a current can be shown.

10. How may we determine the direction of a current by means of a magnetic needle?

11. Sketch the field of a current-bearing loop. How is the magnetic strength affected by multiplying the number of turns of wire so as to make a compact coil?

12. What general fact may be inferred by the resemblance between the field of any closed electric circuit and that of a magnet?

13. Suggest a series of experiments by which the fact thus inferred may be verified.

14. What is an electromagnet? What advantages has it over a permanent magnet?

15. Diagram an electric call bell, trace the current, and explain its action.

16. What is the principle of the galvanometer? State its uses.

17. Diagram a D'Arsonval galvanometer and explain its action.

18. Describe the modifications that will convert D'Arsonval's combination into an apparatus producing continuous rotation.

19. What are some of the most important changes that must be made in the details of this apparatus in order to make an efficient electric motor?

20. Describe the experiment of the rotating charged disc and tell what conclusions we may draw from it.

21. Show that a magnetic system can have energy and do work only when energy has been supplied to it.

PROBLEMS

1. Write the expression that represents the magnitude of the force f in dynes, between a north-seeking magnetic pole of strength s and a south-seeking pole of strength s' placed d cm apart. Is this force attractive or repulsive?

2. When two south-seeking magnetic poles of 2 and 3 units strength, respectively, are placed 6 cm apart, with what force do they affect each other? Is the force attractive or repulsive?

3. What is the strength of a magnetic pole which exerts an attraction of 1,000 dynes on another pole which is distant 20 cm and has a strength of 25 units?

4. With the aid of outline diagrams, describe the construction and operation of the telegraphic key, the sounder, and the relay.

5. Diagram a telegraphic circuit of two stations, without relays, trace the current by arrows, and explain the operation of sending a signal.

6. Diagram a complete telegraph circuit for two stations, with ground connections, battery, keys, relays, local circuits, local batteries, and sounders. Trace the main line circuit around with black-ink arrows, and the local circuits with red-ink arrows. Explain the action of all the instruments when a key is worked.

7. Remove the cover from the push button of your electric door bell, examine the contact spring carefully to find out how it works, and make a diagram explaining its action.

8. Diagram a door bell battery of two cells in series, as you will find them connected (*cf.* Art. 218). To this diagram add one of the bell, and connect the battery with the bell by a wire. Add to the diagram a wire returning from the bell through a push button to the

battery, so as to make a complete circuit when the button is pressed. Trace the current through the circuit by means of arrows, and explain the action.

9. To your bell diagram, add several push buttons, as they would be used to ring the bell from different points of the house.

10. Make a diagram representing a bell circuit in which the current from the battery divides among several bells and reunites in a single wire which conducts it back to the battery. Put into the diagram one push button by which all the bells may be rung together.

11. Diagram an arrangement in which five bells are placed in a row, and a current may go from a battery through any one of the bells, from thence through a push button in a distant room, and from the push button to a common return wire which conducts it to the battery. This is the arrangement of the annunciator seen in the offices of hotels. There is a push button and an indicator for each room.

SUGGESTIONS TO STUDENTS

1. With a ten-cent toy magnet, a few sewing needles, knitting needles, some bits of broken watch springs, which your jeweler will give you, a few bits of cork, some sealing wax, and a small amount of ingenuity, you may verify all the properties of magnets, mentioned in Arts. 199-206.

2. If you have a little shop of your own with a few good tools, you may easily make yourself a simple D'Arsonval galvanometer, like that shown in Fig. 128, a working telegraph set, a small motor, etc.

3. You will find much helpful information about making such things in Hopkins's *Experimental Science; Electric Toy Making*, by T. O'Connor Sloan (Norman W. Henley & Co., New York), and in a series of little handbooks of the Bubier Publishing Company, Lynn, Mass.

4. Get the necessary information from an electrical supply store, and take charge of the electric bells in your home, keeping the battery in order and the bells in adjustment.

5. Examine the field windings of a toy motor. Is it shunt wound or series wound? See if you can change the connections so as to convert it from one style of winding to the other. A shunt winding requires many turns of fine wire a series winding few turns of coarser wire.

6. Connect a toy motor with a battery and a galvanometer, and note the deflection. Now hold the armature so it can not rotate, and see if the deflection is greater. If the first deflection is too great connect a shunt across the galvanometer terminals (*cf.* Art. 268, Chap-

ter XIII). Does the experiment show that the motor takes energy from the current while running?

7. If there is a shop in your city where electrical apparatus, such as motors, is built, or a store where such things are sold, visit it and find out what you can. Many interesting electromagnetic devices, toy motors, etc., are sold even in small electrical supply stores, and tradesmen are usually willing to explain them to any one who is interested. Make a short written report of what you learn.

8. What can you find out about trolley car motors, and how their power is transmitted to the car wheels?

9. The Central Scientific Co., Chicago, sell the parts of a small motor, ready to put together. If you can not make a motor entire you can easily assemble one of these.

10. If you know of a new house, where wires for electric bells and annunciators are being put in, go in and find out how the wires are arranged.

11. Find out, if you can, what changes should be made in the connections of a motor in order to make it turn in the opposite direction.

12. Read *A Century of Electricity* by Prof. T. C. Mendenhall (Houghton, Mifflin & Co., Boston), a very attractively written book, with much about the history of discovery.

CHAPTER XII

INDUCED CURRENTS

229. Source of Current. In the preceding chapter we have seen how the modern electric motor might have been developed from principles that had all been discovered as long ago as 1825. Why was it that sixty years elapsed before it really grew into a practical machine, and came into general use? The answer is, that it had to wait for its counterpart, the dynamo electric machine. And why? Because no matter how efficient the motor itself may be, it must get its energy from the electric current. The only means then known of supplying electric currents were the various forms of voltaic cells, all of which derive their energy from the chemical combination of zinc with oxygen, just as the steam engine gets its energy from the chemical combination of the carbon and hydrogen of the fuel with the oxygen in the air. But the cost of zinc is so much greater than that of coal, gas, or oil, that it costs a great deal more to do mechanical work with a motor that is run by burning zinc in a battery, than it does to do it with an engine that is run by burning coal or gas under a boiler. Thus the motor is of little practical value unless we can generate electric currents in large quantity and at reasonable expense. Where shall we look for the solution of this problem?

230. Current and Magnetic Field. In our studies thus far we have found that many physical processes are reversible. Thus a current of air will turn a windmill and do mechanical work. Conversely, if we do mechanical work in turning a windmill backwards, we can make it act as a rotary fan, and produce a wind for ventilating purposes. Heat may be converted into mechanical work. Conversely, mechanical work may produce heat. Therefore the question naturally arises: *Since a current generates a magnetic field, can not a magnetic field be made to generate a current?*

231. Faraday's Discovery. The discovery of how a current can be generated with the help of a magnet was made by Michael Faraday (1791–1867) in 1831. We shall be able better to appreciate this great discovery if we repeat some of Faraday's experiments. In order to do this we shall need a coil *S*, of many turns of fine wire, a bar magnet *M*, a couple of voltaic cells or other source of steady current, a sensitive galvanometer *G*, and a pocket compass. This apparatus, Fig. 137, differs in no essential way from that used by Faraday.

Before making experiments, let us see if our previous study will

FIG. 137. THE MOVING MAGNET GENERATES A CURRENT

enable us to foretell what results we may expect. We know that if we pass the battery current through the coil *S*, one of its ends will become a north-seeking magnetic pole, and the other a south-seeking pole. Lines of force will emerge from the former and enter the latter. Let us send the current from the battery in such a direction through the coil *S* that its upper end repels the north-seeking pole of the compass needle, and is therefore, itself a north-seeking pole.

Now insert the galvanometer *G* into the circuit, and note its deflection. Suppose it is toward the right. We then know

that when the galvanometer is deflected to the right, the current circulates in such a direction in the coil that its upper end is a north-seeking pole. Therefore, when the current is passing in the coil, it is able to do the work of pulling a south-seeking pole into the coil. If the phenomenon is reversible, we may expect that if there is no current flowing in the coil, and if we do the mechanical work of pulling the south-seeking pole out of the coil, we shall generate in the coil a current, and that it will flow in the same direction as the current from the battery flowed.

232. Current Induced by a Moving Magnet. In order to verify this conclusion, we must remove the battery from the circuit, leaving the coil and galvanometer connected as before. Then place the magnet inside the coil with its south-seeking pole down, and see that the galvanometer is at rest in the zero position. When the magnet is quickly pulled out, the galvanometer gives a quick right-handed deflection. Our prediction was correct. While the magnet is moving, a current is passing in the coil. Since the deflection is right-handed, we know that the current which we induced in the coil must be in the same direction as the battery current. That current made the upper end of the coil a north-seeking pole. The induced current, therefore, does the same.

But since the upper end was a north-seeking pole, it tended to pull the magnet in, i.e., to stop its motion; so the induced current did what it could to oppose the motion by which it was generated. This is exactly what we ought to expect; for have we not learned long ago that a perpetual motion machine is impossible, and that if we produce some energy, as we have just done in generating this current, we must do some extra work in order to produce it? The extra work that we do is that of overcoming the magnetic attraction between the induced current and the inducing magnet pole.

Faraday was much puzzled, at first, by the fact that the induced currents were but momentary, for in his time the principle that the energy stored and the work done in storing it are always exactly equivalent to each other, was not so well known. It ought to be perfectly plain to us, however, that the induced

current can last just as long as the work continues, and no longer.

We have just learned that when a south-seeking pole is withdrawn from the coil, the upper end of the coil becomes a north-seeking pole, which thus opposes the motion. What will happen if we push the south-seeking pole of the magnet back into the coil? If a current is induced in the coil, and if the direction of this current is such as to oppose the motion, it should make the upper end of the coil a south-seeking pole. When we try the experiment, we find that the galvanometer gives a quick deflection—not to the right, but to the left. But since a deflection to the right means that the upper end of the coil is a north-seeking pole, this left-hand deflection tells us that this upper end was a south-seeking pole, as we predicted.

If we reverse the magnet and push its north-seeking pole into the coil, we get a right-hand deflection, indicating an induced current which makes the upper end of the coil a north-seeking pole, and which thus repels the approaching north-seeking magnet pole. When we withdraw the north-seeking pole of the magnet from the coil, we get a left-hand deflection, indicating an induced current, which makes the upper end of the coil a south-seeking pole, and thus tries to attract the north-seeking magnet pole and oppose the motion of withdrawing it.

Hence we conclude that when a magnet pole is pushed into a coil of wire, or withdrawn from it, a current is generated in that coil. This current lasts only while the motion lasts, and is always in such a direction that its magnetic field opposes the motion.

233. The Number of Lines of Force is Changed. A little reflection will enable us to see clearly that by all the four motions which we made with the magnet, we either pushed lines of force into the coil or pulled them out. We may, therefore, often find it convenient to conceive that the induced current is generated by changing the number of lines of force that pass in one or the other direction through the closed conducting circuit.

We should always remember, however, that when we vary the number of lines of force in any of these ways, we must expect to

expend some energy; for if we could vary them without energy, and thus induce a current, we should be able to design a successful perpetual motion electrical machine—a thing which all competent minds agree is impossible.

234. Currents Induced by Currents. In the last chapter we learned that a coil through which a current was flowing had magnetic poles, just like a magnet. We may, therefore, expect that if we move such a current-bearing coil either into or out of another coil, we shall get effects precisely similar to those obtained by moving the magnet. The apparatus for the experiment is shown

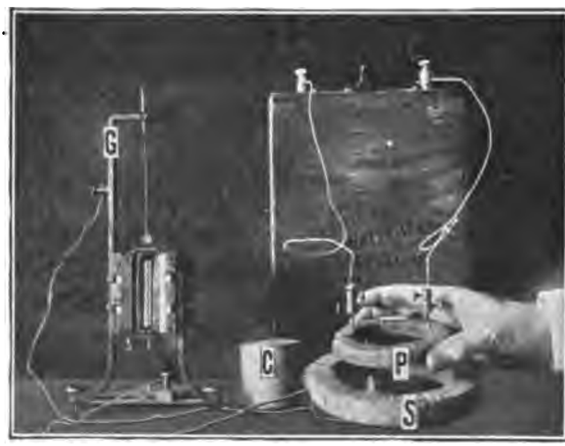


FIG. 138. THE MOVING COIL MAY GENERATE A CURRENT

in Fig. 138. When we pass the current through the coil *P*, and then bring it quickly near the coil *S*, the galvanometer gives deflections as before, and these deflections again indicate induced currents, which in all cases oppose the changes that we make in the lines of force.

We have learned, then, that a current in one closed circuit can be made to induce a current in another closed circuit. The first current is often called the PRIMARY OR INDUCING CURRENT, and the second the SECONDARY OR INDUCED CURRENT.

235. Iron Core. Since we know that more lines of force can be gathered into the primary coil P by placing soft iron therein. a new question is suggested, What effect will this stronger field have on the secondary circuit? Will it vary the directions or the magnitudes of the induced currents? The answer ought to be easily forthcoming, for we know that there will then be more lines of force to move into the coil or out of it, but that their directions will be the same as before. Therefore it is fair to infer that a greater number of lines, moved in the same time, will induce a stronger current each time, but that its direction will be the same as that induced by the primary current without the soft iron core C. Again the appeal to experiment confirms our predictions. The greater deflections of the galvanometer indicate the presence of greater secondary currents.

236. Currents Induced by Making and Breaking Circuits. There remains still another thing to try. By changing more

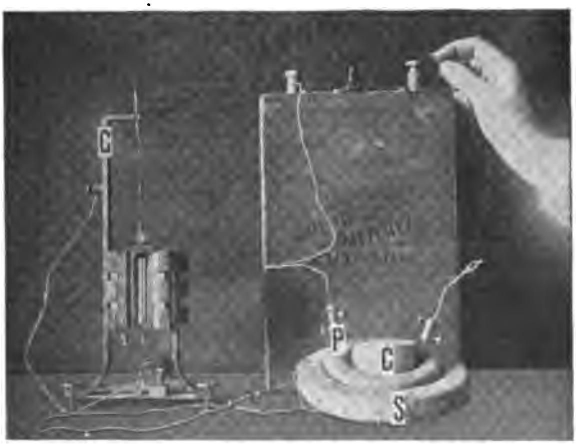


FIG. 139. CURRENTS ARE INDUCED WHEN THE CIRCUIT IS CLOSED OR OPENED

lines in the same time, we change the number of lines at a greater time rate, i.e., more quickly. Is there any way in which we may further increase this rate of change? Evidently we can do so by

moving the primary coil more quickly, and on trial we find that the induced currents are still greater.

But we have not yet reached the limit of increasing the rapidity with which the number of lines is changed. We can change them from zero to maximum, or *vice versa*, in a very small fraction of a second by placing the primary inside the secondary, and simply making and breaking the primary circuit. We may verify this deduction as we did all the others (Fig. 139).

237. The Laws of Induced Currents. We are now ready to formulate the results obtained by this interesting series of experiments. They are:

1. *Whenever the number of lines of force that pass in a given direction through a closed conducting circuit is changed, a current is induced in that circuit.*

2. *Other things being equal, the magnitude of the induced current is directly proportional to the rate at which the number of lines of force is changed.*

3. *The direction of the induced current is always such that its magnetic force opposes the motion which produces it.*

The third law is known as LENZ'S LAW, from the name of the Russian physicist, Heinrich Lenz, who first announced it. It may here be mentioned that when a current is induced in a secondary coil by starting a current in the primary, the reaction of the secondary current stops the primary, if it can not push it away mechanically; and the current induced by stopping the primary tends to keep the primary going. The reaction, therefore, may be electrical as well as mechanical.

We are now in possession of all the principles necessary for the invention of the dynamo, the induction coil, the alternating current transformer, and the telephone; therefore we shall now take up each of these inventions in turn.

238. The Dynamo Principle. In the last chapter we learned about the construction and action of an electric motor, and saw how it may convert electrical energy into mechanical work. In the present chapter we have learned that this process is reversible,

and that we can convert mechanical energy into electrical energy.

But how are the laws of induced currents applied in the construction and operation of the dynamo? With the aid of Fig. 140, let us try to find out. In the diagram, N and S represent the two poles of the field magnet, their lines of force being indicated by the long vertical arrows. FF_1 represents a single coil armature, CC' a split ring commutator, and $B+$ and $B-$ a pair of brushes attached to the terminals of the external circuit around which the current is to be sent. The shaft and bearings (*cf.* Figs. 129 and 130) are omitted for the sake of clearness.

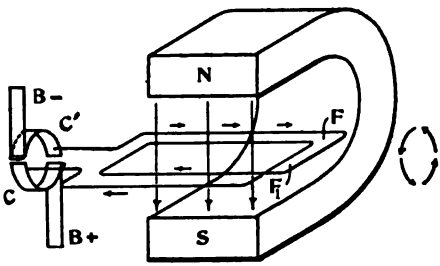


FIG. 140. THE DYNAMO DIAGRAM

The armature coil being in the position shown, the greatest possible number of lines of force pass into the upper face F. But when the armature is turned through a quarter of a revolution in the direction of the curved arrows (seen at the right of the diagram), its plane will be vertical. The number of lines of force passing into the face F will have been reduced from the maximum to zero. Therefore an induced current will circulate around the coil in such a direction as to make this face F a south-seeking pole, which, by its attraction, opposes the rotation.

This current, whose direction is shown by the short, horizontal arrows, will charge brush $B+$ positively and $B-$ negatively, and will flow onward from $B+$ around the external part of the circuit, returning to the armature through brush $B-$ and commutator segment C'. When the armature has turned through the second quarter revolution, its plane will again be horizontal, and it will again embrace the maximum number of lines of force. But since the lines now enter the face F_1, and since pushing the lines into the face F_1 has the same effect as has withdrawing them from F,

the resulting induced current will continue to flow around the coil in the same direction as before. Therefore the brush $B+$ will again be positively charged.


Fig. 141. Six-Pole Field

During the third and the fourth quarter revolution the lines are withdrawn from the face F_1 and pushed into the face F. Therefore the induced current around the coil is reversed during these two quarter turns. But it does not reverse at the brushes, for at the instant when the third quarter turn begins, the commutator segments reverse their contacts with the brushes, and so $B+$ continues to be charged positively, and $B-$ negatively. Therefore the current flowing around the external circuit is in the same direction throughout the rotation, i.e., from $B+$ to $B-$. This is the principle of the direct current dynamo.


Fig. 142. Armature and Commutator

239. The Dynamo. The power and efficiency of a dynamo are increased by the means previously described in the case of the motor. The field magnets are electromagnets, and instead of two poles there may be four or more. They are designed so as to give as strong and dense a field as possible (Fig. 141).

The armature consists of many coils wound on a soft iron core. Not only must the armature be carefully balanced mechanically, but the distribution of the coils must be such that the moments of the magnetic forces are also symmetrically balanced about the axis; otherwise the rapidly rotating armature will wabble like an ill-balanced flywheel (*cf.* Art. 91). Furthermore, the coils must be wound in slots in the core, and strongly bound in their places; for if they were not held firmly in the slots, the magnetic forces that tend to stop their motion would combine with the centrifugal force to pull them out of their places (*cf.* Art. 90). The insulation of the coils should also be as perfect as possible. The iron between the slots also serves to fill the air gaps between the coils, and conduct the lines of force into the space where they are most effective. Fig. 142 shows the armature, with the commutator on the left. In Fig. 143 the assembled machine is shown with the commutator, brushes, shaft and one of the bearings on the right. Fig. 144 shows how an armature core for a very large dynamo is built up of thin slotted plates of soft iron.

FIG. 143. COMPLETE DYNAMO

240. Winding of the Field Magnets. In the direct current dynamo the current generated by the armature is used to excite the field magnets. Accordingly these are called SELF-EXCITED, to distinguish them from the alternating current machines, which must be SEPARATELY-EXCITED by a direct current from a small separate dynamo. In the direct current dynamo, as in the motor, the armature current may be carried around the field coils from

the positive brush, then to the external circuit, and thence back to the negative brush (series winding, *cf.* Figs. 134 and 152); or it may divide at the brushes, one branch going around the field coils, and the other around the external circuit (shunt winding, Figs. 135 and 153); or both styles of winding may be used together (compound winding, Fig. 136). Fig. 141 shows the long, shunt coils, next the iron-clad frame and the short, series coils close to the ends of the pole-pieces. It is therefore a compound wound field.

FIG. 144. BUILDING AN ARMATURE CORE

241. How the Field of Force is Built up. As the field magnet cores are of soft iron, it may be asked, Since the cores are not magnetized unless the current is flowing around them, and since a current can not be induced in the armature unless lines of force from the field magnet pass through it, how is it that the current can start at all? The answer is, that the field cores always retain a little of their magnetism after having once been strongly excited. This RESIDUAL MAGNETISM is sufficient to generate a small induced current in the revolving armature, and this small current, in turn, increases the magnetic strength of the field magnets. They are then able to induce a still stronger current in the armature. The magnetism of the field cores and the resulting current in the armature thus add gradually each to the strength of the other, until the field magnets are saturated.

242. Magnetos. The early dynamos were magnetos, i.e., their field magnets were permanent steel magnets, resembling that in Fig. 140. Such machines are still much used in operating call bells on private telephone lines, and in producing sparks for the ignition of the gases in gas engines.

243. Alternating Current Dynamos. For many purposes, an alternating current has very decided advantages over a direct current. In general principle the "alternator" resembles the direct current machine, but it has collecting rings (*cf.* Fig. 128) instead of commutator segments, so that the electric impulses sent out to the line change direction every time a pair of its coils passes a pair of its poles. As there are usually several pairs of poles and as many pairs of coils, there will be several alternations at each revolution.

244. The Induction Coil. The induction coil (Fig. 145) is an instrument frequently mentioned in the papers and maga-

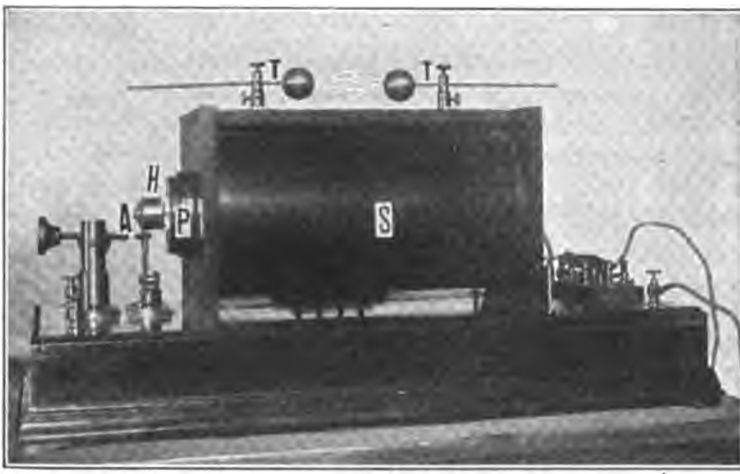


FIG. 145. THE INDUCTION COIL

zines, because it is used in producing X-rays, and in starting the ether waves by which messages are sent through space without wires. It was invented by an American, Charles G. Page, in 1838. It consists of a primary coil P, of a few turns of coarse wire, containing a soft iron core, and surrounded by a secondary coil S, of many turns of fine wire, whose ends lead to a pair of insulated knobs or points TT. Thus far it is precisely like our apparatus for investigating induced currents (Fig. 139).

But for convenience and speed in making and breaking the primary circuit, there is usually added an automatic contact breaker *H*, which keeps itself vibrating, and automatically opens and closes the primary circuit exactly as the armature of the electric call bell does (Art. 218). Alternating currents are thus induced in the secondary coil.

Since a current impulse or pressure, called ELECTROMOTIVE FORCE, is started in every turn of the secondary coil, every time that the primary circuit is made or broken, it follows that these impulses in all the turns will be added together. Therefore, up to a certain limit, the pressure of the induced current increases with the number of turns in the secondary coil. The induced electromotive force is also proportional to the suddenness with which the primary current is started or stopped (*cf.* Art. 236). As the alternating induced currents surge back and forth in the secondary coil, the electrical pressures at the terminals *TT* become so great that disruptive discharges occur between them. A 40-inch spark coil produces a pressure equal to that of from 60,000 to 100,000 voltaic cells and contains over 250 miles of wire in the secondary coil. Large induction coils must be designed with great care, especially with regard to the insulation, which would otherwise be punctured by the great electrical pressures.

245. The Alternating Current Transformer. Fig. 146 represents the apparatus used by Faraday in one of his earliest experiments with induced currents. It will be seen that when a current is started in one of the coils *P*, it will send its lines of force around through the iron of the ring *C*, and that when these lines of force enter the other coil *S*, they will induce in it a current whose direction is opposite to that of the primary and which may be detected by the galvanometer *G*. If the primary current stops, a cur-

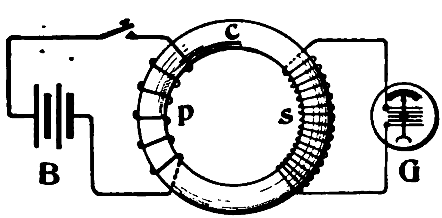


FIG. 146. FARADAY'S RING

rent is induced having the same direction as the primary. If the primary current is alternating instead of intermittent, its lines of force will enter the secondary coil first from one direction and then from the other alternately, and will thus produce alternating currents in the secondary. Since the electromotive force or pressure in the secondary coil is proportional to the number of turns of wire in it, it follows that the electromotive forces in the primary and secondary coils are proportional to the corresponding numbers of turns of wire in the two coils. For example, if the primary coil has 100 turns and the secondary 100,000, the electromotive force of the induced current will be 1000 times as great as that of the inducing current.

Fig. 147. A Transformer

Thus we may send an alternating current of low pressure and large quantity into the short coil of such a "transformer," and get out of the long coil an alternating induced current at high pressure, and of proportionally smaller quantity. Conversely, we can send an alternating current of high pressure into the long coil, and get out of the short coil an alternating current of lower pressure and proportionally larger quantity. If used in the former way, the apparatus is called a "step-up" transformer; if in the latter way, a "step-down" transformer. The induction coil is a "step-up" transformer.

Fig. 148. Construction of the Transformer

Transformers are used extensively in electric lighting and car service, because the current can be transmitted with far greater economy at high pressure than at low pressure. High pressure currents, however, are not suitable for use in lamps, and are not permissible in buildings, because of the danger of fire and loss of

life. High pressure currents from alternating dynamos are, therefore, distributed to transformers similar in construction to Faraday's ring. These are placed on poles outside the buildings, Fig. 147, and the low pressure currents are carried into the buildings, for service either in electric lamps or in alternating current motors. Fig. 148 shows such a transformer with the outside case taken off.

246. Alternating Current Motors. There are two kinds of alternating current motors. SYNCHRONOUS MOTORS resemble alternating current dynamos in construction. They are so called because the alternating currents in their fields and armatures keep time, or step, with those in the dynamo that furnishes the current to them. The armatures of *induction motors* are turned by the magnetic forces acting between the field currents and the resulting induced currents in the armature. They require very little care, because the armature currents have no electrical connection with the supply wires, and therefore they have no sliding electric contacts.

The study of alternating currents and induction motors, though exceedingly interesting, is beyond the scope of an elementary course.

247. The Telephone. How is it that sounds so complex as those which are produced by spoken words, with all their variations of loudness, pitch, and quality of tone, can be taken up by a small piece of sheet iron and transformed into electrical waves? And how is it that these electrical waves, after traveling along hundreds of miles of wire, can be retransformed into sound that is a close copy of that produced by the voice of the speaker? This is, indeed, the most marvelous of all the facts that our studies in Physics have yet brought to our attention. And the more we learn of sound, and the great complexity of the motions which it impresses on the air, the more we shall be led to wonder, that a pair of such simple contrivances as a telephone transmitter and receiver can work such a miracle.

The construction of the telephone is easily understood. The parts are shown in Fig. 149. First, there is a funnel-shaped

mouthpiece *M*, into which we talk. This mouthpiece keeps the sound from spreading into space, and directs it against a diaphragm or disc of sheet iron *D*, placed just at the end of the funnel. Behind the diaphragm *D*, and attached to it, is a smaller disc of carbon *E*, and just behind *E* is a second disc of carbon *E*, which is attached at its back to a metallic plate *B*. The small space between *E* and *E* is filled with grains of hard carbon.

The metal supports on which the carbon discs *E* are mounted are insulated from each other; but one is electrically connected with one of the terminals of a voltaic battery *Ba*, or other source of current, while the other is connected with one of the terminals of the primary wire of a small step-up induction coil *I*. The other

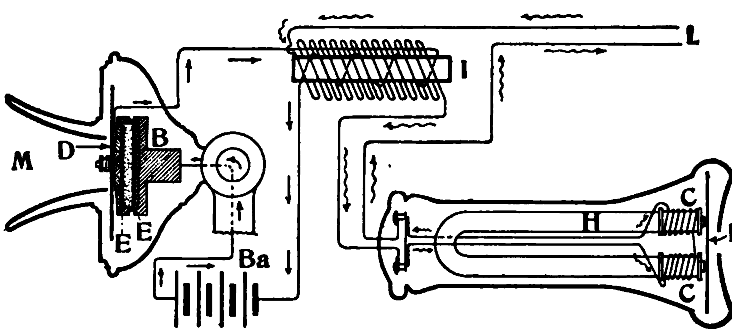


FIG. 149. TELEPHONE TRANSMITTER, RECEIVER, AND CIRCUIT

terminal of this primary wire is connected with the other terminal of the battery. A current of electricity is thus always passing between the carbon discs *E*, across the loose contacts of the carbon granules, and completing its circuit back to the battery by the way of the primary wire of the induction coil.

Now, a loose contact between two pieces of conducting matter has the very remarkable property of conducting a current better when the pressure on it increases. Very small variations in pressure at the points of contact produce changes of considerable magnitude, in the strength of a current of electricity that may be passing across the contact. Such a loose electrical contact is called a *microphone contact* because it enables us to hear very faint

sounds. The two carbon discs with their microphone contact, the battery, and the induction coil, are the only essential parts of the telephone transmitter; but the metallic case is of course necessary to protect these parts and hold them in their places.

The receiving instrument, or Bell telephone, is even more simple. Its only essential parts are a strong steel magnet H, whose poles are surrounded by coils of fine wire CC, and a diaphragm D' of sheet iron like that of the transmitter. Even this diaphragm is not absolutely necessary, for the sounds can be heard without it. The instruments are connected, as shown, the diagram for the other station being exactly like Fig. 149, except that it is reversed. Their action is partially explained as follows:

1. The sound sets the diaphragm D of the transmitter into vibration. 2. In some way not yet completely understood, these vibrations, when communicated to the microphone contacts, produce variations in the electric current flowing through them. These variations are counterparts in every detail of the sound vibrations. 3. These variations of current strength produce corresponding variations in the number of lines of force passing through the secondary of the induction coil. 4. The result is that induced electrical surges are sent chasing one another along the line wire to the distant receiving instrument. 5. These electrical waves, surging backwards and forwards in the little coils of the receiver, produce corresponding variations in their magnetic field. These variations in the number and direction of the lines of force that pass through the coils cause mechanical vibrations of the diaphragm D'. 6. Finally, the vibrations of the diaphragm are transmitted to the air, reproducing all the modulations of tone quality, pitch, and loudness that belong to the sounds emitted by the speaker.

SUMMARY

1. The electrical energy of a battery is obtained by consuming zinc, and the voltaic cell is therefore too expensive for generating electricity on a large scale.

2. Many physical processes are reversible, and we find that

this is true of the conversion of electrical energy into mechanical work.

3. Whenever the number of lines of force that pass through a conducting circuit is changed, an induced current is started, which lasts only while the change is going on. The magnitude of the induced electromotive force is proportional to the rate of change in this number of lines of force, and its direction is always such that its reaction—either mechanical or electrical—opposes the change that causes it.

4. The dynamo electric machine is a contrivance for the conversion of mechanical energy into electrical energy. It consists of a field magnet and an armature, one or the other of which rotates, and a sliding contact device.

5. In the direct current dynamo the sliding contact device is a commutator and brushes; in the alternator it is a pair of collecting rings and brushes.

6. Each coil of the armature, as it rotates, generates alternating induced currents, which are sent out through collecting rings, or a pair of commutator segments, and thence by a pair of brushes to the external circuit wherein the electrical work is to be done.

7. The field magnets of a direct current dynamo are excited by the current from its own armature, and they may be series wound, shunt wound, or compound wound.

8. The field magnets of an alternator must be excited by a separate direct current dynamo.

9. An induction coil consists of a primary coil which contains a soft iron core and is surrounded by a secondary coil. The primary current is made and broken by a contact breaker. Such a coil may be made to give long, powerful sparks.

10. Alternating current "step-up" transformers are used to transform currents of low pressure and great quantity into currents of higher pressure and smaller quantity for transmission to a distance. "Step-down" transformers are used to reconvert these into currents of low pressure and large quantity for use in lamps and motors.

11. A telephone transmitter consists essentially of a diaphragm,

a microphone contact, and a small induction coil, in circuit with a local battery or other source of direct current.

12. A telephone receiver consists of a steel magnet, a small coil (or two coils) of wire, and a diaphragm.

13. Sound impulses are transformed by the transmitter into electrical waves, which are sent along a wire to a distant receiver.

14. The receiver reconverts these electrical waves into sound impulses similar in every detail to that which was spoken against the diaphragm of the transmitter.

QUESTIONS

1. Mention some physical processes that are reversible.

2. Describe the four ways in which currents may be induced in a coil of wire by means of a steel magnet.

3. In each of these cases, what is the kind of pole induced at the end next the magnet, the direction of the force between the induced and inducing poles (i.e., attraction or repulsion), and finally, the effect on the motion (i.e., assistance or opposition)?

4. In a similar manner, describe the eight different ways in which an induced current can be started in a secondary coil by means of a primary coil and a battery.

5. What variation in these effects will result from the use of a soft iron core? From increasing the suddenness of the motions?

6. Explain why the electromotive forces of induced currents are of short duration.

7. State the laws of induced currents, in which the results of all such experiments are summed up.

8. What are the essential parts of a dynamo-electric machine? Point out the resemblance in construction, and the difference in action, between a direct current dynamo and a direct current motor.

9. Briefly explain how the act of rotating an armature coil sets up therein an induced current which changes direction at every half turn.

10. Explain how these alternating currents may be led out to the external circuit as alternating currents by means of collecting rings, or converted into direct currents by means of a commutator.

11. How are the magnitude and the uniformity of such direct currents affected by having the armature made up of several coils instead of one?

12. Mention advantages gained in the design of a dynamo by the following features: (*a*) increasing the number of field poles; (*b*) perfectly balancing the coils electrically and mechanically; (*c*) slotted armature core; (*d*) perfect insulation; (*e*) good ventilation.

13. Describe the three modes of field magnet windings.

14. Explain how a self-excited machine builds up its own magnetic field.

15. What are magnetos, and what are some of their uses?

16. Diagram an induction coil, trace the primary current through its circuit, and show how the induced currents of the secondary coil are started. Explain the action of an automatic break hammer.

17. Describe the construction and operation of the alternating current transformer. State the relation of the electrical pressures to the numbers of turns in the two coils. What is the great advantage of such transformers in alternating current lighting and power circuits?

18. Diagram a telephone circuit with a transmitter and receiver at each end of the line.

19. Trace the local current in the transmitter, and describe the manner in which its strength is varied in correspondence with the variations of air pressure due to the sound. Describe the induced currents that result from these variations of the local or primary current.

20. Describe the results of these induced currents when they reach the receiving telephone.

PROBLEMS

1. Examine a "spark coil," or "kicking coil," such as is in common use for lighting gas by electricity. It has a soft iron core and only one coil, of many turns. When placed in a circuit with several battery cells, it gives a strong spark on breaking the circuit, whereas if the wire were not coiled no spark could be obtained. Does this imply that an induced current is generated "at break" which adds its electrical pressure to that of the battery? May a current in each turn of this coil induce a current in every other turn? On "making" the circuit, would the induced current be in the same direction as the battery current, or in the opposite direction? This added current is called *a self-induced current*. Does it differ essentially from any other induced current?

2. A break hammer induction coil is subject to troublesome sparking at the break. Is this spark due to the same cause as that of the kicking coil? This spark, by forming an arc, like that of an arc light, bridges the gap, and not only burns the contacts, but also prolongs the time of breaking the primary circuit. Will the induced current in the secondary be as strong as if the arc were not formed? The arc may be partially prevented by connecting the opposite coatings of a condenser across the gap. Why?

3. When two telephone wires, whose circuits are completed through the ground instead of by return wires, are placed on poles parallel to one another, the conversation on one line may be heard on the other. Can you explain why? The noise of the trolley cars and of telegraph

instruments is often heard in the telephone. May this be due to the same cause?

4. Look carefully at Plate III. Do you see a dynamo direct-connected to it? Find the iron-clad field magnet, and the armature Also examine Plate VII. This is an alternating current generator. Find the armature and the field poles. In this dynamo, is it the field or the armature that revolves?

SUGGESTIONS TO STUDENTS

1. If you have made yourself a galvanometer, as was suggested in Chapter XI, repeat the experiment of Faraday's ring (Art. 243); and also one made by Henry at about the same time (*cf.* Hopkins's *Experimental Science*, pp. 467–476).

2. Visit the power house of the electric lighting company or the street railway company, and make a brief report on the results of your investigation. (In most cases it will be necessary to write a letter to the manager, stating why you wish admittance and requesting the favor of a pass.)

3. Belt a toy motor to the flywheel of a sewing machine, so that you can rapidly turn the armature by means of the treadle. Connect the terminals of the motor with your galvanometer and work the treadle. Does the galvanometer indicate that the motor is operating as a dynamo? If you have not made a motor you can buy one at an electrical supply store for a dollar or less.

4. Unscrew the diaphragm end of a receiving telephone case, remove the diaphragm and look at the end of the magnet and its coil.

5. Read Tyndall's *Faraday as a Discoverer* (Appleton, N. Y.), and S. P. Thompson's *Life of Faraday* (Macmillan, N. Y.).

6. Look up the biography of Joseph Henry. See, "A Study of the Work of Faraday and Henry," by Mary A. Henry (*Electrical Engineer*, N. Y., Vol. 13, p. 28), also Cajori's *History of Physics*. This last book will give you references to books on the lives of all the discoverers in Physics.

7. Forbes's *Elementary Lectures on Electricity and Magnetism* (Longmans, N. Y.), and Wright's *The Induction Coil in Practical Work* (Macmillan, N. Y.), are especially interesting in their descriptions of the phenomena of induced currents.

8. If Faraday's *Experimental Researches in Electricity* is in your city library, read some of it. It is one of the most remarkable books that was ever written. It will be worth your while to become at least a little acquainted with the mind of this great man.

CHAPTER XIII

THE ELECTRIC CURRENT AT WORK

248. Questions for Further Study. In the preceding chapters, we learned how a dynamo and a motor work; but some questions of great interest still remain unanswered. How do arc lamps and glow lamps work? How much power is required to operate them? What are the methods of distributing the current? How do electricians measure currents and calculate their power? How much loss is caused by the resistance of the wires? What uses can be made of the heat developed by the current? How is electroplating done? How does a storage battery differ from a voltaic battery? These are things that everybody wants to know something about, and a little further study will enable us to understand them.

249. The Arc Light. In 1808, Sir Humphry Davy, the predecessor of Faraday at the Royal Institution, produced the first arc light with a powerful battery and two pencils of carbon. When these two carbons, connected with the terminals of the battery, were brought into contact and then slightly separated, the current was not broken, because an arc, composed of white hot vapor of carbon, was formed. The carbons, being in contact with the air, are gradually burned up, just as is the carbon in burning illuminating gas or oil. In all arc lamps an ingenious arrangement of electromagnets "feeds" the carbons together automatically as fast as they burn away (*cf.* Art. 270).

250. Pressure and Current in the Arc Lamp. An ordinary street lamp is equivalent nominally to 2000 candles. It is found not to burn satisfactorily unless it is fed by a current having a constant strength of about 9.5 amperes, the electrical pressure

at its terminals being maintained at about 50 volts. Lamps of greater candlepower must, of course, have more current. We have already learned something in a general way about current strength, resistance, and pressure; but if we wish to know anything of the way in which electrical calculations are made, we must learn how to express our ideas more precisely. Some definitions and precise statements of relations, therefore, are necessary.

251. The Current Strength in a conductor is the rate of flow of the current, i.e., *the quantity of electricity passing per second at a given cross-section of the conductor;* and the AMPERE, the practical unit of current strength, is defined as the steady current which deposits silver by electrolysis from a solution of a silver salt at the rate of .001118 grams per second (*cf.* Art. 208). The ampere is named in honor of André Marie Ampère (1775–1836), who was professor in the Polytechnic School at Paris, and who followed up the discovery of Oersted with valuable researches on relations between currents and magnets.

252. Resistance. We are accustomed to conceive that a conductor offers RESISTANCE to the passage of a current of electricity, and that an ELECTROMOTIVE FORCE, or ELECTRICAL PRESSURE, is required to force the current through it, because *in transmitting the current, the conductor absorbs some of the electrical energy and gives off this energy again in the form of heat.* The unit of resistance is the OHM, which is defined as the resistance at 0° C. of a column of pure mercury 106.3 centimeters long and of a uniform cross-sectional area of 1 square millimeter. Such a column should weigh 14.45 grams. The ohm is named in honor of Georg Simon Ohm (1789–1854), an eminent German physicist, who was teacher of mathematics and physics at the Gymnasium at Cologne, and afterwards professor at the University of Munich. Ohm investigated, both experimentally and mathematically, the resistances of conductors and their relations to current strength. By his experiments and those of others, the following relations have been established:

253. The Laws of Resistance. The resistance of a conductor is

1. *Directly proportional to its length.*
2. *Inversely proportional to its cross-sectional area.*
3. *Directly proportional to a constant whose value depends on the material of the conductor and on the units in which its length and cross-section are expressed.* This constant is called the RESISTIVITY of the substance, and it represents the resistance at 0° C. of a conductor of the given substance having unit length and unit cross-section.
4. *Other things being equal, the resistance of a given conductor depends on the temperature.* For most metallic conductors, the resistance diminishes as the temperature is lowered; and it is interesting to note that according to some recent experiments, this diminution appears to take place at such a rate that at the absolute zero they would have no resistance (*cf.* Art. 123). For carbon, and for those substances that are broken up or electrolyzed when conducting a current, the resistance is diminished by raising the temperature.

254. Ohm's Law. One of the most important contributions of Ohm to our knowledge of electric currents is the law known by his name, and stated as follows: *The current strength in any conducting circuit is directly proportional to the electric pressure or electromotive force, and inversely proportional to the corresponding resistance.* Letting C represent the current strength, E the electric pressure, or electromotive force, and R the resistance, we may express Ohm's law by the equation $C = \frac{E}{R}$ or

$$\text{Current in amperes} = \frac{\text{Pressure in volts}}{\text{Resistance in ohms}}. \qquad (11)$$

This equation defines the volt for us; for if the current strength and resistance are each made equal to 1 unit, the pressure, according to the equation, is 1 volt.

A VOLT, therefore, is that electrical pressure or that electromotive force which will maintain a current of one ampere in a conductor whose resistance is one ohm. The volt is named in

honor of Volta (*cf.* Art. 207). A simple voltaic cell has an electromotive force of nearly one volt. A difference in electrical pressure is often spoken of as a DIFFERENCE OF POTENTIAL.

255. Ammeters and Voltmeters. It will interest us to learn how the current strength and pressure required by a lamp or a motor are measured. The instruments most widely used for this purpose are made on the principle of the D'Arsonval galvanometer. The coil is pivoted in jeweled bearings, and balanced against a hairspring after the manner of the balance wheel of a watch. In all voltmeters and ammeters, the number of volts or amperes is indicated by a pointer attached to the coil and moving over a scale which has been accurately graduated or "calibrated" by reference to currents of known strength, so as to read volts or amperes. The voltmeter has a very high resistance, and the ammeter a very low resistance; the voltmeter is connected as a switch or "shunt" across the terminals of the lamp or motor, where the pressure is to be measured; the ammeter is placed "in series" in any part of the circuit around which the current is flowing. Fig. 150 shows the proper method of connecting them. Fig. 151 shows a portion of the switchboard in a power-house, with ammeters, voltmeters, and switches.

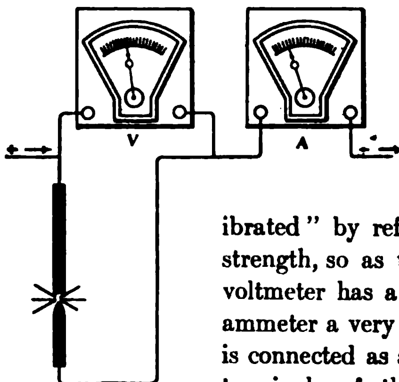


FIG. 150. VOLTMETER AND AMMETER CONNECTIONS

256. To Calculate the Power. The next problem that claims our interest is the calculation of the power used in the lamp. In Chapter IX we learned that we could calculate the amount of work done per second in the cylinder of a steam engine, not only by taking the product of the average force of the steam and the distance traversed per second by the piston, but also, more conveniently,

by taking the product of the average steam pressure and the quantity (volume) of steam used per second. It is easy to show that similarly the work done per second by an electric current is proportional to the product of the electrical pressure and the quantity of electricity used per second. If we measure the pressure in volts and the quantity-per-second, or current strength, in amperes, it is obvious that the power will be one unit when the pressure is one volt and the current strength one ampere; therefore, the following definition is adopted for the unit of power:

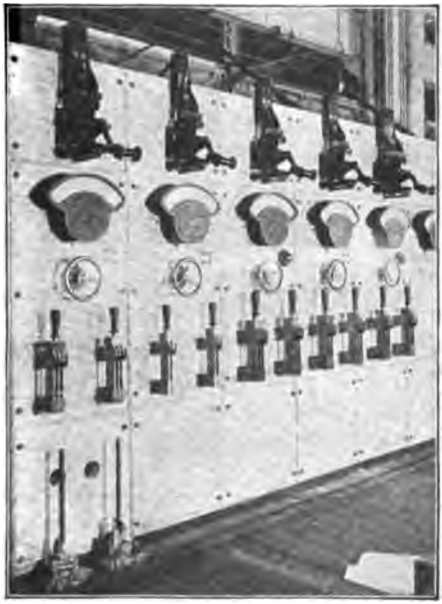


FIG. 151. SWITCHBOARD

The unit of electrical power or activity is the power of a current of one ampere under a pressure of one volt. This unit is called the WATT, in honor of James Watt, the inventor to whom we are most indebted for the modern steam engine. A watt is found to be equal to 10^7 ergs per second, or $\frac{1}{746}$ horse-power (*cf.* Art. 43). With these units, the equation that expresses the power of a current is:

Power in watts = Current in amperes $\times$ Pressure in volts (12)

Or, if A represent the number of watts, C the current strength, and E the pressure, $A = CE$.

With this equation we can now calculate the power consumed in our 2000 candlepower arc lamp, for since $C = 9.5$ amperes and $E = 50$ volts, the power $A = 9.5 \times 50 = 475$ watts. Let the student find the number of ergs per second and the horse-power that correspond to this number of watts.

257. Watt Meters. When large quantities of electrical energy are used in varying amounts, it is most convenient to measure the total amount of it in watt-hours.

The watt-hour is the amount of energy furnished in one hour at the rate of one watt; and it therefore equals $10^7 \frac{\text{erg}}{\text{sec}} \times 3600$ sec $= 36 \times 10^9$ ergs; 1000 watts is called a kilowatt (1 K. W.). The number of watt-hours used by a consumer is measured by an interesting instrument called a watt meter. A common form of watt meter is a little motor having no soft iron cores. Its field coils have few turns, and are placed in series in the circuit whose energy is to be measured; the armature coils have many turns, and are connected as a shunt across the mains or feeders, like a voltmeter. The instrument is ingeniously regulated, so that the number of revolutions of the armature is proportional to the energy supplied. Therefore, if a train of clock wheels is geared to the armature, the total number of watt-hours of energy that have passed the meter up to any given time may be indicated on a series of dials by index hands attached to the gear wheels, just as the number of cubic feet is indicated on the dials of a gas meter.

258. An Arc Light Plant. We now have at command the knowledge that is necessary in making the calculations for a small arc lighting plant. Suppose that we wish to light a shop with ten 2000 C. P. (candlepower) arc lamps, and must know the necessary engine and dynamo power. How shall we attack the problem? In a case like this the lamps are ordinarily placed IN SERIES as represented in the diagram, Fig. 152; therefore the total resistance is the sum of all the resistances in the circuit.

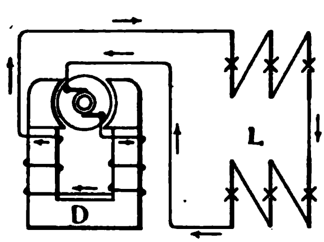


FIG. 152. LAMPS IN SERIES

The loss in pressure, or fall of potential in any part of the circuit is proportional to the corresponding resistance, in accordance with Ohm's law, i.e., $E = CR$. (*cf.* Art. 254). Therefore, THE

NUMBER OF VOLTS USED IN THE LAMPS is equal to that for one lamp multiplied by the number of lamps. So the pressure needed for the 10 lamps is $50 \times 10 = 500$ volts.

We must now find the voltage necessary to overcome the resistance of the line. Since this voltage is not available for use in the lamps, it is called the LINE LOSS or "DROP." Let us suppose that the total length of wire from the dynamo through all the lamps and back again to the dynamo is 600 ft. The fire insurance regulations require us to use at least a Number 14 wire to carry 9.5 amperes (Table I, page 298). This, by reference to Table I, is found to have a resistance of 2.565 ohms per thousand feet. Since the resistance is proportional to the length, that of 600 ft. is 0.600 of 2.565, or 1.5 ohms, nearly. The loss of pressure in the line is, therefore, 9.5 amperes $\times$ 1.5 ohms $=$ 14.25 volts.

We have found that there are 500 volts required for the lamps, and that the line loss or "drop" is 14.25 volts; hence the electromotive force demanded from the dynamo is 514.25 volts, and at this pressure it must send out 9.5 amperes. The output of power by the dynamo is 514.25 volts $\times$ 9.5 amperes $=$ 4885 watts. Since more lamps may be needed, as the requirements of the shop increase, it is customary to provide for this increase of power. Let us suppose, therefore, that we are to order a dynamo capable of giving 750 volts and 9.5 amperes. The power of this machine will be 7125 watts or, say, 7.5 K. W. The equivalent of 7.5 kilowatts in mechanical horse-power is $\frac{7500}{746} = 10$ H. P., nearly.

Since THE EFFICIENCY of a good dynamo is about 90 per cent, we must allow for a 10 per cent loss of energy in the dynamo itself. There is also a further loss of about 5 per cent in transmitting the mechanical power from the engine to the dynamo. Hence, 10 H. P. is 85 per cent of the power that must be furnished by the engine to provide for the greatest load that it will get from the electric plant. This extra power to be provided by the engine, then, is $\frac{100}{85}$ of $10 = 11.8$ H. P. Our engine must, therefore, be big enough to take care of a load of about 12 H. P. in addition to the power furnished by it for running the machinery of the shop.

259. Incandescent Lamps. The incandescent or glow lamp consists of a slender thread or filament of specially prepared carbon, enclosed in a glass bulb and mounted on a pair of terminals that pass through a glass plug at the bottom of the bulb.

The air is pumped from the bulb, which is then fastened into a base. This base may be screwed into a socket in such a way that when the terminals of the socket are connected with the supply wires, the current is conducted through the filament. The carbon has a relatively high resistance, and when a sufficient current passes through it, the resulting heat makes it white hot or incandescent (*cf.* Art. 153). It can not burn, however, because the air has been removed from the bulb, and no oxygen is there for it to combine with.

A glow lamp is usually so adjusted that it gives 16 candle-power when a current of 0.5 amperes is passing through it. Its resistance is 220 ohms; hence the fall of pressure through it is 110 volts, and it takes energy at the rate of 55 watts. The student may easily verify the last two statements by calculation from equations (11) and (12), Arts. 254, 256.

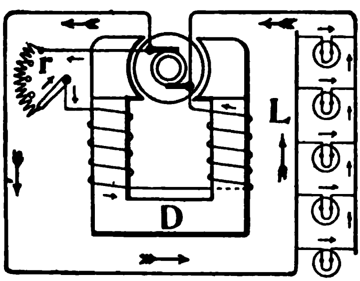


FIG. 153. LAMPS IN PARALLEL

260. The Parallel Method of Distribution. The diagram, Fig. 153, shows the method generally employed for distributing the current to glow lamps.

The dynamo is designed and operated so as to maintain a constant difference of potential at its terminals, whether the current taken from it is large or small; and in good practice the wires for the mains are so chosen that the pressure lost in traversing them shall not be more than about 10 per cent of that furnished by the dynamo.

Since the loss in pressure through the lamps is 110 volts, the dynamo must at least have an electromotive force equal to $\frac{100}{90}$

of 110 = 122.2 volts. For small plants the usual machine is wound for 125 volts.

261. An Incandescent Light Plant. Let us suppose that a certain man has on his country place a good-sized waterfall, and wishes us to tell him whether he can use this power for lighting his house. How shall we apply our knowledge of physics so as to solve his problem for him?

We must ascertain: (1) The greatest number of lamps that will be in use at any one time. (2) The distance from the fall to the center of distribution, or the point where the branches to the various rooms are to be taken from the main wires or "feeders." (3) The height of the falls. (4) The least volume of water per second that will be available for use. This makes it necessary to measure the cross-section and velocity of the stream above the falls.

Let us suppose that he wishes to light 200 lamps, 150 of which are likely to be in use at any one time, and that the falls are located 1500 feet from the center of distribution.

The lamps will require a pressure of 110 volts, and if a 125 volt dynamo is used, the line loss allowable will be 15 volts, or 12 per cent. Since the main current divides amongst the lamps it must equal the sum of the currents in all the lamps. Therefore the number of amperes required for 150 lamps of 16 C. P. each, is $0.5 \times 150 = 75$. The output, therefore, must be 125 volts $\times$ 75 amperes = 9375 watts. In designing a lighting plant it is always well to allow for a few more lamps, so we will figure on a 10 K. W. dynamo, giving an output of 10,000 watts, or, in mechanical units, $\frac{10000}{746} = 13.4$ horse-power.

Allowing an efficiency of 90 per cent for the dynamo, the mechanical H. P. supplied to it by the water wheel must be:

$$\tfrac{100}{90} \text{ of } 13.4 = 14.89 \text{ H. P., or, say, } 15 \text{ H. P.}$$

The efficiency of a good turbine water-wheel is about 80 per cent, and if we allow for an additional loss of about 5 per cent in transmitting the power from the turbine to the dynamo, the power furnished by the water must be $\frac{100}{75}$ of 15, or 20 H. P.

Now let us suppose that the falls are 300 cm high and that by floating a stick on the water just above the falls and timing it with a watch, we find that it passes over a measured distance of 200 cm in 2 sec.

The velocity is, therefore, $100 \frac{\text{cm}}{\text{sec}}$. If in measuring the depth and width of the stream at the place where it goes over the falls, we find that a fair average of each dimension gives us, depth 60 cm, width 300 cm, the average cross-sectional area of the stream is: 18,000 cm^2, and at the speed of $100 \frac{\text{cm}}{\text{sec}}$ the volume passing the falls in 1 sec is 18×10^5 cm^3. The mass of the water (*cf.* Art. 32) is 18×10^5 gm, and its weight is therefore $18 \times 10^5 \times 980 = 1764 \times 10^6$ dynes. Since the distance through which this weight can act is 300 cm, the potential energy of the fall is $1764 \times 10^6 \times 300 = 5292 \times 10^8$ ergs each second. Therefore, since 1 H. P. $= 746 \times 10^7 \frac{\text{erg}}{\text{sec}}$, its horsepower is: $\frac{5292 \times 10^8}{746 \times 10^7} = 72$ H. P., nearly.

We see that the inaccuracy in our method of measuring the falls is not serious, for the result of the calculation shows us that we have a large margin. We may, therefore, safely assume that the stream will furnish us the necessary 20 H. P.

The sizes of wire for the feeders and branches still remain to be calculated. We have seen that our allowable line loss is 15 volts. We must not have more than 3 per cent drop in voltage in our distributing wires, because the drop varies with the current used, so that when only a few lamps are lighted, it will be less than when all are lighted. Unless the drop allowed for were small, there would then be too much pressure for these lamps and their life would thus be shortened. We shall therefore take 3 volts for the drop in the distributing wires and leave 12 volts for drop in the feeders. Now, since the line drop is to be 12 volts and the current required is 75 amperes, we have from Ohm's law: $R = \frac{E}{C} = \frac{12 \text{ volts}}{75 \text{ amperes}} = 0.16$ ohms. Since the feeders are 1500 feet long, the length of wire in them is $1500 \times 2 = 3000$ feet; and the number of ohms per thousand feet is $\frac{0.16}{3} = 0.0533$. Consulting the wiring tables, we find that a Number 0000, Brown and

Sharpe's gauge wire has a resistance per 1000 ft. of 0.04966 ohms, which is the nearest to 0.0533 ohms and is, therefore, the size to be chosen.

The sizes of wire on each of the various branches are chosen by a similar calculation, in accordance with the number of amperes taken and the length of the branch wires, so as to give a drop in each group of nearly 3 volts, as demanded by the conditions stated.

Such a lighting plant as this would be rather expensive; but the energy would cost nothing; the repair bills would not be large, no high-priced attendance would be necessary, and the only important cost would be the interest on the money invested. The power could be utilized in the daytime, by means of motors, for operating a threshing machine, feed chopper, cream separator and churns, elevators, sewing machines, electric fans, and, in fact, for everything in which power is needed on the place, including the charging of electric automobiles. A few storage battery cells could also be kept charged by the current and used for operating door bells, burglar alarms, signal bells, and other household apparatus, and an electromagnetic device for stopping and starting the water-wheel by means of a switch, located in the house.

Heating coils also might be used in the house for cooking, ironing and the like. Thus, considering the great convenience, cleanliness, and wide range of usefulness afforded by such an electric plant, the capital invested in it would certainly be very advantageously employed.

262. Heating Effects of the Current. In the transmission of electric power it is desirable to have as little of the energy transformed into heat as is possible. On the other hand, when we want to use the energy as heat we should plan our apparatus so as to have the electrical energy liberated in the particular limited space where it is wanted.

It is thus very important to know definitely the relations that the number of heat units bear to the numbers of volts, amperes, and watts.

These relations were investigated by James Prescott Joule, who made the first determination of the mechanical equivalent of heat. Joule placed a small coil of platinum wire in a calorimeter with a weighed quantity of water and a thermometer (Fig. 154), and in the usual manner (*cf.* Art. 126) measured the quantity of heat given up to the water when currents of different strengths were passed through the coil.

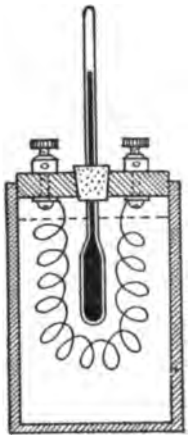


FIG. 154. JOULE'S CALORIMETER

Besides measuring the pressure and current strength, he also measured the time intervals during which the current had been passing. As a result of these experiments, which were subsequently repeated with greater accuracy by the late Professor Rowland of Johns Hopkins University, the following relations were established:

263. Joule's Law. The number of heat units generated by a current of electricity is directly proportional:

1. *To the square of the current strength.*
2. *To the resistance of the conductor.*
3. *To the time during which the current passes.*

If H represents the number of calories liberated in t seconds, C the current strength, and R the resistance, Joule's law may be expressed by the equation, $\frac{H}{t} = .24\ C^2R.$ Or,

$$\text{calories in one second} = .24 \times (\text{amperes})^2 \times \text{ohms}. \tag{13}$$

The factor .24, therefore, represents the number of calories per sec. corresponding to one watt.

This is apparent from Ohm's law: for $C = \frac{E}{R}$ or $E = CR$;

Also, from the equation for electrical power, $A = CE$; so, substituting CR for E, this equation becomes $A = C^2R$; or watts = (amperes)^2 $\times$ ohms. This, when multiplied by .24, gives the number of calories in one second as stated by Joule's law (*cf.* equations 11 and 12, Arts. 254 and 256).

An inspection of these equations, $E = CR$ and $A = C^2R$ (or $\frac{H}{t} = .24\ C^2R$), will tell us what to do when we want to transmit electrical energy with the least possible loss by heating, and also, on the other hand, what to do when our purpose is to get heat from the current.

264. The Heat Loss in Transmission. When considering the energy lost in transmission, we note from Ohm's law, $E = CR$, that *the drop in voltage, caused by the constant resistance of the line, is proportional to the current* C. Also from Joule's law, *the energy dissipated in the line as heat is proportional to* C^2. Therefore we may make these losses small by making the current as small as possible.

Now, the power delivered by the line is measured by the product of the current C and the pressure E at which it is delivered; and since the electrical power is the same so long as this product CE remains constant, we can deliver the same amount of it with less line loss by making C smaller and E larger in the same proportion. For example, suppose we wish to deliver 1000 watts to a certain house. We can do this by means of a current of 10 amperes at a pressure of 100 volts, or by a current of 5 amperes at a pressure of 200 volts. If the resistance of the line is 2 ohms, then, in the first case, the power lost in the line by heating is, watts $= C^2R = 10^2 \times 2 = 200$. In the second case the loss is watts $= 5^2 \times 2 = 50$. Thus, by doubling the voltage, we have reduced the watts lost in the line to one-quarter of its former value; and we can understand why it is more economical to transmit electrical energy at a high voltage and a low amperage.

If it is desirable to save copper instead of energy, we see that we may in the second case get the same efficiency as in the first, using a wire of one-fourth the cross-sectional area. For if the wire is reduced to one-fourth its former size, its resistance will be 4 times as great, or 8 ohms. Then the heat loss in the wire is, $5^2 \times 8 = 200$ watts, as at first.

From these examples we see that by doubling the voltage while the total output of the dynamo remains the same, we can

save either three-fourths of the energy that would be lost in transmission, or three-fourths of the copper, as we may elect. Thus again it appears that *it is more economical to transmit electrical energy at high pressure and small current strength.*

This superior efficiency of high voltage transmission is taken advantage of in lighting and power circuits in some very interesting ways, two of which we will now consider.

265. The Three-Wire System. Fig. 155 shows a method of wiring much used for motors and incandescent lamps. It will be seen from the diagram, that if equal numbers of lamps are glowing on the two branches, the current will flow through the pairs of lamps in series, and that a given number of lamps will take twice the voltage, but only half the current taken by the same number of lamps on the two-wire plan.

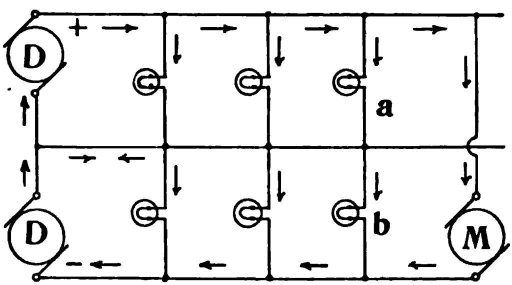


FIG. 155. THE THREE-WIRE SYSTEM

As shown in the preceding section, the line loss then will be only one-fourth what it would be on the two-wire plan. If the two sides are balanced, i.e., if they are equally loaded, no current will traverse the middle or neutral wire. The lamps, however, are independent of one another and of the motors. This is clearly shown by the diagram, for if a lamp *b* is cut out of the negative side, then the current from *a* can no longer pass through *b* to the negative wire, so it returns by the neutral wire. And if one of the lamps *a* is cut out of the positive side, the current necessary to supply its counterpart *b*, although now no longer fed to *b* through *a*, is supplied *via* the neutral wire from the dynamo $D-$ on the negative side. When the two sides are balanced, the two dynamos work strictly in series, giving double the voltage of one; and the second, $D-$, sends all its current through the first, $D+$;

but when the demand is greater on the negative side than on the positive, the dynamo $D-$ sends a sufficient part of its current out along the neutral wire, and so supplies this extra demand. With this arrangement, for a given number of watts delivered and a given line loss, the + and − feeders can be reduced to one-fourth the weight required by the two-wire plan; and since the neutral wire has the same cross-section as one of the feeders but only half the length (and weight) of the two feeders, the total weight of copper used is $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ of that required by the two-wire plan.

FIG. 156. MOTOR-DRIVEN LATHE

Fig. 156 shows how electrical power is utilized when transmitted to a large shop. The motor is direct-connected to a lathe which is turning axles for car wheels. Figs. 157 and 158 show the advantage of transmission by wires over transmission by belts and shafting.

FIG. 157. MOTOR-DRIVEN SHOP

FIG. 158. BELT-DRIVEN SHOP

266. Alternating Current Transmission. We are now prepared to appreciate the value of the alternating current transformer (Art. 245), for by means of a transformer a current can, with very little loss of energy, be "stepped up" from 125, 250, or 500 volts in the generator to 5,000, or 10,000, or even to 50,000 volts in the feeders, and then "stepped down" again to 220 or 110 volts by the means of another transformer. Thus the current is at a high voltage during transmission, and the losses in the line are reduced to a minimum. These high voltage currents are very dangerous and easily escape by leakage unless the conductors by which they are carried are thoroughly insulated.

Since alternating currents can be transmitted with so much greater economy than can direct currents, the former are rapidly displacing the latter for most purposes.

267. Electrical Heating. Having found in the preceding paragraphs that the heating effect of the current increases as the resistance and as the square of the current strength, it is clear that when we want to convert the current energy into heat we must have either a large current, or a large resistance, or both. By thus converting electrical energy into heat, very high temperatures may be obtained; so that the process is much used in electric welding, and in the reduction of ores.

In the reduction of aluminum, for example, a current of 2,500 amperes, under a pressure of only about 8 or 9 volts, is passed from a large carbon terminal, through the ore in a carbon-lined crucible, which forms the other terminal. The reduction of the metal from the molten ore is effected partly by the intense heat and partly by electrolysis. Carborundum, which is much used instead of emery for grinding edge tools, and calcium carbide, which is used for producing acetylene gas for lighting purposes, is made in electric furnaces.

For heating suburban cars, and also soldering tools, cooking utensils, flatirons, chafing dishes, and even curling irons, current at ordinary pressure is passed through coils of highly resisting metal, such as iron, German silver, or platinoid. Electric heating, as compared with direct heating by burning the fuel without

transformation through a steam engine and dynamo, is altogether too expensive to come into general use in cases of this sort, but will always be appreciated when small quantities of heat are needed, when the greater convenience and cleanliness offset the disproportionate cost.

268. Divided Circuits. In considering the distribution of current in parallel conductors, in connection with glow lamps, it is found that if the branches have equal resistances they get equal portions of the current. In electrical engineering, it is often necessary to know the amount of current on each of several branches whose resistances are not equal; or sometimes it is desirable to arrange the resistance of a branch or "shunt" so as to switch off a definite fraction of the current.

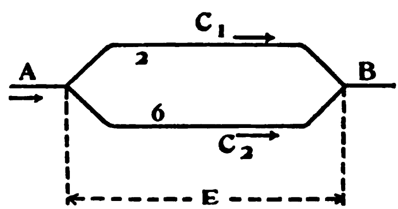


FIG. 159. A DIVIDED CIRCUIT

We must, therefore, know how the resistances of the branches govern the distribution of the current among them. In order to find this relation, let us consider the conductor, Fig. 159.

It is evident from this diagram that there is a definite difference of potential, or loss of pressure, between A and B, and that this must be the same for each branch.

Let us call this difference of potential E volts.

The current on AC_1B is then $C_1 = \dfrac{E \text{ volts}}{2 \text{ ohms}}$, or $E = C_1 \times 2$, and the current on AC_2B is $C_2 = \dfrac{E \text{ volts}}{6 \text{ ohms}}$, or $E = C_2 \times 6$. Since the difference of potential is the same for both branches, $C_1 \times 2 = C_2 \times 6 = E$ or $\dfrac{C_1}{C_2} = \dfrac{6}{2} = \dfrac{3}{1}$.

Of the 8 amperes, therefore, the 2 ohm branch gets 6 amperes or $\frac{3}{4}$, and the 6 ohm branch will get 2 amperes or $\frac{1}{4}$, i.e., *the current strength on each branch is inversely proportional to the resistance of that branch.* It may be proved, both mathematically

and experimentally, that this is true for any number of branches whatever.

269. Shunts. We may apply this principle when we are using a delicate galvanometer with a strong current, and wish to send only $\frac{1}{1000}$ of the current through it, so as not to injure it. We then connect a shunt across its terminals, and the resistance of the shunt must be $\frac{1}{999}$ of the resistance of the galvanometer branch. The current will divide between the galvanometer coil and the shunt as it does between AC_1B and AC_2B, Fig. 159; so the galvanometer will then get $\frac{1}{1000}$ of the current and the shunt the remainder, or $\frac{999}{1000}$.

270. Arc Lamp Regulation. Another very important application of the shunt principle is found in the regulating magnets of the arc lamp.

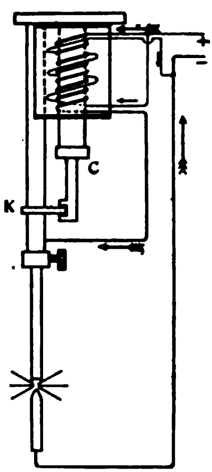


Fig. 160. The Arc Lamp Regulator

The diagram, Fig. 160, shows on the regulating magnet two windings, which are carried around it in opposite directions, one of a few turns of thick wire and in series with the carbons, and the other a shunt coil of many turns of fine wire. When the current is turned into the lamp, the carbons are in contact, and their resistance is small; so that the current is correspondingly large. This current, going around the series coil, magnetizes it strongly. This causes it to pull up the core C, which in its turn acts on the clutch K so as to pull up the + carbon. When the + carbon is pulled up too far, the current through the carbons is reduced, because of the greater resistance of the lengthened arc; but because of this increased resistance a greater proportion of the supply current goes around the shunt coil. This, being wound in the opposite direction, counteracts the magnetic pull of the series coil and lets the clutch down, so that the carbon slips through it and the arc is shortened. If the arc gets

too short, the diminished resistance allows more of the current to go through the series coil and less through the shunt coil, so the series coil pulls the clutch up and lengthens the arc.

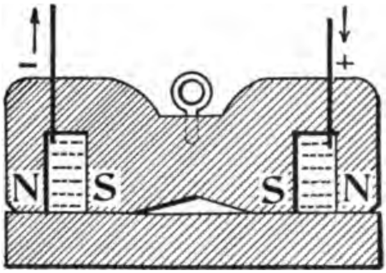


FIG. 161. SECTION OF A LIFTING MAGNET

If the resistances of the two coils are properly proportioned, the carbons will be kept a constant distance apart, as mentioned in Art. 249. This principle of divided circuits has important applications in the shunt windings of the field magnets for dynamos and motors, and in connection with controllers used in starting motors.

271. Lifting Magnets. Another important application of electro magnets remains to be considered. This is the lifting magnet. Fig. 161 represents a cross-section of one, showing how the poles are arranged so as to have a complete magnetic circuit. Anyone who has seen one of these magnets in operation, lifting heavy steel plates or girders and carrying them about a shop, will appreciate their convenience. Fig. 162 is a photograph of such a magnet holding a mass of iron that weighs about 5 tons. This mass of iron is called a "skull cracker," and it is used to break up old castings before remelting them. The mass is lifted by means of the magnet and then dropped on the castings.


FIG. 162. THE LIFTING MAGNET AT WORK

272. Voltaic Cells. We have found the answers to most of the questions that were raised at the beginning of this chapter,

and we shall now try to answer the others. Of the discovery of the voltaic cell and the important fact that it supplies a continuous current of electricity, we learned something in Chapter XI. But we were then interested chiefly in the discoveries in electromagnetism, which immediately followed the discovery of currents, and so we deferred the study of voltaic and electrolytic action until now.

Battery cells, as well as other electric generators, must have a COMPLETE CONDUCTING CIRCUIT in order to do work. Fig. 163 shows a good way of representing a circuit in which, for example, an electromagnet is operated by several cells in series (*cf.* Art. 258 and Fig. 152). In such a case it is found that both the voltages and the resistances of the circuit are added up, which is what we should expect, since the current goes through one cell after another. It follows, therefore, that if we have any number of equal cells in series, with some external resistance:

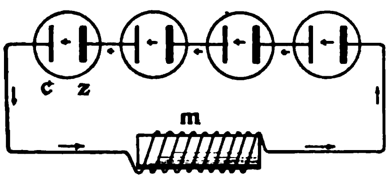


FIG. 163. BATTERIES IN SERIES

(1) *The total electromotive force of the circuit is that of a single cell multiplied by the number of cells.*

(2) *The resistance of the battery itself, which is called the internal resistance, is that of a single cell multiplied by the number of cells.*

(3) *The total resistance of the circuit is the internal resistance plus the sum of all the external resistances.*

(4) *The current strength, therefore, may be found by dividing the total number of volts E. M. F. by the total number of ohms resistance.*

273. Energy of the Cell. When a voltaic cell is sending a current around a circuit, it will be noticed that chemical changes are going on. Bubbles of hydrogen gas are seen to be liberated at the copper plate. Also, if we weigh the two plates before operating the circuit, and after some time remove them and

weigh them again, we shall find that the zinc plate has diminished in mass, while the copper plate has not.

If we were to determine the chemical composition of the fluid, we should find less sulphuric acid in it than before, and we should find a new compound, zinc sulphate. The relations between the quantities of the substances taking part in this chemical change are found to be constant. Thus, for every 65 grams of zinc that disappear, 98 grams of sulphuric acid are used up. At the same time 2 grams of hydrogen are liberated, and 161 grams of zinc sulphate are made. These relations may be represented by symbols, as follows:

	and	produce	and
Zinc	Sulphuric acid	Zinc Sulphate	Hydrogen
Zn	$+ H_2SO_4$	$= Zn\ SO_4$	H_2
65 gm	2+32+64=98 gm	65+32+64=161 gm	2 gm

The symbols, S and O_4 represent sulphur and oxygen in the proportions of 32 and 64 gms, respectively, to 2 of hydrogen or 65 of zinc. From an inspection of the relative weights of the substances, it is evident that the zinc replaces the hydrogen in the sulphuric acid and combines with the SO_4 (or sulphion, as it may be called).

Now, we know that we get energy from carbon by burning it, i.e., causing it to combine with oxygen from the air. Similarly, it ought now to be quite plain that we get energy from the voltaic cell by causing zinc to combine with the oxygen in the sulphine of sulphuric acid, as was stated in Art. 229.

Thus chemical energy is transformed into electrical energy.

274. Polarization of a Voltaic Cell. A very troublesome defect in Volta's cell is that the hydrogen bubbles stick to the copper plate and diminish the current in two ways:

1. Hydrogen is a bad conductor, as all gases are, and it insulates that part of the plate which it covers, so the internal resistance is increased.

2. It tends to recombine with the acid and displace the zinc. This tendency gives rise to an electromotive force which tries to send a current backwards through the cell. This counter electro-

motive force weakens the current. When in this condition, the cell is said to be POLARIZED.

275. Why is the Hydrogen Liberated at the Copper Plate? We have seen that the metal which combines with the SO_4 and releases the hydrogen is the zinc, not the copper; for the zinc is found to be used up, while the copper is not. It seems, then, that although the chemical action starts at the zinc plate, yet the copper plate is the one at which the hydrogen is actually liberated. In order to explain this, a hypothesis has been advanced, which will be understood by reference to Fig. 164.

It is supposed that when the sulphuric acid is dissolved in the water, some of its molecules are always broken up into parts, each of which carries a charge of electricity, one kind of part being positively charged hydrogen; the other kind, negatively charged sulphion.

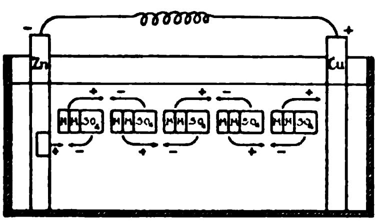


FIG. 164. THE IONS CHANGE PARTNERS

It is conceived that the solution is in an unstable condition. Some of the molecules are always broken up into these electrically charged parts, or IONS, as they are called, and the ions are continually recombining to form molecules. If this is really the case, it is easy to see that as a result of electrical attractions between the oppositely charged ions, a series of combinations would take place, as indicated by the arrows in the diagram, and there would be a procession of positively charged hydrogen ions toward the copper, and of negatively charged sulphions toward the zinc.

276. The Ion Hypothesis. This hypothesis not only gives us an idea as to why the hydrogen comes off at the zinc, but also gives us some conception of a mechanism by which electricity may be conducted through a liquid and how the liquid may be broken up while conducting the current. It is also found to fit well with the molecular hypothesis, which we found it convenient to adopt while

studying heat, for it explains many phenomena of solutions, which would be very difficult to understand otherwise.

The plate toward which the + ions travel, the copper in this case, is called the CATHODE, and the one toward which the − ions travel, the zinc in this case, is called the ANODE. *The direction of the current is conceived to be from anode to cathode through the liquid, and from cathode to anode along the wire.*

277. Local Action. Another defect of the simple voltaic cell arises from the fact that the zinc continues to waste away, even when the circuit is open, so that no current is passing. This waste may be almost entirely prevented by amalgamating the zinc with mercury.

The reason for this will be understood from Fig. 165. Millions of little particles of carbon and iron exist as impurities in the plate, and, with the neighboring zinc particles, they form diminutive voltaic cells which give rise to little local currents, as represented by the arrows in the diagram. As these little currents travel in short circuits, and never get out to the conducting wires, all their energy is transformed into heat and is wasted.

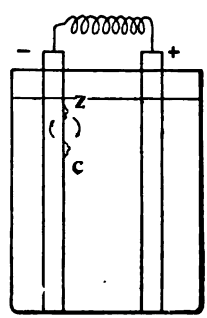


FIG. 165. LOCAL ACTION

The mercury prevents this LOCAL ACTION by dissolving zinc to form a zinc amalgam, which spreads over the zinc plate and covers up the impurities.

278. Commercial Cells. Very early in the history of the simple voltaic cell, it was found to be inefficient. Accordingly, several modifications have been made to increase the output.

1. Since the voltage depends on what substances are used for the plates and the active fluid, various metals and fluids have been tried. Carbon or copper is now used almost exclusively for the anode, zinc for the cathode, and either sulphuric acid or ammonium chloride (sal ammoniac) for the active fluid.

2. The internal resistance is diminished (*a*) by giving the plates as large a surface as possible, and (*b*) by diminishing the distance between them? Why?

3. Polarization is remedied by using some oxidizing agent, i.e., a substance that easily breaks down chemically and gives off oxygen, which combines with the hydrogen to form molecules of water. Polarization is prevented by introducing a second fluid, which deposits a metallic ion instead of hydrogen.

279. The Chromic Acid Cell. This is the best kind of cell for amateur use and for schools that are not equipped with a direct current dynamo, or storage battery. The anode is carbon, the cathode zinc, the active fluid sulphuric acid, and the depolarizing agent chromic acid.

280. The Leclanché Cell. Many forms of this cell are sold under various trade names. The anode is carbon, the cathode zinc, the active fluid ammonium chloride, and the depolarizing substance a black powder, called manganese dioxide. This is compacted around the carbon, or inclosed with it in a porous cup of either carbon or earthenware. This type of cell polarizes very rapidly in spite of the action of the manganese dioxide, and hence should never be used in a circuit that is to be kept closed for any considerable time. It is a very good cell for door bells, electric gas lighting, local battery for telephone transmitters, etc., when the current is used during short and infrequent intervals. The so-called dry cells are of this type.

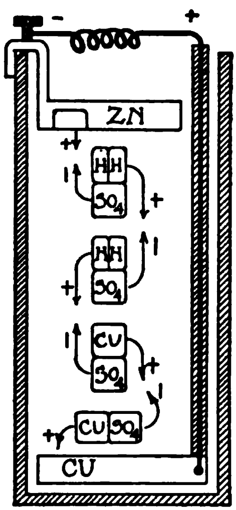


FIG. 166. GRAVITY CELL

281. The Gravity Cell. This is a two-fluid cell. The anode is zinc, and is placed at the top of the cell; the cathode is copper, and is placed at the bottom. The active fluid is very dilute sulphuric acid, and floats on the depolarizing fluid (a solution of copper sulphate), which is heavier than the sulphuric acid, and so remains at the bottom. Polarization can not occur when the cell

is in good condition, as will be seen by reference to Fig. 166. This diagram shows that since the copper cathode is surrounded by a copper sulphate solution, there are no hydrogen ions to be liberated there. Copper ions are liberated instead, and these can do no harm.

Gravity cells must be kept in a closed circuit with a large external resistance, and if the zincs are large and an excess of crystallized copper sulphate is kept in them, they last for a long time and without attention, and give a very steady current. They are in common use for telegraphic work, except on long distance lines, where dynamos are now used.

282. Electrolysis. In the voltaic cell, ions combine and give up their charges, producing an electric current. This process is a reversible one, for if a current from a dynamo or voltaic battery be sent between two plates of metal immersed in a solution that contains a salt of any metal, the ions of this salt immediately begin to progress in opposite directions, the metallic or positively charged ions going toward the cathode, and the non-metallic, or negatively charged ions, going to the anode.

This process of breaking up a liquid compound by passing a current through it, is called ELECTROLYSIS (*cf.* Art. 208).

The liquid is called an ELECTROLYTE, OR ELECTROLYTIC CONDUCTOR. The anode and cathode plates are called ELECTRODES. The relations which exist in connection with electrolysis were first thoroughly investigated by Sir Humphry Davy, who discovered metallic sodium and potassium by this process, and by means of it, made many important advances in chemical science. Davy's work was followed up and greatly extended by Faraday, who discovered and announced the following relations:

283. Faraday's Laws of Electrolysis. 1. *The amount of chemical action is the same in all parts of the circuit.*

2. *The mass of any kind of ion liberated at an electrode is proportional to the quantity of electricity that passes, i.e., to the product, amperes × seconds.*

3. *The mass of any kind of ion liberated by a given quantity of electricity is proportional to its chemical equivalent, i.e., to the*

number of grams of it that combine with a gram of hydrogen (or replace one gram of hydrogen in combination).

Knowledge of the laws of electrolysis has been employed in many ways, some of the most interesting of which will be described presently, but none of its applications is so important as its use as a means of investigation in theoretical chemistry, upon which all practical chemistry is based. Among the other uses of electrolysis we may mention the measurement of current strength for the purpose of standardizing galvanometers, voltmeters, and ammeters, the reduction of metals from their ores, the refining of crude copper for electric conducting wires, the making of electrotype plates from which books are printed, electroplating, and the storage battery.

284. Electroplating. Fig. 167 shows an electroplating bath. It is usually a large vat, containing a solution of some compound of the metal which is to be deposited. Thick copper rods, or "bus bars," are laid along its length, and two of these, $B+$ $B+$, are connected with the + feeder from a dynamo, the other with the − feeder.

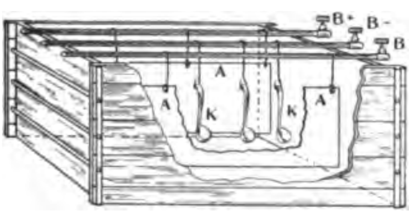


FIG. 167. AN ELECTROPLATING BATH

From the + bus bars are suspended, as anodes, large plates AA of the metal to be deposited, and from the − bus bar are hung, as cathodes KK, the articles to be plated. When the current is passed, the procession of metallic ions goes toward the cathodes, giving them the desired coating, while for every ion deposited on the cathode, an ion from the anode goes into the electrolyte and takes its place.

The electrolyte thus remains constant in strength, while the anodes lose as much metal as the cathodes gain.

285. Storage Batteries. The storage battery is the converse of the electroplating cell; the electrolyte is dilute sulphuric acid and the electrodes are perforated lead plates, photographs of

which are shown in Figs. 168 and 169. The perforations in these plates are filled with oxides of lead. When a current from a dynamo is passed through this cell, hydrogen ions go to the cathode and reduce the lead oxide there to metallic lead. Simultaneously, sulphions go to the anode and there give up oxygen, which combines with the lead oxide to form peroxide of lead, i.e., an oxide with a greater proportion of oxygen in it. When all the oxides are thus changed, the cell is said to be fully charged; and this condition is indicated by a copious evolution of hydrogen and oxygen gases at the electrodes. This charged cell is in a highly polarized condition (*cf.* Art. 274); it has a counter

FIG. 168. STORAGE BATTERY, POSITIVE PLATE

FIG. 169. STORAGE BATTERY, NEGATIVE PLATE

E. M. F. of polarization equal to about 2 volts, which tends to send out a reverse current.

In fact, if 55 of these cells in series are charged by a small 110 volt dynamo, and the belt by which the dynamo armature is driven be thrown off, this reverse current will go back through the dynamo and run it as a motor. *The storage battery, then, is a battery in which electrical energy is transformed into chemical potential energy and stored up for future use.* The reversed procession of the ions begins whenever the electrodes are joined through an external conductor, and the chemical energy is reconverted into electrical energy, ready to do any kind of work.

Storage batteries are used for running automobiles, but their greatest use at present, is in large power plants, where they are employed to store energy when the demand is light and pay it out again when the demand is extra heavy. Fig. 170 shows such a storage battery, belonging to a large power plant. They have not yet come into very extensive use for automobiles because of their great weight and liability to deterioration when not properly cared for. There ought, however, to be a great future for them in connection with windmills, because energy could be stored up in them when the wind was blowing strongly and taken from them when the wind was too light to operate the mill.

Fig. 170. Storage Battery Plant

286. Retrospect. Before leaving the study of electricity and magnetism, it may be useful to review some of the important things we have learned about them. We have discovered that when two bodies, made of different substances, are brought in contact and then separated, each is electrically charged, one positively, and the other negatively. These electrically charged bodies have been found to act on other bodies not in contact with them; and this action takes place not only through dielectrics, but also through a vacuum. We have seen that a magnet acts on another, or on a magnetic substance, although air, or wood, or other substances are between them. We have learned how an electric current is generated by chemical action in a voltaic cell, and how such a current is equivalent to electrostatically charged particles in rapid motion. Finally, we found that a magnetic field may be obtained by passing a current through a wire or coil; and conversely, that a current may be produced by moving a magnetic

field or changing its strength in the neighborhood of a closed conducting circuit.

All of these effects, produced by charged bodies, currents, and magnets, take place without apparent connection between the bodies that so act. Since it is hardly conceivable that these actions are effected without any connection whatever, we are constrained to assume that they are produced by stresses in some medium. If we have to suppose that a medium exists for these phenomena, is it not simpler to conceive that this medium and the one that transmits the heat waves are the same? Let us then adopt the hypothesis that these electric and magnetic phenomena are manifestations of stresses of some kind in the ether. We shall have occasion to recall this hypothesis before we finish the study of the other branches of this subject.

SUMMARY

1. Current strength is measured in amperes, electromotive force in volts, and electrical resistance in ohms.

2. The resistivity of a substance is the resistance at 0°C. of a conductor of the substance having unit length and unit cross-sectional area.

3. The electrical resistance of any substance may be found from the equation:

$$\text{Resistance} = \frac{\text{Resistivity} \times \text{Length}}{\text{Area of Cross-section}}.$$

4. Ohm's Law is expressed by the equation:

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}}.$$

5. Joule's Law. The power consumed in an electrical conductor is found by the equation:

$\text{Watts} = (\text{Amperes})^2 \times \text{Ohms} = \text{Volts} \times \text{Amperes}$: the heat developed, by the equation:

$$\text{Calories} = \text{Watts} \times .24.$$

6. One horse-power = 746 watts, or $746 \times 10^7 \frac{\text{erg}}{\text{sec}}$.

7. Electricity is distributed in series and in parallel circuits; sometimes by a combination of both.

8. Economy of electrical transmission is secured by using high electrical pressures.

9. Electrical power is utilized by means of motors. Electrical heating is done by means of coils of high resistance, or by suitable electric furnaces in which the heating action is somewhat similar to that of the arc light.

10. In divided circuits, the current in each branch or "shunt," is inversely proportional to the corresponding resistance.

11. Delicate electrical apparatus may be used with large currents if suitable shunts are employed.

12. In a voltaic or electrolytic cell the electricity is conducted by a progression of + ions toward the cathode and of − ions toward the anode.

13. The polarization of a voltaic cell may be remedied by oxidizing the hydrogen as in the chromic acid or the Leclanché type; or it may be prevented by employing a second solution from which are deposited ions of the cathode metal, as in the gravity type.

14. When cells or other electric generators are joined in series, the total voltage is the sum of the voltages of all, and the total resistance is the sum of all the resistances of the circuit.

15. Faraday's Laws of electrolysis state that the mass of an ion liberated is equal to its electro-chemical equivalent multiplied by the product of the current strength and the time.

16. The most common and important applications of electrolysis are: 1. Current measurement. 2. Electroplating and electrotyping. 3. Reduction of metals from their ores, and the refining of crude metals. 4. Storage batteries.

PROBLEMS

1. How many amperes are there in a current which deposits 16.77 gm silver in 50 minutes? How much silver will be deposited by a 2 ampere current in 30 minutes?

2. What electromotive force will send a current of 10 amperes through a lamp whose resistance is 4.8 ohms? What is the resistance of an arc lamp which takes a current of 15 amperes at a pressure of 55 volts?

3. What is the rate (watts) at which each of the lamps of problem

2 consumes energy? Find the mechanical equivalent (horse-power hours) of the energy used by each in 12 hours.

4. Find the voltage and the power (K. W.) of a dynamo that will operate a series arc lamp circuit, the data being as follows: Number of lamps, 25; volts for each lamp, 45; current strength, 10 ampères; length of line circuit 2500 ft.; resistance of wire (ohms per thousand ft.), 2.5. What horse-power must the engine supply to the dynamo, if we allow an efficiency of 80 per cent for the dynamo and belt?

5. How many gm cal of heat may be obtained from a current of 2000 amperes at a pressure of 12 volts? How many Kg of copper may be raised from 14° C. to its melting point (1054°) by this heat if the mean specific ht. of copper for this temperature range is 0.105?

6. The following problem illustrates the way in which automobile engines are tested in a certain large factory. A car was lifted on jack screws, and the driving wheels were belted to a dynamo. When the angular velocity of the drivers corresponded to a linear car speed of 25 $\frac{\text{miles}}{\text{hour}}$ the dynamo was able to light 128 16-candle-power glow lamps connected in parallel circuit. The meters showed that the lamps were taking half an ampere each at 110 volts pressure. They were so near the dynamo that there was no line drop. What were the total current strength and the electromotive force of the dynamo? Its output (watts)? What H.P. did it take from the engine, allowing for 25 per cent loss in the dynamo and belt? The answer corresponds to the horse-power developed by the engine when the car has the given speed on a smooth, level road.

7. A waterfall 6.10 m high delivers 15,000 Kg water per minute. What is its H.P.? What H.P. may be delivered from it by a water-wheel having an efficiency of 70 per cent through a dynamo having an efficiency of 85 per cent? The output of this dynamo will equal how many watts? If its electromotive force is 125 volts, what current will it supply?

8. The same fall, measured in foot and pound units, gives; pounds of water per minute 33,000, height 10 ft. Answer the questions and compare the answers with those of question 7.

9. The dynamo, problems 7 and 8, delivers its current to a number of glow lamps in parallel, with a 10 per cent drop in voltage through the feeders. What is the voltage through the lamps? The current delivered by the feeders? Allowing 0.5 amperes per lamp, how many lamps may be operated? Calculate the line resistance and the watts lost in the line, from the data here given. If the line is 500 feet long, consult the wiring table, p. 298, and find the gauge number of the proper size of wire to use?

10. Suppose in the lamp circuit, problem 9, a pair of branch

wires have a length of 100 ft. and supply 20 lamps, what current must they carry? They are to be chosen so as to have a 2.5 volt drop. What is their resistance? From the wiring table find the gauge number to be selected. Find the rate (watts) at which energy is lost in these branch wires, and in the feeders. Find the heat developed (gm cal) in each case.

11. An electric bell has a resistance of 10 ohms and works perfectly when connected with short wires to one dry cell having an electromotive force of 1.5 volts, and an internal resistance of 0.3 ohm. What current is it then using? The same bell is connected on a door bell circuit of 150 ft. of number 20 copper bell wire, and although the current is found to be complete, it does not work. Mention two ways in which the trouble might easily be remedied. From the wiring table find the resistance of the line wire and again calculate the current strength. Suppose two more cells just like the one in the last problem were connected in series with it and the circuit. Calculate the resulting current.

12. In table II, p. 299, the resistivities of some metals are given, the resistivity for this table being defined as the resistivity in ohms of a conductor one meter long and one square millimeter (1 mm^2) in cross-sectional area. From this table and the laws of resistance (Art. 253), find the resistances of the wires for which the following data are given: 1. A silver wire 3 m long and 0.1 mm in diameter. 2. A German silver wire 20 m long and 0.5 mm^2 in cross-section. 3. A lead wire 0.6 m long and 0.8 mm^2 in section.

13. From the table of resistivities express the resistances of each of the materials in terms of the resistance of copper as a unit; thus, other things being equal, the resistance of a platinum conductor is how many times that of a copper one?

14. A galvanometer has a resistance of 5 ohms, and it is desired to use it with a current of 10 amperes; but the greatest current that can be sent through it without either injuring it or causing its deflections to be too great to read is 0.1 ampere. What must be the resistance of a shunt that will produce the desired result when connected across its terminals?

15. In Fig. 171 a current divides at A and reunites at B. A galvanometer G is connected across, as shown. If the current on $A\ C_1\ B$ is C_1, and that on $A\ C_2\ B$ is C_2, show that the pressure on the part $A\ C_1$ is $e_1 = C_1r_1$, and that on the part $A\ C_2$ it is $e_3 = C_2r_3$. Now, if the point of contact C_2 be moved along $A\ C_2\ B$ toward A or B till a place is found such that no deflection of the galvanometer occurs, show that under this condition the points C_1 and C_2 are equi-potential; i.e., there is no difference in electrical pressure at these two points. Prove that when this is the case $e_2 = e_3$, hence $C_1r_1 = C_2r_3$ (a). In a

similar way, prove that under the same conditions $C_1r_2 = C_2r_4$ (*b*). Divide the equation (*a*) by the equation (*b*) and simplify. What relation has been proved to exist among the four resistances, when no current passes through the galvanometer?

16. Suppose in the last problem the resistances r_2, r_3, r_4 can be varied at will, and that r_1 is a certain unknown resistance whose value we wish to find. For example, we make $r_3 = 1$ ohm, $r_4 = 100$ ohms, r_1 the unknown, and then begin to vary r_2 until we have found a value for it which will leave the four resistances so adjusted that no current passes through the galvanometer. Now r_1 is the only unknown quantity, and the equation obtained in the previous problem applies, for the conditions are the same as there supposed. What is the numerical value of r_1 when $r_2 = 250$?

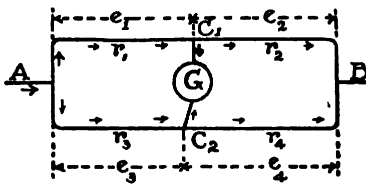


FIG. 171

17. Show that if *A C B* is made of a wire of uniform cross-sectional area and material, the ratio of the lengths of the two parts $A\ C_2$ and C_2B may be substituted for the ratio of the actual resistances, and give the same numerical value as before for the unknown resistance. Apparatus arranged to measure electrical resistance in the ways suggested by problems 15–17 is called a Wheatstone bridge, and is of great service in practical electrical work as illustrated in this chapter.

18. Does the electromagnet, Figs. 161, 162, really do any "lifting," or does it simply *hold* while the crane-hoist to which it is attached does the lifting? Then does such a magnet use a great amount of energy?

SUGGESTIONS TO STUDENTS

1. If you are permanently interested in electricity or expect to go farther in the study of it than this course can take you, you ought to own these two books; *Elementary Lessons in Electricity and Magnetism*, by S. P. Thompson, and *Elementary Electricity and Magnetism*, by D. C. and J. P. Jackson (both published by Macmillan, New York).

2. Find out from books, or the bulletins of electrical manufacturers, or from some friend who is an electrician, what a "booster" or rotary transformer is, and how it is useful in connection with a storage battery for an electric railway power plant, or an electric light station. If you can get the information, write a short paper about it.

3. From similar sources, get information about the use of a starting resistance with a shunt motor, as shown at *R*, Fig. 135, and put the results into the form of a written report. Can you find out anything about the counter-electromotive force of a motor in this connection?

About the danger of a "field discharge" when a shunt motor is suddenly shut off, without having in circuit with the armature a suitable starting resistance like that mentioned? See *Practical Electricity*, by J. C. Lincoln (published by the Cleveland Armature Works, Cleveland, Ohio), which is excellent on the subjects treated in this chapter.

4. Find out also how the electromotive force of the shunt coils of a shunt or compound wound dynamo may be regulated for varying loads by means of a variable resistance, placed in the same branch of the circuit with the shunt coils of the field magnets, as in Fig. 150.

5. Visit an electrotype foundry or electroplating shop, find out what you can and make a report.

6. Find out what you can about electric elevators and hoists.

TABLE I

Characteristics of Copper Wire

Number.	Diameter.	Cross-Section.	Weight	Resistance.	Amperes.	Diameter.	Number.
Brown & Sharp's Gauge.	Mils 1 mil = .001 inch.	Circular Mils. d^2.	Pounds per 1000 ft.	Ohms per 1000 ft. at 75° Fahr. = 24°C.	Safe Carrying Power.	Millimeters 1 mm = 0.001 cm	Brown & Sharp's Gauge.
0000	460	211600	641	0.04966	210	11.684	0000
000	410	168100	509	0.06251	177	10.405	000
00	365	133255	403	0.07887	150	9.266	00
0	325	105625	320	0.09948	127	8.254	0
1	289	83521	253	0.1258	107	7.348	1
2	258	66564	202	0.1579	90	6.544	2
3	229	52441	159	0.2004	76	5.827	3
4	204	41616	126	0.2525	65	5.189	4
5	182	33124	100	0.3172	54	4.621	5
6	162	26244	79	0.4004	46	4.115	6
8	128	16384	50	0.6413	33	3.264	8
10	102	10404	32	1.01	24	2.588	10
12	81	6561	20	1.601	17	2.053	12
14	64	4096	12.4	2.565	12	1.628	14
16	51	2601	7.9	4.04	6	1.291	16
18	40	1600	4.8	6.567	3	1.024	18
20	32	1024	3.1	10.26	Rubber	0.812	20
24	20.1	404	1.2	26.01	Covered Insulated	0.511	24
28	12.6	158.8	0.48	66.18	Wire.	0.321	28
30	10.0	100.0	0.30	105.1		0.255	30
32	8.0	64.0	0.19	164.2		0.202	32

The following table gives the resistivities of some metals, the resistivity for this table being defined as the resistance at 0° C of a conductor of the metal having a length of one meter and a cross sectional area of one square millimeter. The resistivity of German silver and other alloys varies with the composition.

TABLE II

RESISTIVITIES

Copper	0.017	Lead	0.210
Silver	0.016	Iron	0.100
Platinum	0.108	German silver	0.34

PRACTICAL USE OF TABLES I AND II

Electric wiremen often use a formula for determining the cross section of a wire of given dimensions and having a required resistance, and express this cross section in *circular mils*. A wire of circular cross section and one one-thousandth of an inch (1 mil) in diameter is said to have a cross section of one circular mil. They also define the specific resistance of a wire as the resistance of one mil-foot (i.e. 1 mil in diameter and 1 ft. long) at 75° Fah. The resistance of a mil-foot of good commercial copper wire is 10.5 ohms. If L represents the length of a wire in feet, CM its cross section in circular mils, show that its resistance $R=10.5\,\frac{L}{CM}$ ohms. Also that $CM=\frac{10.5\,L}{R}$. Wiremen also determine the cross section of a wire of a certain length, which will cause a given drop with a given current by the equation, $CM=\frac{10.5\,L\times A}{V}$ wherein CM represents circular mils, L the length in feet, A the amperes carried, and V the drop in volts. Show from the preceding formula and from Ohm's law that this one is correct.

From these formulas and tables I and II you can make up and solve as many wiring problems of this sort as you like.

Another good exercise will be to find the specific resistances in mil-foot units of the substances in table II by multiplying the resistance of a mil-foot of copper (i.e. 10.5) by each of the ratios obtained in problem 13, p. 296.

If you are taking the commercial course of your school, an interesting exercise will be to calculate the cost of the copper, also to get data from a friend who is an electrical contractor and draw up specifications and estimates for the two electrical plants described in this chapter.

CHAPTER XIV

WAVE MOTION

287. Of Waves. In Chapter VIII, while studying the transfer of heat from one body to another when there is no visible contact between them, we were led to adopt the hypothesis that heat energy is propagated across apparently empty space by some form of invisible wave motion. Similar reasons lead us to adopt a wave hypothesis to describe the phenomena of sound and light, and we can best appreciate how beautifully this theory fits all the facts together into an intelligible story, if we first make a little study of the waves with which we are all familiar. Let us, then, ask: How do waves originate? What properties must a medium possess in order to be capable of transmitting waves? What is the mechanism by which they are propagated?

288. Water Waves. We are all familiar with water waves; for who has not amused himself by throwing stones into still water and watching the charmingly symmetrical figures produced on its surface? There are few of us who are not in possession of some vivid mental pictures that may aid us in studying this most fascinating portion of our subject.

289. Origin of Waves. If we throw a stone into a pond and watch closely when it strikes the water, we notice that its first effect is to push aside the water at the place where it falls. This action lowers the level of the water surface, thus leaving a hollow behind the stone as it sinks from sight. Since the free surface of a liquid always strives, as it were, to remain level, the water, which has been thus thrust aside, rushes back as if to fill the hollow left by the stone. But when the particles rush in from all sides behind the sinking stone, they acquire kinetic energy, which carries them further than they intend. The result is that

water is now piled up in a little heap over the place where the stone fell.

The water particles then hastily retrace their steps, but again they are irresistibly carried past the position they desire to occupy, and again a hollow is formed in the surface of the pond; but this time it is not so deep as before. This process of piling up and retreating is repeated several times, the elevation being less marked each time, until the motion at the point where the stone struck ceases altogether. A back and forth motion of this sort is called a vibratory motion. We thus reach the conclusion that *waves originate at a point where a vibratory motion is forced on the medium by some outside body.*

290. Characteristics of Waves. Are the waves that spread out on the pond different when produced by a large stone from what

FIG. 172. WAVES FROM A LARGE STONE

FIG. 173. WAVES FROM A PEBBLE

they are when produced by a small one? On trying the experiment, we find that the waves caused by a large stone are larger than those produced by a small one. If we wish to compare them, we must agree on a method of measuring them. In order to appreciate what the characteristics of waves are, let us suppose that a succession of these water waves is suddenly frozen solid; the shape of the surface resembles the curve in Fig. 174. Examination of this curve shows that some parts of the wave are above the normal level of the water, while other parts are below it. The parts above the normal level are called CRESTS, those below it are called TROUGHS. The distance of the top of the crest from the

normal level is equal to the distance of the bottom of a trough from the same level and is called the AMPLITUDE of the wave. The WAVE-LENGTH is the distance between two successive crests, or between two successive troughs.

Applying these definitions to the cases of the large and small stone, we see that the waves started by the large stone have both a greater amplitude and a greater wave length than those about the small stone. Thus there is a relation between the nature of the vibration that started the waves and the characteristics of the waves, so that we can judge of the vibration by observing the waves.

FIG. 174. SHAPE OF A SIMPLE WAVE

291. What Waves Can Tell Us. We have just learned that we can form some idea of the magnitude of the disturbance that started the waves by noting the wave length and the amplitude of the wave. Furthermore, we can form some idea of the direction in which the point of disturbance lies by noting the direction in which the waves are traveling. Finally, we can infer something about the nature of the disturbance from the shape of the waves. Hence we see that waves may bring us four kinds of information concerning the source of the vibrations, viz.: 1. *The direction in which the waves are traveling indicates the direction of the source.* 2. *The length of the waves informs us as to the rapidity of the vibration.* 3. *The amplitude of the waves tells us of the violence of the disturbance.* 4. *The shape of the wave allows us to infer something concerning the nature of the vibrations of the source.*

292. Wave Motion. Another important fact may be learned from watching the water waves. If we throw a small chip on the surface of the pond, well out from the shore, and observe its motion when the waves pass it, we see that the chip is not carried along in the direction in which the waves move, but that it merely rises and falls while the wave motion passes beneath it. Since the chip indicates the motion of the water particles about it, we may conclude that the water does not move forward with the wave, but merely rises and falls as the wave passes.

This fact leads us to an important conception as to the mechanism of wave motion. Thus, let us consider a row of particles held together by cohesion or some other elastic force (Fig. 175). If the first particle is displaced in a direction perpendicular to the row, the force that holds the two together will compel the second particle to follow. But since the connection between the two particles is elastic, not rigid, the second will always lag a little behind the first. Hence, when the first particle has reached the end of its trip, the second will not have traveled quite so far, the third will lag a little behind the second, and so on. Therefore the condition of the row of particles, when the first one has reached its position of greatest displacement, will be that shown in Fig. 175.

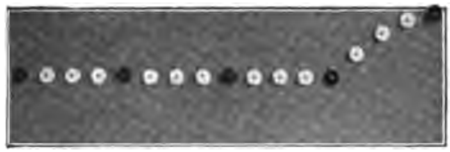


Fig. 175. Particle 1 has Executed ¼ vibration

Particle 1, having reached its position of greatest displacement, pauses there for a brief instant and then begins to retrace its steps. While this particle is stationary, 2 catches up with it and reaches its position of greatest displacement as 1 starts downward. Particle 3 follows 2 in the same way, and so on. Thus we see that the successive particles reach their positions of greatest displacement one after another. We may say that this position of greatest displacement is passed along from one particle to the next. But the position of greatest displacement constitutes the crest of the wave, and so we get a conception of the mechanism by which waves are propagated along a row of particles that are held together by cohesion, or any other elastic force. The positions of the particles when number 1 has returned to the starting point are shown in Fig. 176.

Fig. 176. 1 has Executed ½ Vibration

Now, when particle 1 reaches the position from which it started, i.e., when it has completed half a vibration, it is moving with considerable velocity. It therefore possesses kinetic energy. This energy will cause it to move past its original position and to make an excursion on the opposite side. Hence it will now move downward, dragging the adjacent particle after it, will reach a position of greatest negative displacement (Fig. 177), and return again to the starting point. The positions of the particles, when this has been done, are shown in Fig. 178. Particle 1 is now in the same condition in which it was when it began to move. Therefore, if nothing interferes with it, it will repeat the operation just described and continue to do so until its energy is expended.

FIG. 177. 1 HAS EXECUTED $\frac{3}{4}$ VIBRATION

293. Relative Positions of the Particles. Several important things are apparent from this discussion. In the first place, we note that when particle 1 has executed a complete vibration, the other particles along the wave have not yet done so. Each has performed only part of one vibration. Each successive particle has executed a smaller portion of one vibration than has the particle ahead of it and a larger portion than has the one behind it. Thus, when particle 1 has completed its first vibration, 17 is just ready to begin moving; 13 has executed $\frac{1}{4}$ of a vibration; 9, $\frac{1}{2}$ a vibration; 5, $\frac{3}{4}$ of a vibration, and the intermediate particles intermediate fractions of one vibration. It is convenient to have a simple word for expressing this relation. Therefore, the particles are said to be in different PHASES of vibration. *The phase thus means the portion of one complete*

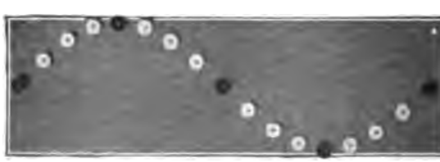


FIG. 178. 1 HAS EXECUTED ONE WHOLE VIBRATION

vibration that any particle has executed. So we may say that particle 16 has a phase zero, 12 a phase of $\frac{1}{4}$ vibration, 8 a phase of $\frac{1}{2}$ vibration, 4 a phase of $\frac{3}{4}$ vibration, and 1 a phase of 1 vibration.

294. Wave Length. Using this term "phase," we may make a general convenient definition of wave length; for we may say that *a wave length is the shortest distance between any two particles that are in the same phase.* Thus the distance 1 to 17, 2 to 18, and so on, is a wave length.

295. Time of Vibration. It is often convenient to consider the time it takes a particle to execute one vibration instead of considering the numbers of vibrations per second themselves. When a vibratory motion continues to be repeated in equal time intervals, it is called PERIODIC, and *the time taken by any particle in executing one vibration is called the period of that vibration.* Hence, when we are considering vibrations from this point of view, we may speak of a phase of a quarter period, of half a period, etc. Further, it is evident that if the source of vibration executes 10 vibrations in a second, the time it takes to execute one vibration, i.e., its period, is $\frac{1}{10}$ sec. Thus, in general, if the number of vibrations per second is represented by n, the period is always $\frac{1}{n}$ sec.

296. Velocity of Propagation. An important conclusion may now be drawn from the discussion of Fig. 178; for we note that while particle 1 has been executing one vibration, the disturbance has traveled a distance 1 to 17. This distance is a wave length. Hence, it is manifest that the disturbance travels along the row just the distance of one wave length while particle 1 executes one vibration. If particle 1 executes n vibrations in a second, how far will the disturbance travel in that second? Evidently n wave lengths. But the distance traveled in one second measures the velocity (*cf.* Art. 2). Hence, we may express this result by saying that *the velocity with which a wave travels i-*

numerically equal to the product of the number of vibrations per second and the wave length. If v represents the velocity, n the number of vibrations, and l the wave length, then

$$v = nl. \tag{14}$$

This simple relation enables us to determine the velocity of the waves when we can measure the wave length and the number of vibrations per second of the source. This equation, however, does not tell us how this velocity depends on the properties of the medium through which the waves travel. We can get a general idea of how the properties of the medium affect the velocity with the help of equation (4), Art. 27. For $f = ma$, therefore $a = \frac{f}{m}$. In this case f is the elastic force acting between two adjacent particles of the medium, m the mass of a particle, and a the acceleration given to the particle by the force f. Since a is proportional to f, the equation shows that if f is increased, particle 2 will have a greater acceleration; so it will follow faster after 1. For the same reason 3 will follow faster after 2, and so on; therefore the disturbance must travel faster along the row of particles.

On the other hand if the density of the elastic medium is greater, each particle will have a greater mass m. In this case the equation shows that the particles will have smaller accelerations, so each one will move more slowly and lag more behind the one just ahead of it; therefore the disturbance will travel more slowly.

The exact relation of the velocity of a wave in a medium to these two factors, elasticity and density, has been determined mathematically and by experiment; and it has been found that the velocity v of waves traveling in a medium having an elasticity e and a density d is

$$v = \sqrt{\frac{e}{d}} \tag{15}$$

297. The Types of Waves. In the discussion thus far we have confined our attention to waves in which the motion of the

particles is perpendicular to the direction in which the waves travel. When this is the case, the waves are said to be TRANSVERSE. We have also pictured the motion of each particle as taking place along a straight line. These restrictions were introduced in order to simplify the discussion, though neither one is essential. Thus we may just as well have waves in which the paths of the particles are circles or ellipses in planes perpendicular to the direction of propagation of the wave; or we may conceive that the particles move back and forth in the direction of propagation of the waves. In this latter case the particles are alternately crowded together and separated, so that we have to deal with condensations and rarefactions of the medium instead of with crests and troughs. Waves of this type are called LONGITUDINAL. We shall learn more concerning both types of waves in the following chapters.

298. Waves of Simple Shape. In this discussion it has been stated that we may draw conclusions as to the nature of the vibration by a study of the shape of the wave. In Fig. 174 we have drawn a wave of particular shape. What sort of vibrations were executed by the body from which these waves proceeded? From the simplicity of the shape of the curve we may imagine that the vibration must be of a simple type. Now, this type of vibration is that executed by a pendulum, as may be easily shown by fastening a small pencil to the bob of the pendulum and drawing a card under it in a horizontal direction, and in such a way that the pencil writes a trace on the card while the pendulum is vibrating. If we do this, we find that the shape of the curve obtained is the same as that shown in the figure. The pendulum has thus been made to construct a graph which represents the relation between the displacements and the corresponding times for its own motions. Vibrations of this type are called SIMPLE HARMONIC VIBRATIONS, and the curves that represent them graphically are called SINE CURVES.

Since every particle in the wave executes the same kind of vibrations as the source does, the shape of the waves that originate from a simple harmonic vibration will be that of a sine curve.

When we have to deal with a simple harmonic motion of one definite period only, the corresponding waves are called HOMOGENEOUS. All other forms of waves are COMPLEX. It is perhaps unnecessary to remark that the ones with which we are actually familiar are in every case complex.

299. Complex Waves. If all waves which we know in nature are complex, why do we study simple homogeneous waves at all? In answer to this question, let us consider what happens when we have two or more simple homogeneous waves traveling through the same medium at the same time. The result is most easily obtained from a diagram. Let us begin with the case shown in Fig. 179, and suppose that the two simple waves have the same period, amplitude, and phase. The disturbance that results when these two waves are traveling through the same medium at the same time is obtained by adding as vectors the displacements of the particles. The result is shown by the lower curve in the figure. We note that the resultant is a wave of the same period and phase, but with twice the amplitude.

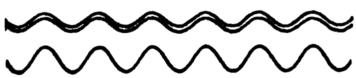


FIG. 179. SAME PERIOD, AMPLITUDE, AND PHASE

Repeating this operation for two waves that have the same period and amplitude, but differ in phase by one-half period, we get the result shown in Fig. 180. This result shows that two such waves may be traveling in the same medium without giving any external sign of their presence.

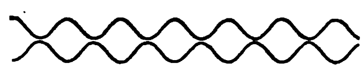


FIG. 180. SAME PERIOD AND AMPLITUDE, OPPOSITE PHASES

If we were to add together two waves of the same period, but differing in phase by $\frac{1}{4}$ of a period, or by any other fraction of a period, we should find that in every case the resultant wave has the same form as the constituent waves. Hence we may conclude that *the addition of any number of simple homogeneous waves of a given period always gives a resultant which is also a simple homogeneous wave of the same period.*

300. Waves of Different Shapes. If the addition of simple waves of the same period always gives as a result a simple wave, how may we produce waves of complex form? Let us see what the effect will be if we add together two waves that have different periods. Take, for example, the two waves drawn in Fig. 181, one of which is twice as long as the other and has twice the amplitude. We note that the resultant *R* obtained by adding them together is a wave differing entirely in shape from the two component waves. Let us now add to this resultant a third simple wave, with period and amplitude each one-third of the first period and amplitude. The result is shown at *R′*. The resultant obtained by adding together waves whose periods and amplitudes have the ratios 1, $\frac{1}{3}$, $\frac{1}{5}$ is shown in Fig. 182. The meanings of the curves will be perfectly clear on careful inspection.

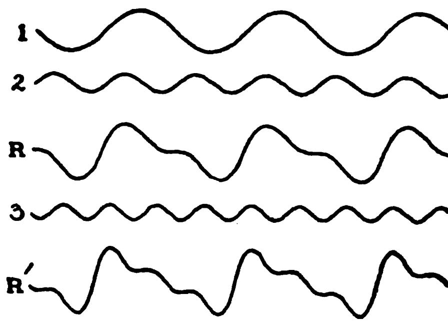


FIG. 181. ADDITION OF WAVES OF DIFFERENT PERIODS

A study of these curves will make it apparent that we can produce waves differing greatly from one another in shape by adding together simple homogeneous waves which differ from one another in period as well as in amplitude and phase. That the number of shapes which may be produced by the addition of such waves must be indefinitely great, may be realized by any one who considers that we have at our disposal, first, an indefinite number of possible periods; second,

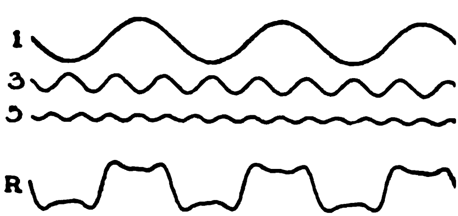


FIG. 182. ANOTHER COMPLEX WAVE

an indefinite number of possible amplitudes; and, third, an indefinite number of possible phases.

But even if the number of shapes that can be so artificially built up is practically without limit, is the converse proposition true, viz., that every shape that actually occurs in nature can be resolved into a series of simple homogeneous waves differing from one another in period, amplitude, and phase? This problem occupied the attention of mathematicians and physicists for a century and a half before its final solution was reached. That solution proved conclusively that this converse proposition is true, and shows us how to proceed in order to separate the compound wave into its component simple homogeneous waves. Hence, the importance of studying the nature of the simple waves becomes manifest, for it follows that *all waves with which we are familiar in nature are built up of these simple homogeneous waves.* The study of SIMPLE HARMONIC MOTION, which gives rise to these simple homogeneous waves will be deferred to the next chapter.

301. Stationary Waves. There is still another kind of waves which we have not yet discussed, but which, nevertheless, merits attention. A jumping rope, one end of which is fastened to a tree, must be turned at a certain rate in order to swing properly. It is a matter of common observation that by turning it faster the rope may be made to break into two equal loops separated by a point where the rope moves very little. If the hand is turned still faster and the rope is long enough, the rope may be made to vibrate in three, four, or more parts. How are these loops formed, and why does the rope stay nearly still in certain places?

An analysis of the operation will give us the answer. When the hand is moved periodically it impresses a certain vibration on the rope. The rope may be looked upon as a row of particles held together by elastic forces, and so the vibratory motion of the hand is propagated along the rope in the form of a wave, until it reaches the other end. What happens then? Does the wave give up all its energy to the tree. or is part of that energy reflected so as to travel back along the rope? You may easily show, by holding the rope rather tight and hitting it suddenly, that the

wave is reflected; for the hump raised on the rope by the stroke may be seen to travel to the far end of the rope and then to turn around and come back. Therefore, when you send a series of impulses along the rope they travel in the form of waves, are reflected at the further end, and return. We see, then, that if the series of impulses be continued, we shall soon have generated two trains of waves, the direct and the reflected, traveling along the rope in opposite directions. We may infer that, when this is the case, the result is the peculiar vibration that we get on the rope. Let us see if this is so.

In the case we are considering, the two trains of waves have the same period and nearly the same amplitude, and are traveling along the rope at the same rate but in opposite directions. They are represented by curves A and B, Fig. 183, A moving to the right and B to the left. When the waves are in the positions indicated at V and D in the diagram, the resultant obtained by adding their displacements is shown by the thick black line.

If now we conceive each of the two trains of waves to have advanced $\frac{1}{4}$ wave length, A to the right and B to the left, their respective positions are those shown at W and E in the diagram; and the resultant will be the black line WE.

Two more advances, of $\frac{1}{4}$ wave length each, bring the two into the position shown at X,F and Y,G respectively. Clearly the resultants will be as there shown. A final advance of $\frac{1}{4}$ wave brings the two into a position similar to their original positions, so that the resultant for Z,H is the same as that for V,D.

When these five resultants are superposed, we obtain the curves shown at R,N,C. The similarity between this figure and the rope in the case under consideration will at once be noted, for certain of the points never leave their positions of equilibrium, while others execute vibrations of greater amplitude. The positions of no amplitude are called NODES, while those of great amplitude are called LOOPS. Since the nodes remain at rest with respect both to vibratory motion and also to the motion of propagation, such waves are called STATIONARY WAVES.

When we analyze the motion of the particles in these waves we see (1) that all the particles that move are in their positions

of greatest amplitude at the same instant; and (2) that they are all in their positions of equilibrium at the same instant as shown by the straight line RNC. We note further (3) that all the particles in one loop are in the same phase at the same time, but that their respective amplitudes are different, for the particles at the

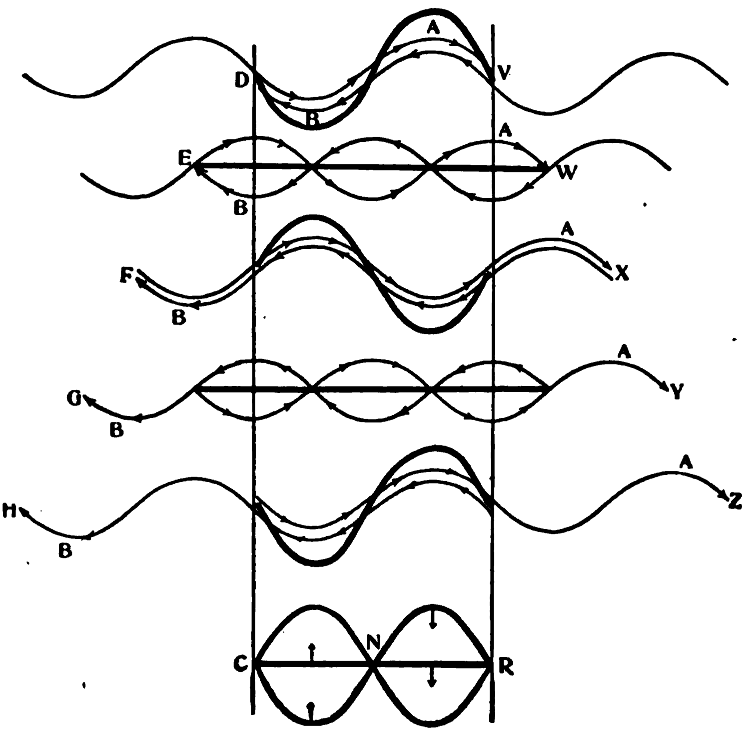


FIG. 183. FORMATION OF STATIONARY WAVES

middle of the loop have a large amplitude, while those near the nodes have a small amplitude. Another fact worthy of remark is (4) that the particles in one loop have a phase that is different by half a period from that of the particles in either of the adjacent loops. And finally, we see (5) that *the distance between two nodes is half a wave length.* Thus it becomes clear that *such stationary waves are actually produced by two*

trains of waves of the same period traveling along the same row of particles in opposite directions. The relations between the period of vibration and the length of the loop will be taken up in a later chapter, for these stationary waves play a conspicuous part in the phenomena of sound, and so we shall have to discuss them further when considering that subject.

SUMMARY

1. Waves originate at a vibrating body.

2. Waves bring us four kinds of information: 1. As to the direction of the source. 2. As to the period of the vibrations. 3. As to their amplitude. 4. As to their complexity.

3. The characteristics of waves are direction of propagation, length, amplitude, and shape.

4. A suitable medium is necessary for the transmission of waves.

5. The particles of a medium do not partake of the progressive motion of a wave, but merely vibrate about their positions of equilibrium.

6. In progressive waves the successive particles are in different phases at the same time.

7. Waves may be transverse or longitudinal.

8. The velocity of propagation of waves is equal to the number of vibrations multiplied by the wave length. ($v = nl$.)

9. The velocity of waves in an elastic medium is equal to the square root of the elasticity divided by the density. $v = \sqrt{\frac{e}{d}}$.

10. When waves are superposed, the resultant is the algebraic sum of the components.

11. The simplest kind of wave is the simple homogeneous or sine wave.

12. The vibrations that produce these simple waves are called simple harmonic vibrations.

13. Waves of complex form result from superposition of simple homogeneous waves of different periods, amplitudes, and phases.

14. Waves of complex form may always be analyzed into a series of simple homogeneous waves.

15. Stationary waves are produced when two trains of waves of equal period, but traveling in opposite directions, are superposed.

QUESTIONS

1. Describe the motions of water at the point where a stone is dropped into it.

2. What sort of information is derived from each of the four chief characteristics of waves?

3. Describe the mechanism of wave propagation. Do the vibrating particles partake of the progressive motion of the waves?

4. What two types of waves may we have? What is the distinctive feature of each?

5. What can you say of the relative phases of the successive vibrating particles along a progressive wave?

6. How may wave length be defined with reference to phase?

7. Does it seem reasonable to suppose that waves can travel through empty space?

8. What is a simple homogeneous wave? From what kind of vibration does it originate?

9. When two or more waves are traveling at the same time in the same medium, how do we find the resultant wave?

10. What sort of vibrations does a pendulum execute? How may we obtain a graph to show this?

11. Explain how stationary waves are produced?

12. What are nodes and loops?

13. How is the distance between two adjacent nodes related to the wave length?

PROBLEMS

1. What is the period of vibration of an oarsman who makes 40 strokes per minute? Of a swing that makes one complete oscillation in 4 sec? In 6 sec? Of a wagon seat that makes two complete vibrations in 1 sec?

2. How many complete vibrations per sec does a tuning fork make, if its period is $\frac{1}{256}$ sec? What is the period of the waves started by a paddle-wheel that has 6 paddles, and makes 20 revolutions per minute? If these waves are 2 ft. long, what is their velocity?

3. Air and hydrogen have the same elasticity, under given conditons, but air is 14.5 times as dense as hydrogen. Supposing sound to be a wave motion, ought it to travel faster or slower in hydrogen than in air? How many times?

4. What is the length of the waves that are traversing a jumping rope 30 ft. long when it is vibrating in 1 loop? In 2? In 3? In 4? In 5? If the period is 1 sec when it is vibrating in 1 loop, what is the period in each of the cases just supposed? What is the velocity of each of the waves?

5. Plot the following cases of simple homogeneous waves traveling in the same direction: 1. Two waves of equal period, amplitude, and phase. 2. Two waves of equal amplitude and period, but of opposite phase. 3. Two waves of equal amplitude and period, but with a difference of phase of ¼ period.

6. Plot the following cases of simple homogeneous waves traveling in the same direction: 1. Two waves of equal phase, one having half the amplitude and half the wave length of the other. 2. Combine this resultant with a third wave whose length and amplitude are each ¼ that of the first wave.

7. Were you ever out in a boat when the waves were running fairly high, or have you ever floated on your back among them? In that case you had an excellent opportunity to observe carefully the kind of motion that the water particles in the wave have. Did the water move you up and down only, or was there compounded with this up and down vibration another that was nearly horizontal? What was the resultant path or *orbit* of the water particles which were carrying you with them as they oscillated?

8. Suggest a way for producing transverse waves in a long rubber tube. How may you produce longitudinal waves in it? Can water react elastically to forces that tend to compress it longitudinally as well as to forces that tend to displace it laterally? Do air particles cling together as water particles do? Can air resist both transverse and longitudinal stresses? If not both, which?

9. In which direction (longitudinal or transverse) does a rubber tube offer the greatest elastic resistance, to a force producing a given displacement? Which kind of waves then (longitudinal or transverse) will travel faster in the rubber? Answer the same questions for water, wood, brass.

SUGGESTIONS TO STUDENTS

1. Throw stones of different sizes into a pool of water, and note the differences in the lengths and amplitudes of the corresponding waves.

2. By moving both hands up and down with a regular period in a tub of water, see if you can produce stationary waves. Keep the hands a foot or two apart, and gradually change the period till the nodes and loops are seen at definite places in the water.

3. With two companions and a kodak, go to a pond or lake and

make a similar experiment. Instead of your hands use as sources of waves two long poles just alike, having nailed to them equal circular pieces of board. These will be easy to keep vibrating with equal periods. When two of you have practiced so that you can maintain the stationary waves, let the photographer of the party take a snap shot of the waves. By shortening one of the poles and reducing the size of the circular board on it, see if you can succeed in getting waves of forms that are more complex, but nevertheless definite.

4. For many beautiful and simple home experiments on wave motion, read Prof. A. M. Mayer's *Sound* (Appleton, New York). By all means read Prof. J. H. Fleming's *Waves and Ripples* (E. & J. B. Young & Co., New York).

This is a series of experimental lectures delivered to young people at the Royal Institution, London (where Davy, Faraday, and Tyndall worked). It is a most fascinating account of waves in water, air, and ether.

5. If you are in the manual training class or have a shop of your own, get together with some of your classmates and make for the school a Kelvin wave model as described in Michelson's *Light Waves and their Uses* (University of Chicago Press) pp. 5 and 6. Read also pp. 1-13.

6. Another excellent wave motion model which you can easily make is described in Jones's *Heat, Light, and Sound*, p. 238. Read also pp. 236-241.

7. Let a companion hold one end of a clothes line while you hold the other, and with it produce the result predicted theoretically by the diagrams in Fig. 183. Let another companion who is a photographer try to get some snap shots of the line while vibrating in 1, 2, 3, 4, or more loops, and make lantern slides of them for the school collection.

CHAPTER XV

SIMPLE HARMONIC MOTION

NOTE.—The authors recommend that this chapter be used only for informal discussion on the first reading. If time is short it may be omitted altogether.

302. Relation to Uniform Circular Motion. The study of simple harmonic motion is made much easier by first considering the relation that exists between this type of motion and uniform motion in a circle. In order to make this relation clear, let us conceive that we have a small body traveling with uniform velocity v, in the horizontal circular path $ABCD$, Fig. 184. To one looking at this motion from above, the path of the body is seen to be circular and its velocity uniform. But if the motion be observed from a point in the plane of the circular path, and at a distance from the circle in the direction CA, the body will appear to be moving back and forth along the straight line BD, and its motion will no longer appear uniform. Thus, when the body is passing the points B and D in its circular path, it will appear to be at rest. On the other hand, when it is passing the points A and C, it will appear to be moving with a speed which is the same as its uniform speed v in the circular path. At intermediate points its speed will appear to vary between these two limits, i.e., between O and v.

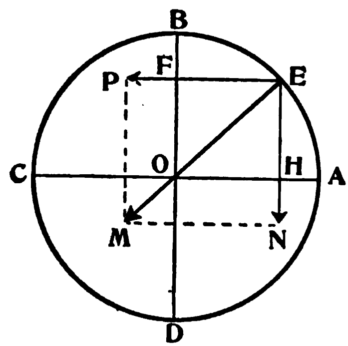


FIG. 184. SIMPLE HARMONIC MOTION

When we observe the uniform motion of the particle in this latter way, it appears at each instant *as if it were projected on the*

straight line BD, i.e., as if it were at the foot of a perpendicular drawn from it to BD. So we can readily understand how the same effect would be produced on the distant observer if we replace the body moving uniformly in the circular path by an equal body that moves back and forth along the line BD in such a way that the position of this second body at any instant is the projection on the line BD of the position of the first body at the same instant. Let us conceive this to be done. It then remains for us to consider what forces must be applied to the body moving in this way along BD, in order to produce the required motion. The problem is not so difficult as at first sight it may appear.

303. Force and Displacement. Suppose the first body is at any point E (Fig. 184) of its circular path. The second body must then be at F, the projection of E on BD. What force is acting on E at this time? We have learned in Chapter V that the force is directed toward the center of the circle, i.e., along EO, and is numerically equal to $\frac{mv^2}{r}$, in which m is the mass of the body, v the uniform velocity in the circumference, and r the radius of the circle. Let the magnitude and direction of this force be represented by the vector EM. We may now conceive this force to be resolved into two components, one in the direction EF, perpendicular to BD, and the other in the direction EH, parallel to BD. These components will then be represented by EP and EN. It is clear that the component EP has no effect on the motion of the body at E in the direction BD. It is also clear that if we allow a force equal to the component EN to act on an equal mass at F, the motion produced along BD will be the same as the motion in the direction BD of the body at E. Hence, the force that must be applied to the second body at F, in order that it may always be at the projection of E on BD, is represented by this component EN.

But what is the value of this component? From the similar triangles, EMN and EOH, $\frac{EN}{EM} = \frac{EH}{EO}$ or, $EN = \frac{EM \times EH}{EO}$.

But EO is the radius v of the circle and $EH = FO$ is the distance of the second body from O. If this distance be denoted by d, $EN = EM \times \frac{d}{r}$, but $EM = \frac{mv^2}{r}$, therefore substituting this value for EM we have $EN = \frac{mv^2}{r} \times \frac{d}{r}$. But EN represents the force that must be applied to the body at F in order to make it move in the required manner. If we denote this force by f, we have finally

$$f = \frac{mv^2}{r^2}\, d.$$

In a given case m, v, and r are constant, therefore the force that must be applied at each instant to a body in order to make it vibrate in the required manner, is proportional to the distance of the body from the center of its swing. If we call this distance the DISPLACEMENT, and define the motion as SIMPLE HARMONIC MOTION, we reach the conclusion that *when a body is vibrating in simple harmonic motion, the force at any instant is proportional to the corresponding displacement.*

304. The Sine Curve. Since the value of the ratio $\frac{d}{r}$ depends on the size of the angle EOH, it is possible to express the force f in terms of this angle instead of the displacement. To do this, we name this ratio the sine of the angle EOH, i.e., we define the SINE of an angle as *the ratio in a right triangle of the side opposite the angle to the hypothenuse.* It is for this reason, and also because the simple homogeneous waves considered in the last chapter are produced by this simple harmonic motion, that the graphs representing their shapes are called SINE CURVES. *The abscissas of the sine curve represent the angles and the ordinates represent the values of the corresponding sines.*

305. Illustrations. We can now realize why simple harmonic motion is of so great importance in science, for vibrations are usually produced by elastic forces, and these are proportional to the displacements. One of the simplest cases of such vibration is that of a weight on the end of a spiral spring, Fig. 185. In this

case we may easily prove that the force is proportional to the displacement; for when we hang a weight of 100 gm on the end of the spring and measure the displacement produced, and then repeat the operation with a weight of 200 gm, we find that the displacement in the second case is twice what it is in the first, and so on.

FIG. 185. FORCE IS PROPORTIONAL TO DISPLACEMENT

306. Period. One other point remains for consideration: How does the time of vibration depend on the mass of the vibrating body and the force acting to bring it back to its position of equilibrium? We may find the answer to this question as follows: When body 2 which moves along the diameter BD, Fig. 184, is passing the point O of its swing, it is evident that its velocity is the same as that of body 1 at A; i.e., the velocity is v. But if T represent the time it takes body 1 to travel once around its circular path, i.e., if T represent the period, this velocity will be equal to the circumference of the circle divided by T, i.e., $v = \frac{2\pi r}{T}$. But since at O the velocity of body 2 is also v, or $\frac{2\pi r}{T}$, its kinetic energy at this point is found by substituting this value in equation (6), Art. 39, i. e.,

$$e = \frac{mv^2}{2} = \frac{m}{2}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr^2}{2T^2}.$$

When, however, body 2 reaches the point B, its velocity is reduced to 0. Hence, at B it has no kinetic energy, but this energy has been converted into potential energy. This potential energy, as we learned in Chapter II, is equal to the work done in bringing the body to the position B. Now, this work is the force $f \times$ the displacement. So if we let F represent the force acting on body 2 to bring it back to O when its displacement is 1 cm, the

force f acting at B to cause its return will be Fr, because it is proportional to the displacement OB, and OB is r cm from O. Now, this force increases from the value 0 to Fr as the displacement increases from 0 to r. Therefore, in order to get the work done in moving the body from O to B, we may assume that this variable force is replaced by a constant one. The numerical value of the constant force that will do the same amount of work in this case is the average of the forces at O and at B; i.e., this force is equal to $\frac{0 + Fr}{2} = \frac{Fr}{2}$. Therefore, the work done in moving the body from O to B is this force $\frac{Fr}{2}$ multiplied by the displacement r, i.e., $W = \frac{Fr^2}{2}$. This work is equal in value to the potential energy of the body at B; and this potential energy is, as just stated, equal to the kinetic energy at O. Therefore, $\frac{4\pi^2 mr^2}{2T^2} = \frac{Fr^2}{2}$. Solving this equation for T^2, we have $4\pi^2 \frac{m}{F} = T^2$, and finally

$$T = 2\pi\sqrt{\frac{m}{F}}, \qquad (16)$$

i.e., *the time taken in executing one complete vibration is equal to* 2π *multiplied by the square root of the quotient obtained by dividing the mass of the body by the force necessary to displace it 1 cm from its position of equilibrium.* This force per cm is called the FORCE CONSTANT of the system.

It is easy to see that the expression on the right-hand side of the equation represents a time, for its symbol in terms of gm, cm, and sec evidently is the square root of gm divided by that for dynes per cm, or

$$\sqrt{\frac{\text{gm}}{\frac{\text{gm cm}}{\text{sec}^2\text{ cm}}}} = \sqrt{\text{sec}^2} = \text{sec}.$$

307. Pendulum. There are several other important cases in which this relation can be applied, besides that of a spiral spring. Most important among these is that of the pendulum. We will now take up the consideration of this case. Suppose our pendulum

consists of a ball of lead A, of mass m, suspended on the end of a wire, Fig. 186. Let the distance between the point of suspension O and the center of gravity of the ball be denoted by l. The mass in this case is clearly m, but what is the force constant?

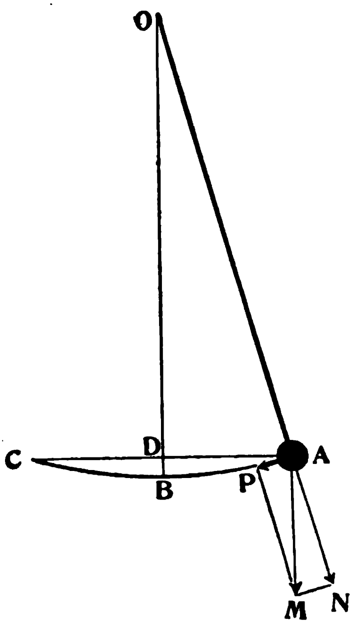


FIG. 186. THE PENDULUM DIAGRAM

As is well known, the pendulum when set in motion moves backward and forward along the arc ABC, while its motion, to be strictly simple harmonic, must be along the straight path AC. But if the arc ABC is small compared with the length l, the difference between this arc and its chord AC becomes so small that we may, without appreciable error, consider that the actual path ABC is equal to the chord AC, and that the motion is harmonic. The displacement of the mass is, then, the distance AD in the figure. The question therefore is, How great a force is acting on the mass m when this displacement equals 1 cm? To answer this question, conceive the pendulum swung to the position A, in which its displacement equals 1 cm. The only force involved is the weight of the ball acting vertically downward. Let the vector AM represent this force. We now conceive this force to be resolved into two components, AN and AP, one in the direction of the wire OA and the other perpendicular to it. The component AN merely causes a tension in the wire, while the other component AP produces the motion along the arc AB. What, then, is the value of the component AP?

Since the triangles AMP and AOD are similar (Why?) we have $\frac{AP}{AM} = \frac{AD}{OA}$, therefore, $AP = \frac{AM \times AD}{OA}$, but AM repre-

sents the weight of the body, i.e., mg, $AD = 1$ cm, and $OA = l$, therefore $AP = \frac{mg}{l}$. This is the force corresponding to unit displacement, therefore it is the force constant for this system. When we have substituted this expression for the force constant F in our equation (16), the result is

$$T = 2\pi \sqrt{\frac{m}{\frac{mg}{l}}} = 2\pi \sqrt{\frac{l}{g}}.$$

We thus reach the conclusion that *the time a pendulum takes to execute one complete vibration is numerically equal to* 2π *multiplied by the square root of the quotient obtained by dividing the length of the pendulum by the acceleration of gravity.* It must not be forgotten, however, that this is strictly true only when the displacement is so small that the chord ADC and the arc ABC are sensibly equal and when the mass of the wire is inconsiderable.

308. Uses of the Pendulum. This is one of the most important relations in physics, for *it furnishes us with a simple and very accurate method of determining g, the acceleration of gravity.* Thus, if we solve this equation for g, we get

$$g = \frac{4\pi^2 l}{T^2};$$

and since we can easily measure the length of a pendulum and also its time of vibration, we obtain the value of g immediately.

This equation is also of use in proving with great accuracy that g is the same for all bodies at a given place; for if we make a series of pendulums all of the same length, but whose bobs are made of different substances, and if we find that they all vibrate in the same time, we must conclude that *at a given place g is the same for all masses.* This experiment was performed by Newton, and later with greatest accuracy by Bessell, and the results show that all pendulums of the same length, no matter of what substance they are made, vibrate at a given place in the

same time. Therefore we are justified in comparing masses by comparing their weights, as stated in Chapter II.

Furthermore, since all pendulums of the same length vibrate at a given place in the same time, *the pendulum furnishes a most convenient method of measuring time.* The enormous importance of the pendulum to mankind in this respect is so familiar, that we need do no more than call attention to it.

FIG. 187. THE PROOF THAT THE EARTH ROTATES

In the case of the pendulum, it is customary to call its period the time taken in moving from one end of its swing to the other, not the time taken to complete a whole vibration. But since this time is half that required for a complete swing, the equation for the pendulum is usually written:

$$t = \pi \sqrt{\frac{l}{g}}. \quad (17)$$

309. The Foucault Pendulum. *The pendulum also furnishes a means of proving that the earth rotates.* The experiment that shows this was first performed by Foucault in 1851. He suspended a ball of lead, having a mass of 28 Kg, on a steel wire 67 m long in the dome of the Pantheon in Paris, Fig. 187. On starting the pendulum into vibration, it was found that the plane in which it swung turned with reference to the building, and the amount of this turning could be measured on the large circle

beneath the bob, for the pendulum at each vibration would knock down parts of a ring of sand which had been piled up around the circumference.

The explanation of this phenomenon is as follows. On account of its inertia the pendulum swings in a plane that has a fixed direction in space; and, therefore, as the earth turns with reference to this fixed plane, this plane appears to turn with reference to the earth. If such a pendulum were suspended directly over the north or the south pole of the earth, its plane of vibration would turn once around in 24 hours. On the equator its plane would not turn at all, and in intermediate latitudes it would oscillate back and forth every day through an angle that depends on the latitude.

SUMMARY

1. When a body moves with simple harmonic motion, the force that acts to return it to its position of equilibrium is proportional to the displacement of the body from that position.

2. Elastic forces are proportional to the displacement.

3. A body vibrating under the action of elastic forces executes simple harmonic motion.

4. The periodic time of a body executing simple harmonic motion is equal to 2π times the square root of the mass divided by the force constant.

5. A pendulum when its displacement is small vibrates in simple harmonic motion.

6. The periodic time of a pendulum for a single swing is equal to π multiplied by the square root of $\frac{l}{g}$.

7. The pendulum furnishes the most accurate method of determining g.

8. With the pendulum we may prove that g is the same for all masses at a given place.

9. The pendulum is our best measurer of time.

10. The pendulum furnishes us with a means of proving that the earth is rotating.

QUESTIONS

1. What relation exists between forces and displacements when a body moves with simple harmonic motion? How is this proved?

2. Do bodies vibrating under the action of elastic forces execute simple harmonic motion? Why?

3. How does the periodic time of a body vibrating with simple harmonic motion depend on the mass of the body?

4. What is the force constant? How does the periodic time depend on it?

5. Upon what two factors does the periodic time of a pendulum depend?

6. How may the pendulum be used: 1, to measure g; 2, to prove that we may compare masses by comparing weights; 3, to measure time; 4, to show that the earth rotates?

PROBLEMS

1. Draw a circle 6 cm in diameter. Beginning at the point corresponding to A, Fig. 184, divide the circumference into 12 equal parts, and draw perpendiculars from the end of each of these arcs to the diameter $A\ C$. Plot a graph in which the abscissas represent the lengths of the arcs, measured from the point A, and the ordinates are the lengths of the corresponding perpendiculars. Does the curve obtained resemble that of Fig. 174? Repeat the construction, using the same scales, but with circles 3 cm and 2 cm in diameter, taking care to have the origin of coördinates for all the curves fall on the same vertical line. Graphically add the three curves together. Does the resultant resemble the curve R' in Fig. 181?

2. Draw a circle of 5 cm radius. From the point corresponding to A, Fig. 184, lay off arcs of 15°, 30°, 45°, 60°, 75°; and drop a perpendicular from each point thus determined to the diameter $A\ C$. Measure the lengths of these perpendiculars in cm, divide the numbers that represent those lengths by 5, and compare the numbers thus obtained with those given in a table of natural sines opposite the same degree number, i e., 15, 30, etc. May the sine of an angle be defined as the length of such a perpendicular in a circle whose radius is unity? Compare Arts. 298 and 304 and problem 1 and see if you can understand the meaning of the term sine curve.

3. What is the length in cm of a pendulum that beats seconds at sea level in New York? How long must a pendulum be in order to vibrate in 2 sec? In 3 sec? Write an equation expressing the relation between the times t and t' of two pendulums and their lengths l and l'.

4. The spring Fig. 185 is elongated 5 cm by a 100 gm weight. What is its force constant? If a mass of 250 gm is suspended on the spring and set vibrating up and down, what will be its period?

5. A circular brass disc is rigidly fastened to a stiff steel wire passing through its center and perpendicular to its surface. The wire is held vertically and its upper end firmly clamped so that the disc hangs in a horizontal plane. If the disc be turned through a small angle about the wire as an axis, and then released, it will execute rotary or torsional vibrations. With the help of Art. 86, tell what will in this case correspond to the force constant. What must take the place of the mass in the equation of vibratory motion in Art. 306? What, then, is the equation for determining the period of vibration?

6. A moment of force whose numerical value is 237×10^5 is required to twist the disc of problem 5 through an angle of 1 radian. If the moment of inertia of the disc has the value of 6×10^5, what is the time of vibration?

7. A disc suspended as in problem 5. has a diameter of 20 cm; a force of 10^6 dynes, acting tangentially at each end of a diameter, is required to give it an angular displacement of 1 radian. If its time of vibration is 1.5 sec, what is the value of its moment of inertia?

8. A bar magnet mounted on a pivot, like a compass needle, and deflected through an angle of 1 radian from the magnetic meridian, tends with a moment of force whose value is 990 to return to that meridian. If the moment of inertia of the magnet has the value 400, what will be the period of oscillation of the magnet when it is released?

9. The moment of force of the bar magnet in problem 8 depends on the mutual action between the magnetic fields of the magnet and of the earth. If the strength of the magnet's field remains constant, how will the period of oscillation of the magnet be changed if the strength of the earth's field is doubled?

SUGGESTIONS TO STUDENTS

1. See how nearly you can determine your own mass by swinging in a swing, determining with your watch your period of vibration, and getting a friend to measure with a spring balance the number of dynes necessary to pull you in the swing a measured number of centimeters from the position of equilibrium. If the spring balance is graduated in pounds, remember that 1 pound-force = 445,000 dynes.

2. Tie both ends of a rope to the branch of a tree about 10 ft. from the ground, so that the rope hangs in a V whose point is about 5 ft. from the ground. From the point of the V suspend by a single cord a tin can, so that it almost touches the ground. Punch a small hole in the bottom of the can, fill it with water, set it to swinging, and see what sort of curves the water will draw on the ground. Similar experiments are described in Mayer, *On Sound* (Appleton, New York).

CHAPTER XVI

SOUND

310. Sources of Sounds. Of all the phenomena of nature none, perhaps, is better known or more universally recognized than the fact that sound always originates at some vibrating body. Even an infant knows that he must shake his rattle to make it sound, and the vibrations of a bell or drum, when they are sounding, are easily felt.

Other familiar facts concerning sound are the following: 1. We can in some way tell in what direction the source of sound lies. 2. We recognize differences in the pitch of sounds, some seeming high and shrill, others low and deep. 3. We recognize differences in the intensity of sounds, some being loud and strong, others soft and weak. 4. We recognize differences in the qualities of sounds, i.e., we are able to distinguish at once between the tone of a violin and that of a piano, and can even recognize one another by our voices in the dark or over a telephone.

311. Sound a Wave Motion. With these facts clearly before us, many interesting questions arise. How does sound get from the vibrating body to us? How do the vibrations of sounds of different pitch differ? What governs the intensity of sound? What characteristics of the vibrations of the source correspond to the differences in tone quality? Let us proceed to find the answers to these questions.

The first conclusion that we draw from the facts just mentioned is that the information which sound brings us concerning a sounding body might be gained from a wave motion. Hence it seems plausible to assume as a hypothesis, that sound travels from its source to us by a wave motion. But if it is a wave motion, what is the medium in which it travels? If we place an alarm clock under the receiver of an air pump, we find that the alarm is no

longer audible when the air is pumped out. This experiment proves that ordinarily air is necessary for the propagation of sound, and if we conclude that sound is a wave motion in air, it lends added weight to the conclusion that waves can not be propagated unless a suitable medium is present to transmit them.

Other facts point to the conclusion that sound is a wave motion in the air; for the presence of these waves in the air may be detected by suitable apparatus, such as membranes and sensitive flames. Another proof of this fact is derived from the velocity with which sound travels, for this velocity can be measured by firing a gun at one place and noting at another, distant place the time that elapses between seeing the flash and hearing the report. It can also be calculated from the properties of air, for in Chapter XIV we have learned that the velocity of waves in an elastic medium is equal to $\sqrt{\frac{e}{d}}$ and both e and d for air are capable of measurement. If the two values obtained from these two different methods agree, we are well justified in saying that sound is a wave motion in air.

312. Sound Waves are Longitudinal. It will be interesting to calculate the velocity of sound waves in air from the formula. In order to do this, we must determine what we mean by elasticity, and this necessitates our knowing what kind of wave motion sound is. Now, in Chapter XIV we have found that waves may be either transverse or longitudinal. We there learned that when a medium transmits transverse waves, the forces brought into play are those that resist a sideways displacement. Hence, since air presents no elastic force that resists a sideways displacement, we conclude that it can not transmit transverse waves; yet since it offers a large elastic resistance to compression, it can transmit longitudinal waves with a large velocity. Therefore we conclude that the sound waves are probably longitudinal, and will proceed on this assumption to find what their velocity is.

313. The Velocity of Sound. In order to calculate the velocity of sound with the help of the equation $v = \sqrt{\frac{e}{d}}$, we must

first consider how the elasticity of the air is measured. The elasticity of any substance may be defined as the ratio of the pressure that produces the change, to the change per cm^3 produced. In the case of air, the change produced by applying pressure is a change in volume; therefore, for air, $e = \dfrac{\text{pressure applied}}{\text{change per cm}^3 \text{ in volume}}.$ The numerical values of these quantities may be found experimentally by applying a measured pressure to air confined in a cylinder and measuring the corresponding changes in volume. The results of such experiments show that, for rapid compressions like those of sound, a pressure of 14200 $\frac{\text{dynes}}{\text{cm}^2}$ is required to produce a change of 0.01 $\frac{\text{cm}^3}{\text{cm}^3}$ in volume. Hence, for air,

$$e = \frac{14200}{0.01} = 142 \times 10^4 \, \frac{\text{dynes}}{\text{cm}^2}.$$

Since the density of air at 0° C and 76 cm atmospheric pressure is 0.001293, we have $v = \sqrt{\dfrac{142 \times 10^4}{0.001293}} = 33150.$ It is to be noted that the numerator is $\frac{\text{dynes}}{\text{cm}^2}$ and the denominator $\frac{\text{gm}}{\text{cm}^3}$, and therefore the quotient is $\frac{\text{gm cm}}{\text{sec}^2\ \text{cm}^2} \times \frac{\text{cm}^3}{\text{gm}} = \frac{\text{cm}^2}{\text{sec}^2}.$ The result is thus seen to have the symbol $\frac{\text{cm}}{\text{sec}}$, as it should have if it is a velocity.

The results of many experiments in which the velocity of sound has been measured by the method of firing a cannon and by other methods, show that this velocity is 33170 $\frac{\text{cm}}{\text{sec}}$ under the conditions of temperature and atmospheric pressure specified. Since the calculated value agrees so well with the observed value, we may conclude that sound waves are longitudinal waves in air.

314. Resonance. Another striking proof of the fact that sound is a wave motion in air may be given with a pair of tuning forks which are tuned so that they have exactly the same periods of vibration. If one of the forks is set into vibration, the other, though placed at some distance from it, will begin to vibrate, so

that it can be plainly heard if the first one is stopped. It must, therefore, have been set into vibration by the regular pulsations of the air that are started by the first fork. The little pushes of the successive waves are applied to it just at the proper time, so that their sum finally produces an appreciable motion of the second fork. Every child who has pushed a heavy person in a swing knows how the little pushes are able to set the swing into vibration if only they are properly timed. So with the two tuning forks; when the two forks have the same period of vibration, the little pushes of the air waves from the first fork reach the second fork at just the proper intervals, and thus set it into vibration.

When a body is thus set into vibration by waves of the same period as those which it is itself capable of sending out, its vibrations are said to be sympathetic, and the phenomenon is called RESONANCE. Every body, when vibrating freely, has a definite period of vibration peculiar to it. This period is called its NATURAL PERIOD. The period of the waves that act on a body to set it into vibration by resonance, is called the IMPRESSED PERIOD. The principle of resonance, then, is generally stated as follows: *A body may be set into vibration by resonance when its natural period agrees with the impressed period.*

315. Effect of Temperature Changes. One further point remains for consideration, viz.: Is the velocity affected by a change in temperature? Evidently it is, since heating the air expands it and thus makes its density less, and a decrease in the value of the density d means an increase in the value of the velocity v. Therefore, sound travels faster the warmer the air is. It is easy to show that the increase in velocity is 60 $\frac{\text{cm}}{\text{sec}}$ for every rise of 1° C. in temperature.

316. Noises and Musical Notes. Having thus proved that sound is a wave motion in air, let us pass on to a study of a vibrating body that produces sound. But before entering on this study, it will be well to make a distinction between noises and musical notes. For a noise is a confused jumble of sounds—

an irregular and mixed phenomenon without definite period of vibration, while in the case of musical notes we have definite periods of vibration; and so the numerical relations are more uniform, and lend themselves better to systematic investigation. Therefore, in what follows, we shall confine our attention solely to the musical notes, and whenever the word "sound" is hereafter used, a continued and regular sound of definite period, i.e., a musical note is meant.

317. The Piano. With this limitation of the meaning of the word, we may safely say that a piano is one of the most familiar of all sources of sound. Let us then begin our investigation by noting some of the features of this instrument. On opening a piano, we find that there are inside it a number of steel wires of varying diameters, lengths, and tensions. If we strike a key, we observe that a small hammer flies up and strikes one or more of these wires. We further note that the wires struck are set into vibration, and that we hear the tone as long as this vibration continues.

Another fact that we notice is that the long and thick wires correspond to the lower keys on the keyboard, and emit, when vibrating, the tones of lower pitch in the musical scale. Such observations as these lead us to ask many questions. What relation exists between the lengths of the strings and the pitches of the corresponding notes? Why are there just eight notes in an octave? What is it in the sound that enables us to distinguish between the tones of a piano and those of a violin? Why do we call certain combinations of notes harmonious and others discordant?

318. Pitch. As has just been stated, we notice that the long strings in the piano are the ones that produce the tones of low pitch. We also observe that these long strings vibrate more slowly than the shorter ones; i.e., they execute fewer vibrations per second. We therefore infer that pitch is in some way related to the number of vibrations per second. That this is really the case may easily be proved by mounting a toothed wheel on an

axis and revolving it. If we hold a card so that it strikes lightly upon the revolving teeth, we notice that the tones produced by the wheel are different for different speeds. Since each tooth causes a vibration when it strikes the card, we must conclude that the difference in the pitch produced is due to the different number of vibrations when the speed of rotation changes.

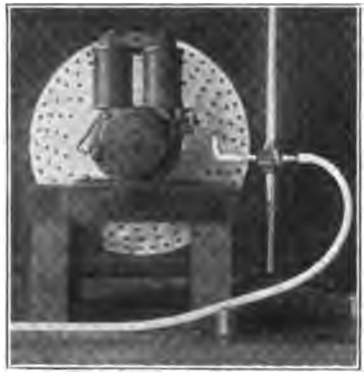


FIG. 188. THE SYREN

Another method of proving this same thing is this: Take a cardboard, or thin metal disk, and punch two or three rows of equidistant holes around its outer edge (Fig. 188). When we blow on one of these rows of holes while the disk is rotating rapidly, we notice that a tone is produced which is different when the number of holes in the rows is different. But since, when the disc is rotating uniformly, a difference in the number of holes means a difference in the number of pulses or vibrations forced on the air each second, it appears that *pitch depends on the numbers of vibrations per second.*

319. Musical Intervals. But we can prove more than this with these instruments; for we can show that definite simple relations exist between the vibration numbers of the notes on the piano. Thus, if the numbers of holes in two different rows on the rotating disc are related as 1 to 2, we perceive that the corresponding notes are one octave apart (from *do* to *do*); if the numbers of holes are related as 2 to 3, we find that the two notes are a fifth apart (*do* to *sol*). Similarly, if the notes given by the two rows of holes are *do* and *fa*, the corresponding numbers of holes in the rows are found to be related as 3 to 4. So it appears that the numbers of vibrations of the notes on the piano are related to one another by simple ratios. But before we take up the question as to the reasons for the existence of these simple

numerical relations among the notes of the musical scale, we must stop to discuss briefly how the lengths and sizes of the strings affect the number of vibrations of the tones.

320. The Laws of Strings. The laws of vibrating strings were discovered experimentally by Mersenne (1588–1648) in 1644. These laws are merely statements of the numerical relations that appear in the well known facts that, other things being equal, the longer a string is, the more slowly it vibrates; the thicker and denser it is, the more slowly it vibrates; and the greater its tension, the faster it vibrates. They are: *Other things being equal, the number of vibrations per second executed by a stretched string is:*

1, *inversely proportional to its length;*

2, *inversely proportional to its thickness;*

3, *directly proportional to the square root of its tension;*

4, *inversely proportional to the square root of its density.*

These laws are all illustrated by the strings of musical instruments. The short, thin, tightly stretched strings on the piano are the ones that give the high notes, while those for the low notes are longer, thicker, and not so tense. The same is also true of the violin, the cello, the banjo, and all other stringed instruments. In these latter instruments, the strings generally all have the same length, and on a given string the notes of higher pitch are produced by shortening the string by pressing the finger on it.

321. Vibrating Rods. Similar relations are found to exist for the case of elastic rods fastened at both ends or supported in other ways. Rods may vibrate either transversely or longitudinally, and the vibration numbers are different in the two cases. This may be shown by clamping a metal or wooden rod about 1 m long and 0.5 cm in diameter in the center and then setting it into vibration first transversely by striking it, and then longitudinally by rubbing it with a damp cloth. In the former case the vibrations will be slow enough to count; and *other things being equal, the vibration numbers are inversely proportional to the squares of the lengths.* In the latter a note of high pitch is

produced and, other things being equal, the vibration numbers are found to be inversely as the lengths. In this case the vibrations are too rapid to be seen, but they may be shown by means of an elastic ball or button suspended so it will just touch the end of the rod. Thus it appears that the different ways in which rods may vibrate are many, since the number of vibrations depends not only on the dimensions of the rod, but also on the way in which it is supported and the manner in which it vibrates.

322. Tuning Forks. One case of vibrating rods is of great practical importance, namely, the tuning fork. This instrument is universally used as a standard of pitch. Its vibrations are simple harmonic, as may readily be shown by fastening a light wire to one of the prongs and allowing the fork to trace its motion on a moving piece of smoked glass. The resulting curve will be found to resemble closely a sine curve (Fig. 174).

323. Organ Pipes. One more class of vibrating bodies remains for consideration, namely, organ pipes. In this case the vibrating body is a mass of air inside the pipe. This column of air may be regarded as a rod of air and its possible vibrations investigated, as in the case of rods. Here also *the vibration number of the note given by such a column of air is inversely proportional to the length of the column.* It varies also with the density of the air, but in all practical cases the changes in the density of the air, due to changes in atmospheric pressure, have so small an effect that they may be neglected.

324. Air Columns as Resonators. Since the air in an organ pipe has a definite mass and shape, it must, like all other bodies, have a natural period of vibration. Therefore, if we impress on this air a vibration whose period agrees with its natural period, the air will be set into vibration by resonance (*cf.* Art. 314). This resonance of an air column may be shown by holding a vibrating tuning fork of a certain pitch over the top of an open organ pipe of the same pitch. The air in the organ pipe is set

into vibration by resonance, thus strengthening the tone given by the fork. The boxes on which tuning forks are usually mounted are made of such a size that the natural period of the air in them agrees with the period of the fork. When the fork vibrates, the air vibrates by resonance, and thus the intensity of the tone is much increased.

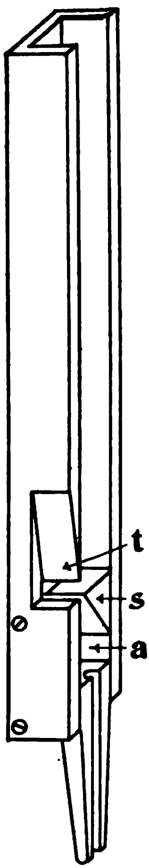


FIG. 189. OPEN ORGAN PIPE

We may now form some idea as to how the organ pipe is made to operate. Air, under pressure, is admitted to the small air chamber *a*, Fig. 189, whence it is blown through the slot *s* in such a way as to strike the tongue *t*. This stream of air blowing across the tongue produces a vibratory motion of the air, and when matters are so arranged that this vibratory motion has the same period as the natural period of the air in the pipe, that air responds by resonance. Toy whistles, flutes, and some other wind instruments work in a similar manner. The air in the tube of such an instrument always acts as a resonator and serves to strengthen the vibrations produced by blowing across an edge of some sort. In the case of the clarinet and saxophone the vibrations are produced by a thin elastic strip of metal called a reed, and in the case of the cornet and other horns, by the lips of the performer, but the resonant air column is also essential.

Organ pipes are made of definite length, and therefore each one has a definite pitch, so that we must have a separate pipe for each note. In the flute, however, the length of the air column may be varied by opening or closing holes in the tube with the fingers; so the flute can be made to produce various notes. In the trombone the length of the air column is varied by a slide, which may be pushed out or in, thus lengthening or shortening the tube. Other wind instruments will be found, on examination, to operate somewhat in a similar manner.

325. Intensity. The pitch of a note has just been found to depend on the vibration number of the source of sound, and this vibration number is inversely proportional to the wave length in the air. Thus pitch and wave length are closely connected with each other. What in the waves corresponds to intensity of vibration in the source? The intensity of a wave must depend on the intensity of vibration of the source of the wave, and this latter is greater the greater the amplitude. Thus the amplitude of the sound waves tells us in a general way of the intensity of vibration of their source. It may readily be shown that the *intensity is proportional to the square of the amplitude.*

The intensity of sound from a sounding body—a stretched string, for example—may be increased by changing the way in which the string is mounted. Thus, if the string is stretched between two heavy blocks of iron, the sound from it is not very intense, because the string has a small area, and so it slips, as it were, through the air without imparting much energy to it. But if the string is stretched over bridges on a large thin board, the bridges and the board are set into vibration by the string, and, since the board has a large area, a large amount of energy is transmitted to the air by it. Therefore the sound from the string is louder. Such a board is called a SOUNDING BOARD. The air in the tubes of organ pipes and other wind instruments serves a like purpose, as has just been mentioned, for the large mass of this air enables them, when set into vibration, to transfer more of their energy of vibration to the surrounding air, and so to increase the intensity of the sound.

SUMMARY

1. Sound originates at a vibrating body.

2. Sound gives us information concerning: 1, the direction of the sounding body; 2, the number of vibrations per second; 3, the intensity; and 4, the nature of the vibrations.

3. Sound is a wave motion of the air.

4. Sound waves are longitudinal.

5. The velocity of sound in air is 331.7 $\frac{m}{sec}$ at 0° C.

6. The velocity of sound in air increases 60 $\frac{\text{cm}}{\text{sec}}$ for a rise in temperature of 1° C.

7. Every elastic body has a natural period of vibration.

8. A body may be set into vibration by resonance when the impressed period is equal to the natural period of the body.

9. The pitch of a note depends upon its vibration number.

10. The vibration numbers of the notes of the piano are related to one another by the simple ratios 1: 2: 3, etc.

11. The length of a sound wave in air depends on the vibration number of the source.

12. The vibration number of a stretched string is inversely proportional to the length and the diameter of the string; and directly proportional to the square root of its tension and its density.

13. The number of vibrations of an air column is inversely proportional to the length of the column.

14. The vibrations of a tuning fork are of the simple harmonic type.

15. The intensity of a sound wave is proportional to the square of its amplitude.

QUESTIONS

1. What is the origin of sound? How is this known?

2. Why are we led to suppose that sound is a wave motion?

3. Why do we believe that sound consists of waves in air?

4. How do we prove that this supposition is correct?

5. How do we define the elasticity of air?

6. What leads us to conclude that sound waves are longitudinal?

7. How fast does sound travel in air at 0° C.? Does its velocity depend on the temperature?

8. Does the velocity of sound in air depend on the pressure of the barometer?

9. Give some familiar examples of resonance, and show how the phenomenon helps us to prove that sound is a wave motion of the air.

10. On what does the pitch of the note depend? How is this proved? What can you say of the lengths of the waves that start from two vibrating bodies of different pitch?

11. On what four characteristics of a string does its number of vibrations depend? In what way does it depend on each? How are these relations determined?

12. Are the transverse vibrations of a rod faster than the longitudinal? How do you know?

13. How is an organ pipe set into vibration? What is the action of the air in it?

14. How may we increase the intensity of the sound emitted by a vibrating string? How is this done in the piano, the violin, the guitar, the cornet, and the trombone?

PROBLEMS

1. Consider a stretched violin or piano string. The string rests on supports near its ends. At these points the string is not free to move. Are they, then, similar to that end of the jumping rope (Art. 301) which was tied to the tree? Will these points be nodes when the string vibrates? Will it vibrate in stationary waves? If the string vibrates as a single loop, like a jumping rope when being used to jump, what is the relation between the length of the string and the distance between the nodes of the stationary wave along it? How does the distance between the nodes in a stationary wave compare with the wave length of the wave? If L represents the length of the string, and l_1 the wave length of the wave formed when the string vibrates in one loop, show that $L = \frac{1}{2} l_1$. Similarly, if l_2 represents the wave length when the string vibrates in two loops; l_3, the wave length corresponding to 3 loops; l_4, that corresponding to four loops; show that $L = \frac{1}{2} l_1 = \frac{2}{2} l_2 = \frac{3}{2} l_3 = \frac{4}{2} l_4$, etc.

2. Draw curves similar to *CNR*, Fig. 183, showing the appearance of a string of length L vibrating in 1 loop; in 2 loops; in 3 loops; in 4 loops.

3. Consider a solid rod 1 m long, 5 cm in diameter, and clamped in the middle. Since the middle point is clamped, there must be a node there when the rod is set vibrating in stationary waves. Strike one end of the rod so as to set it vibrating transversely and watch its motion. Do the points at the ends vibrate with greater amplitude than those near the middle? If so, and the rod is vibrating in stationary waves with a node in the middle, where do the centers of the loops lie? In a stationary wave, is the distance between the centers of the loops equal to that between the nodes? What is, then, the relation between the length of the rod and the length of the stationary wave on it?

4. In the case of the rod of problem 3, the distance from the end of the rod to the node was $\frac{1}{4}$ of the wave length of the stationary wave. Where must the nodes lie if the ends of the rod are free and it vibrates so that the length of the rod is a whole wave length? In this case, must the middle of the rod be left free to vibrate as a loop? Draw diagrams showing the nodes on the rod when vibrating so that its length is $\frac{1}{2}$ wave; $\frac{3}{2}$ waves; $\frac{5}{2}$ waves. If the rod is always left free at its ends, can the ends ever be nodal points?

5. In the open organ pipe, Fig. 189, the air is free to move at both ends of the pipe. When the air column vibrates in this way, where will the node lie? What will be the longest stationary wave which the column of air in the pipe can form? If you draw diagrams showing the positions of the nodes when the air column is vibrating so that its length $= \frac{1}{2}$ wave. length, $\frac{3}{2}$ wave lengths, etc., will these diagrams differ from those of problem 4 in any way except that the vibrations of the particles of air are along the pipe instead of transverse, as the vibrations of the rod's particles are?

6. The longest wave of an open organ pipe is twice the length of the pipe. Sound travels with a velocity of 1120 $\frac{\text{ft}}{\text{sec}}$ at 15° C. What is the length of an organ pipe that gives the tone middle c (ut_3) of 256 vibrations per second?

7. The longest open pipes in church organs are 32 ft. long. What is the pitch of the tone given by one of them?

8. We have seen that in an open pipe the free ends are always the places where the air particles vibrate with greatest amplitude, i.e., the open end always corresponds to the middle of a loop. If a pipe is closed at one end and open at the other (a stopped pipe), will the closed end be a node? Then how does the length L of the stopped pipe compare with the length l_1 of the stationary wave? If you blow very gently across the mouth of such a closed tube, it sounds its fundamental or lowest tone; but if you blow harder it gives a higher tone. Does this indicate that another node has been formed, so that the air column is vibrating in shorter stationary waves? Diagram the condition of the air when there are two nodes, one, of course, at the closed end and the other between the two ends. If l_2 now represents the length of the stationary wave, what fraction of l_2 is L? Blowing still harder across the open end, you may get a still higher note, which corresponds to stationary waves when there are three nodes, including the one at the closed ends. Diagram this condition of the air column, and state the relation of L to l_3.

9. How long must an open pipe be in order that the longest stationary wave in it shall be equal to that in a closed pipe 20 cm long?

10. Can you compare the stationary waves on a rod, clamped at one end and free at the other, with those of a closed organ pipe, and show that $L = \frac{1}{4}l_1 = \frac{3}{4}l_2 = \frac{5}{4}l_3$, etc.? Show, by diagrams, where the nodes ought to be. Clamp a long, flexible, and elastic rod in a vise and see if you can make it vibrate transversely in 1, 3, and 5 half-loops.

11. The numerical value of the elasticity of water is found to be 205×10^7, its density 1 gm per cm^3; what is the velocity of sound in it?

12. When a tuning fork vibrates, its center of gravity remains at rest. How must the prongs move with reference to each other in order that this may be true?

13. When the two prongs of a tuning fork are approaching each other while vibrating, they compress the air between them, thus starting a condensation in the wave. At the same instant is the air on the outer sides of the prongs compressed or rarified? If the fork thus starts a condensation and a rarefaction at the same time, why do not the two destroy each other's effects so that we hear no sound? Hold a vibrating tuning fork near your ear, turn it about its long axis, and see if you can find any positions in which no sound is heard. If you find them, explain their presence.

14. What is an echo? Suggest a way of determining approximately, with the aid of a watch, the distance of a hill which gives an echo.

15. Can you prove by geometry that when sound spreads out from a small source, the intensity of the energy received on one cm^2 of surface is inversely as the square of the distance? If you can do this, explain the use of speaking tubes and megaphones.

SUGGESTIONS TO STUDENTS

1. Have you ever noticed the tones given by telegraph wires when the wind is blowing across them? How do these tones arise? Make an Æolian harp and put it in your window.

2. If you have a flute, measure the distance from the mouthpiece to the hole that gives a certain tone and calculate the vibration number of the tone.

3. Perhaps you have noticed that when a rapidly moving locomotive is whistling as it passes you, the pitch of the whistle changes at the instant when it reaches you. Does the pitch rise or fall while the train approaches? While it recedes? Can you apply your knowledge of the composition of motions to explain why this is so? Suspend an electric bell by wires from 10 to 30 ft. long, connected with a battery and push button. Swing the bell through a long arc and keep it ringing. What changes occur in the pitch? Why?

4. The vibrations of organ pipes are well presented in Sedley Taylor, *Sound and Music* (Macmillan, New York). You will also find a great deal of interesting information about sound and music, and about fog signals, in Tyndall *On Sound* (Appleton, New York).

5. For much information in very concise form, see Jones's *Heat, Light, and Sound* (Macmillan, New York). Blaserna's *Sound and Music* is also good (Appleton, New York). For home experiments, see Mayer's *Sound*, and Hopkins's *Experimental Science*.

CHAPTER XVII

THE MUSICAL SCALE

326. Development of the Musical Scale. The first important problem concerning the musical scale is that of finding why we have selected certain particular pitches and put them together in a certain way to form the gamut of the piano. From the discussion in the last chapter, it appears, that within certain wide limits, strings may be made to execute any number of vibrations; and, therefore, with a large number of strings differing from one another in diameter, length, and tension, such as we have in the piano, we are able to produce a series of tones whose vibration numbers shall be related to one another in almost any way that we may choose.

In the preceding chapter we proved, with the help of the punched disc, or syren (Art. 318), that the vibration numbers of the familiar notes, *do, fa, sol, do,* were related by the simple ratios $\frac{4}{3}$, $\frac{3}{2}$, $\frac{2}{1}$. It therefore becomes of interest to try to find out why we pick out a certain particular set of notes whose vibrations are related to one another in such a simple and definite way. The answer to this question is in one way very simple, and in another very complex; but before we can answer it, we must find out what the relations between the numbers of vibrations of the different notes of the piano scale are, i.e., we must discover the manner in which that scale is constructed.

The history of music helps us here; for from it we learn that mankind has not always had a musical scale, and that different peoples select different scales. We, for example, would find it difficult to recognize the productions of a Chinese orchestra as music. But even nations of our own type of civilization have not always had harmony as we now know it. The music of the early centuries of our era sounds harsh and ofttimes discordant when compared with modern compositions. Thus we learn that the

present musical scale was not used in early times, and that it has gradually developed into its present form, this form having been reached during the 16th century. Since Johann Sebastian Bach was the first who composed masterpieces in the modern scale, he is often called the "father of modern music."

327. The Related Triads. Go to the piano and play the two notes, middle *c* and *g*. Together they form a compound tone that pleases us, so we call it harmony. But this combination of *c* and *g* does not sound rich and full. We like the effect better when we add the note *e* and play together the three notes *c-e-g*. This combination of three notes satisfies us somehow; and if we strengthen the effect by playing also the octave of some or all of the three notes, we are still better pleased. Since the combination of these three notes produces such an effect on us, we make great use of it in musical compositions. We call this combination, i.e., the combination *do-mi-sol*, a MAJOR TRIAD.

Now, although the major triad is a pleasing combination, it becomes monotonous when played continuously. Hence, we must seek for other triads for variety. When we try various other triads on the piano, we find that there are two others that seem to harmonize with the first. Thus, if we play *c-e-g*, *g-b-d*, *c-e-g*, we recognize that the two triads are in some way related; similarly, if we play *c-e-g*, *f-a-c*, *c-e-g*, which are recognized as the familiar amen at the end of hymns. If now we play the three triads thus found in succession, viz., *c-e-g*, *f-a-c*, *g-b-d*, *c-e-g*, we perceive not only that we have played a pleasing succession of chords, but also that we have been left with a sense of repose. We know that the piece has ended and we are satisfied. Therefore we conclude that in some way these three triads define a scale or key. These three triads, which together define a scale, are called the TONIC (*do-mi-sol*), the DOMINANT (*sol-si-re*) and the SUBDOMINANT (*fa-la-do*) triads.

328. The Vibration Numbers. Having thus discovered that these three triads define a scale and that we select them solely because they please us and give us a sense of harmony and repose, let

us next find out if there are any numerical relations between the vibration numbers of the notes that compose them. This may be done in a number of ways, but is accomplished most easily by taking a string of given substance and tension and finding how its length must be changed in order to produce the tones of the triad (*cf.* Art. 320). The experiment is easily performed with a guitar, banjo, or mandolin string; for we have but to measure the length of the string from the nut to the bridge and then measure the distance from the bridge to the frets that give the tones *mi* and *sol*. When we do this, we find that these lengths are related to the lengths of the string by the ratios $\frac{1}{1}$, $\frac{4}{5}$, $\frac{2}{3}$, i.e., if the whole string gives the note *do*, the note *mi* is given by $\frac{4}{5}$ of the string, and the note *sol* by $\frac{2}{3}$ of that length. But since the vibration numbers of strings are inversely proportional to the lengths of the strings, we see that the vibration numbers of the notes of the triad are related to one another as are the ratios $\frac{1}{1}$, $\frac{5}{4}$, $\frac{3}{2}$; or, what amounts to the same thing, by the ratios, 1, $\frac{5}{4}$, $\frac{3}{2}$. Thus we prove that *the vibration numbers of the notes in a triad are related to one another as are the simple ratios* 1, $\frac{5}{4}$, $\frac{3}{2}$.

329. The Major Scale. If we assume that the note *c* executes 24 vibrations each second, we see that the numbers of vibrations of the three notes of the triad *c-e-g* are $24 \times 1 = 24$, $24 \times \frac{5}{4} = 30$, $24 \times \frac{3}{2} = 36$. What will then be the vibration numbers of the notes of the second triad *g-b-d*? Since the lowest note of this triad is the same as the upper note of the other, the vibration numbers of its notes will clearly be $36 \times 1 = 36$, $36 \times \frac{5}{4} = 45$, $36 \times \frac{3}{2} = 54$. To get the corresponding vibration numbers for the triad *f-a-c*, we note that it contains a note *c* which is an octave above the *c* in the triad *c-e-g*. But we learned in the last chapter that the octave executes twice as many vibrations a second as the lower note. Therefore the number of vibrations of the upper *c* is 48. Since this note is the third note in this triad, this number corresponds to $\frac{3}{2}$. Hence, the note *f* in this triad will be $48 \times \frac{2}{3} = 32$, and the note *a*, $32 \times \frac{5}{4} = 40$, or $48 \times \frac{5}{6} = 40$.

We thus find the following vibration numbers for the notes in these triads: $c = 24$, $e = 30$, $g = 36$, $b = 45$, $d = 54$, $c = 48$,

$a = 40$, $f = 32$. We note that all the numbers lie between 24 and 48, excepting d, which corresponds to 54. In order to bring this number within the desired octave, we transpose this note down one octave and thus find the vibration number of the lower d to be $\frac{54}{2} = 27$. We now arrange these notes in the order of their vibration numbers and get the series:

c	d	e	f	g	a	b	c
24	27	30	32	36	40	45	48

On inspecting this series we find that it contains all the notes of the musical scale which correspond to the white keys on the piano, i.e., the notes *do, re, mi, fa, sol, la, si, do.* But this series of notes is composed only of those notes which appear in the three triads which we have found necessary to define a scale or key. Hence we see that the musical scale is selected as it is, in order that it may contain all the notes necessary for the production of the three major triads which we have selected for the reason that they produce, when played together, a feeling of satisfaction and repose. It appears, then, that these particular notes have been adopted for a musical scale because something connected with our perception of sound leads us to pronounce certain combinations of tones harmonious or pleasing, and others discordant or disagreeable.

It is interesting to observe that the ratios of these numbers can always be expressed as the ratio of small whole numbers. This fact was clearly perceived as long ago as B.C. 525 by Pythagoras, who propounded the problem in the question, Why do we call a combination of tones harmonious when the vibration numbers of the component tones are related to one another by the ratios of simple whole numbers? This problem of Pythagoras remained without answer for over two thousand years. Helmholtz, in 1871, finally solved it. But before passing to his solution of it, we must complete the definition of the musical scale, for we have only found the ratios that exist among the vibration numbers that correspond to the notes of the white keys of the piano. We have yet to find out why there are black keys also. Further, we must determine the actual numbers of vibra-

tions of the different notes, for the numbers that have just been given express merely their ratios.

330. The Complete Scale. The necessity for the black keys becomes apparent when we wish to play a set of triads beginning with *e* instead of with *c*. Since the vibration number of the note *e* is represented by 30 in the table just given, we see, by applying our ratios 1, $\frac{5}{4}$, $\frac{3}{2}$, that the relative vibration numbers of the notes in the triad beginning with *e* would be $30 \times 1 = 30$, $30 \times \frac{5}{4} = 37\frac{1}{2}$, $30 \times \frac{3}{2} = 45$. The note represented by 45 already exists in the scale at *b*, but we have no note corresponding to $37\frac{1}{2}$. Since this number falls nearly half-way between 36 and 40, it has been found necessary to add another note to our scale about half-way between *g* and *a*. This note is called *g* sharp, and it is clearly added to enable us to play scales that begin on *e* instead of on *c*, thus increasing the number of scales that can be played on the instrument. Similarly, if we wish to begin a scale on *a*, which is represented by 40, the second note of the triad would be represented by $40 \times \frac{5}{4} = 50$, or by $\frac{50}{2} = 25$; and the third by $30 \times \frac{3}{2} = 60$, or by $\frac{60}{2} = 30$. Now, the note 30 already exists in our series, but an extra note corresponding to 25 has to be added between 24 and 27. This note is called *c* sharp. Similarly, by figuring the numbers of vibration of the triads that begin on the notes *d* and *b*, we find it necessary to add other notes between *f* and *g* and between *d* and *e*. The reason for adding the black keys is therefore apparent. They enable us to play scales that begin on notes other than *c*.

But as we proceed with this addition of notes our series soon becomes very complex. For when we construct the triad that begins on *d*, we find the corresponding numbers to be $27\text{-}33\frac{3}{4}\text{-}40\frac{1}{2}$. We can supply the note represented by $33\frac{3}{4}$ by adding one between $f = 32$ and $g = 36$. But the number $40\frac{1}{2}$ does not agree with $a = 40$, though it comes pretty near it. Similarly, when we come to supply the triad which shall have $c = 48$ for its middle note, we find the numbers $38\frac{2}{5}\text{-}48\text{-}57\frac{3}{5}$, or reducing the latter one octave $28\frac{4}{5}\text{-}38\frac{2}{5}\text{-}48$. Now, we have already added one note between 36 and 40, viz., *g* sharp $= 37\frac{1}{2}$, and

this differs slightly from the one that now appears to be necessary, viz., $38\frac{2}{5}$. If we carry this process of working out triads further, we find that very many notes would have to be added in order to make it possible to play scales which begin on all the notes of the scale of *c*. A keyed instrument of the piano type would require about 70 keys to the octave and would soon become too complicated to manage. Yet every one knows that we can play all the different scales on the piano. How, then, is the difficulty avoided?

331. Tempered Scale. The answer is simple. We insert extra notes which do not exactly satisfy either of the required conditions, i.e., when two notes have vibration numbers that are very nearly equal, we take an average note and let it do for both. Thus, instead of having a note 40 and another $40\frac{1}{2}$, we make one note do for both by tuning the strings so that the number of vibrations shall correspond to about $40\frac{1}{4}$; similarly with the other cases. We do not have on the piano a note $37\frac{1}{2}$ and another $38\frac{2}{5}$, but one note corresponding to about 38, etc. By doing this we do not produce the triads in perfect tune, but we approach nearly enough to perfect tune for all practical purposes. The scale in which these adjustments have been made is called a TEMPERED SCALE, to distinguish it from a scale in which the notes are related by the correct ratios.

All keyed instruments, like the piano, the organ, the clarionet, in which each key corresponds to a note of definite pitch, must be tuned to the tempered scale. On the other hand, stringed instruments, like the violoncello and the violin, may be played in the pure scale. It is for this reason that many musicians find the piano music disagreeable. It is related of Handel that he could not bear to hear music played in the tempered scale, so that he had constructed for himself an organ which had keys for every one of the notes demanded by the theory. A musician like Handel might be able to play upon a keyboard as complicated as this, but less gifted individuals would evidently be able to do nothing with it.

In tempering the scale, what method is employed? Do we

simply guess at the probable location of the notes desired, or do we adopt a fixed principle which shall render the departures from accurate tuning as small and as evenly distributed as possible? Evidently the latter procedure is the only strictly scientific one. The principle which is adopted is that of dividing the interval of the octave into twelve equal parts. Since the numbers that represent the series of notes express ratios merely, this division into twelve equal parts must be done by finding a number such that, if we multiply 24 by it twelve times in succession, the result will be 48. This number has been found to be 1.059, and if we multiply 24 by it twelve times we get for the numbers that correspond to the notes on the piano scale those indicated in the following table. The numbers that indicate the true intonation are added in order to make clear just how great the departures of the tempered scale from the theoretically correct one are:

	Natural	Tempered
c	24	24
c♯ *d*♭		25.43
d	27	26.94
d♯ *e*♭		28.55
e	30	30.25
f	32	32.05
f♯ *g*♭		33.96
g	36	35.98
g♯ *a*♭		38.12
a	40	40.38
a♯ *b*♭		42.80
b	45	45.33
c	48	48

If we examine these numbers we see that the notes *d* and *g* are but slightly out of tune; while some of the others, like *a*, are badly so. However, for an instrument like the piano and the organ, in which the notes are fixed, this distribution of error seems to be the best that can be made without unduly increasing the number of keys.

332. Standard Pitch. One more factor remains to be determined before the scale is completely defined. The numbers that have been given express merely the ratios between the vibration numbers of the different notes. In order to fix the scale completely, therefore, we must state how many vibrations some particular note gives. Two definitions of this sort are in common use. The physicist says, I will define the note middle *c* to be that note which executes 256 vibrations per second. The numbers of vibrations of the other notes of the scale may then be found by multiplying 256 by the ratios given in Art. 329. Thus, the vibration number of middle *g* is $256 \times \frac{36}{24} = 384$, etc. The musician, however, uses a different absolute pitch, for he defines the note *a*, which is the *a* of the violin, to be that note whose number of vibrations is 435 per second. This definition, viz., $a = 435$, is called the INTERNATIONAL STANDARD PITCH.

333. Forced Vibrations. Having thus found what the numerical relations are between the notes of the musical scale and learned that they have been so chosen because of something connected with our perception of sound, we will now proceed to see if we can find out what that something is. Although our ears are complicated structures, yet the physical principle that finds application in their operation is rather simple. It is none other than that of resonance, with which we have become familiar in Art. 314.

We there learned that a strict agreement between the natural period of the body and the impressed period of the wave is necessary for the production of resonance. While this is true in many cases, it is not always true, for light flexible objects will often vibrate by resonance when the impressed period coincides only approximately with the natural period. In such cases the vibrations will be most violent when the agreement between the periods is exact, and will diminish in intensity as the difference between those periods increases. The vibrations produced by resonance when the natural and the impressed periods are not the same, are called FORCED VIBRATIONS.

334. The Ear. The next step in solving the riddle of Pythagoras is indicated by the question, does the ear perceive sound by resonance? Are there in the ear a series of bodies whose natural periods of vibration are different, so that they would be set vibrating by different impressed periods? The answer to this question can, of course, be found only by an anatomical investigation of the construction of the ear. This has been done, and it is found that there are in the ear a large number of fine fibers of different lengths. These fibers are fastened at one end to a membrane which is inside a tiny cell that looks like a small snail shell, and is therefore called the cochlea. The cochlea is full of liquid, so that the fine fibers—called FIBERS OF CORTI, after their discoverer—are surrounded by the liquid. The membrane in which these fibres end is connected to the auditory nerve which carries the sensation to the brain. The arrangement of the cochlea and the other parts of the ear is shown in Fig. 190.

The main thing that interests the physicist in the construction of the ear is the presence of this series of fibers of Corti; for these may be a series of bodies which have natural periods of vibration, and which may, therefore, be set into vibration by resonance by notes of different pitch. And this is what we believe them really to be—a veritable set of resonators, each tuned to one of the notes which we are able to distinguish within the range of the musical scale. But how many would that be? Experiment tells us that we can not hear sounds whose numbers of vibrations are less than about 30 per second or more than about 30,000. So these little resonators in our ears must be tuned to notes that fall within this range. Attempts have been made to count them, and there are found to be about 3,000 altogether. Therefore there must be one for each difference of about 10 vibrations. It is probable that there are more than this in the middle of the musical scale and fewer at the outside limits; but however this may be, it is

FIG. 190. THE EAR

certain that there is not one for each difference of one vibration. And yet we can detect differences of less than this amount in pitches and can hear tones with all conceivable numbers of vibrations within the range just mentioned. How is this possible if there is only one resonator in the ear for each difference of 10 vibrations?

To account for this, Helmholtz holds that because these Corti fibers are flexible and light, they vibrate by resonance in response to notes whose periods are nearly the same as their own natural periods. If this is so, then a fiber whose natural period is $\frac{1}{160}$ sec will respond to impressed periods that lie, say, between $\frac{1}{140}$ and $\frac{1}{180}$ sec. Hence we see that any particular note must affect several of the little resonators in the ear, acting most strongly on that one whose natural period is nearest to the impressed period.

To sum up what we have thus far learned, we see that *the ear contains a large number of tiny resonators (fibers of Corti) which are tuned to different notes throughout the range of audible tones; when a sound wave of definite period falls on these resonators several adjacent ones are set into vibration.* This knowledge of the construction of the ear is essential if we are going to understand at all the reasons for harmony and discord.

335. Beats. We may now ask what sort of excitement of these fibers of Corti would be disagreeable. We can imagine a sort that would probably prove disagreeable by considering the similar case of light; for we all know well that a steady light is necessary for any comfort in seeing, while a flickering light, provided the number of flickers is neither very great nor very small, is intolerable. May it not be that a flickering sound would be as intolerable as a flickering light? But what is flickering sound? Clearly one in which periods of sound and silence follow one another closely, just as the flickering light is one in which periods of light and darkness follow one another closely. Do we ever have flickering sounds? Let us see.

Take two tuning-forks, or organ pipes, or other sources of sound whose vibration numbers differ slightly, one being, say, greater than the other by one. Conceive them to be started at

once in opposite phases. Then the two waves (Fig. 191) which they send out will start in opposite phases, and an observer will hear no sound. But since one of the waves is shorter than the other, and since they both travel with the same velocity, the phase of the shorter wave will gradually gain on that of the longer wave until the phases coincide(*a*, Fig. 191). When this condition has

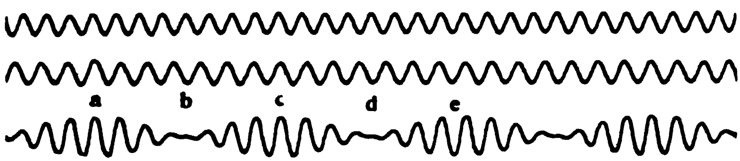


FIG. 191. BEATS

been reached, the two waves add together their effects, and a period of loud sound results. If there is in each second one more of the shorter waves than of the longer, the loudest sound will occur at *a*, in the middle of the second. As the waves proceed further the shorter again outstrips the other in phase, until at the end of the second *b*, they are again opposite in phase and we hear no sound.

If the difference in the vibration numbers of the two notes is 2, then there will be two periods of silence in each second, and so on. If N_1 and N_2 represent the numbers of vibrations per second of the two notes, then $N_1 - N_2 = n$ will be the number of periods of silence in a second. Two tones that produce flickering sound in this way are said to give BEATS, and the number of beats per second is equal to the difference in the numbers of vibrations of the two.

336. Discord Due to Beats. Having thus found out how a flickering sound may be produced, let us see if the result is disagreeable. The experiment may be tried in the laboratory by sounding together two organ pipes, or two tuning forks of slightly different pitch. They are also readily audible when two adjacent lower notes on the piano are sounded together. When they are slow they can be counted. When they get faster they become disagreeable, and when they become very rapid—more than

about 30 beats per second—they fail to be distinguishable and the disagreeable sensation ceases. Therefore Helmholtz concludes that discord is due to beats, and that we call two tones discordant when their combination produces between four and thirty beats per second.

But even with this explanation of discord we are still far from our goal. For if notes that give not over 30 beats per second are discordant, why do we object to the combination *c-f#* and prefer the harmony *c-g*? Since the numbers of vibrations of *c*, *f#* and *g* are 256, 376, and 384, the numbers of beats in these two cases are 120 and 128. Since these numbers of beats both fall outside of the disagreeable limit of 30, why should we judge one of the combinations of tones harmonious and reject the other as discordant? Before we can answer this question we shall have to discover the reasons for differences in quality between the tones of different musical instruments. As this inquiry is somewhat long, we shall devote the next chapter to its study.

SUMMARY

1. The musical scale has not always existed in its present form.

2. The notes whose vibration numbers are related by the ratios $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$ are called a major triad.

3. We choose the ratios $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$ for the triad because we find by experiment that the combination of the corresponding notes is pleasing.

4. There are three triads whose relationship is very close, viz., the tonic, the dominant, and the subdominant.

5. These triads together contain all the notes of the musical scale, and therefore define the scale.

6. The ratios of the vibration numbers for the notes of the major scale are expressed by the following numbers:

$$\frac{c}{24} \quad \frac{d}{27} \quad \frac{e}{30} \quad \frac{f}{32} \quad \frac{g}{36} \quad \frac{a}{40} \quad \frac{b}{45} \quad \frac{c}{48}$$

7. Intermediate notes have to be added to this scale if we wish to be able to play scales beginning on notes other than *c*.

8. This addition of intermediate notes makes tempering necessary.

9. Pianos and organs are tuned to the tempered scale.

10. The physicists' standard of pitch is $c = 256$, while the musicians' standard is $a = 435$.

11. The ear contains a series of resonators called fibers of Corti, whose natural periods lie within the limits $\frac{1}{30}$ and $\frac{1}{5000}$ sec.

12. A Corti fiber is affected by a vibration even though the agreement between its natural period and the impressed period is not exact, so that one impressed period produces vibrations in more than one fiber.

13. Two notes of different numbers of vibrations produce beats. The number of beats per second is equal to the difference between the numbers of vibrations of the two notes.

14. When two notes produce from 4 to 30 beats per second, the sound flickers and we call it discord.

QUESTIONS

1. How can we find the ratios of the numbers of vibrations of the three notes in a triad? What are those ratios?

2. Why do we say that the tonic, dominant, and subdominant triads are related? Why do they define a key?

3. How do we derive the relative numbers for the second and third triads from the first? What are these numbers?

4. What is needed for defining a scale in addition to these numbers?

5. Why is it necessary to add the black keys to the piano keyboard?

6. Why do we temper the piano notes? Upon what principle is it done?

7. Will a continuous force set a body vibrating? What sort of force will?

8. What relation must exist between the period of the impressed force and the natural period of a body in order to produce sustained vibration? When this relation is not exact, can resonance occur?

9. Can a stretched string detect a sound? If so, when?

10. What do we suppose to be the action of the fibers of Corti in the ear when a sound is heard?

11. Do we believe that one or that more than one fiber of Corti vibrates by resonance when a note of definite period is impressed on the ear? State the reasons for your answer.

12. What sort of sound is disagreeable to the ear? How is such a sound produced?

13. If two notes have relatively N_1 and N_2 vibrations per second, how many beats will they produce when sounded together?

14. Why are beats disagreeable only when we have more than 4 or less than 30 per sec?

PROBLEMS

1. Suppose a banjo string to be 90 cm long from the bridge to the nut. Calculate the distances from the bridge to the frets that give the various tones of one octave of a major scale, using the ratios of the vibration numbers as given in Art. 329.

2. Harmonics are produced on a violin string by lightly touching the string at points $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of the length of the string. This forces a node on the string at the point touched, and causes the string to vibrate in 2 loops, 3 loops, 4 loops, etc. What are the tones that may be obtained from the string in this way?

3. Pythagoras proposed to determine the tones of a musical scale by taking only intervals of a fifth (do–sol), starting from a given note. Beginning with $c = 24$ vibrations, find the number corresponding to its fifth, g. Then find the fifth of g, by multiplying by the ratio $\frac{3}{2}$. Continue the process, reducing each number that falls outside the limits 24–48, by $\frac{1}{2}$, and see if you can find out why such a scale is impracticable. This process amounts to raising $\frac{3}{2}$ to the power n. Is $\frac{3}{2}$ commensurable? Will $(\frac{3}{2})^n$ be commensurable? Will you ever by this process reach a note that is an octave of the one from which you started?

4. Calculate the vibration numbers of the triad beginning on $a = 40$ and those of the one whose middle note is $f = 32$. What numbers must be added to those in the scale in Art. 329 to enable you to play these triads? How nearly can you produce these triads with the tones of the tempered scale, Art. 331? Repeat the calculation for the triad beginning on $b = 45$ and the one whose middle note is $g = 36$. How do these triads fit the tempered scale?

5. A string under a tension 0+.600 gms force, gives middle C (C_1). Under what tension will it give E_1? G_1? C_2?

6. A string 60 cm long and 0.5 mm in diameter gives F_1, what must be the diameter of a string of the same length and under the same tension in order that it may give A_1? C_2? A_2?

7. A violin bow is drawn across the top of a narrow strip of spring brass 10 cm long, and it gives a certain tone, say C_2. Consider it as a rod: to what length must it be reduced in order to give the octave C_3?

8. A steel rod 80 cm long, clamped at the middle and rubbed with a rosin cloth gives the tone A_3. What changes in its length will cause

it to give successively the other seven notes of the scale beginning on this note?

9. An organ pipe is 8 feet long. What must be the length of a pipe, all other things being equal, that will give the fifth below the note given by the first? The octave above?

SUGGESTIONS TO STUDENTS

1. If you have a banjo, a guitar, or a mandolin, measure the distances from the frets to the bridge and see if they satisfy the laws of vibrating strings and the vibration numbers of the major scale. Can you find out whether the frets are tuned to the tempered or the pure scale?

2. If you play a violin, or have a friend who does so, play the harmonics and get your friend to measure the distance of your finger from the nut or bridge. Is this distance always an aliquot part of the length of the string? Can you recognize the pitches of the harmonics produced, using, if necessary, a piano to assist you? Do the vibration numbers of these notes "check up" with the lengths of the strings?

3. Can you find out, with the help of a standard tuning fork, whether your piano is up to concert pitch? Are all pianos and organs really tuned to the same pitch? When your piano is being tuned, consult the tuner and find out if he uses beats to determine where the two strings are in tune. Can you find out how organs are tuned?

4. Can you make a musical instrument that will play the scale, by driving pieces of knitting needles into a board? If you succeed, measure the lengths of these rods, and see if they follow the law for transverse vibrations as stated in Art. 321. Examine a musical box and see if it is made on this principle.

5. See if you can cut a long rod of dry, elastic wood into pieces of such lengths that they will play the scale when you lay them across two wedge-shaped sticks and strike them with a light hammer. Look at a xylophone in a music store and see if this principle applies to it. Does the tone depend on where the supports are placed?

6. How are the vibrations of a violin string communicated to the body? What part has the air in the body in producing and sustaining the tone?

7. Try to get a loud sound from a wire stretched between two iron gate posts. Does the result indicate that the air is set into vibration by the string of a violin, or is it the body that does this work?

8. What can you find, by examining the sound-board of a piano, as to the way in which it is adapted in shape and construction so as to give resonance to notes of various pitches?

CHAPTER XVIII

HARMONY AND DISCORD

337. Wave Shape and Tone Quality. We shall devote this chapter to the discussion of the last point in our investigation into the physical basis of harmony and discord. This point is involved in the question, why do we call notes like *c* and *f#* discordant when they produce, when sounded together, as many as 120 beats per second, while no discord results when the beats are less than 4 or more than 30? In order to answer this question, we must recall some of the facts presented in the preceding chapters.

First, waves bring us information as to: 1, the direction of the source; 2, the number of vibrations of the source; 3, the intensity of vibration of the source; and 4, the nature of the vibration of the source. We have further identified these four kinds of information with the characteristics of waves, as follows: 1, the direction of propagation depends on the direction of the source; 2, the length of the wave is connected with the number of vibrations or pitch of the source; 3, the amplitude of the wave is derived from the amplitude of the vibration of the source; and 4, the shape of the wave varies with changes in the nature of the vibration of the source.

When we apply these facts to sound, we see that the direction of propagation tells us of the direction of the source of sound, that the wave length tells us of its pitch, and the amplitude tells us of its intensity. But what information do we derive from differences in the shape of the sound waves? How do we detect these differences in shape? Since the only characteristic of sound remaining for determination is its quality, and the only characteristic of waves remaining undetermined is their shape, we may conjecture that our perception of differences in qualities in sound is dependent on our perception of differences in the shape of the sound waves.

Let us then adopt the hypothesis that tone quality is connected with wave shape, and see whether it will help us in getting the answer to the problem before us. The first step in the discussion of this question is that of determining how differences in the shapes of waves are produced. This we have already done, for we have learned that waves of complex shape are produced by adding together simple homogeneous waves of different lengths, amplitudes and phases (*cf.* Art. 300). Hence we can conceive that a sound wave of complex shape would result from the addition of two or more simple sounds differing from one another in pitch and intensity.

338. The Vibrating Flame. That this conception corresponds with the facts may easily be shown by experiment. We have but to devise a scheme for rendering the motion of the air particles visible, and then to bring several sources of simple homogeneous waves together, to see if the resultant motion of the air does not indicate that we have a complex wave. Probably the simplest method of doing this is the following: A thin rubber membrane *AB* (Fig. 192) is mounted between two rings of wood. A flexible tube *C* leads the sound waves up so that they can act on one side of this membrane. The membrane will then follow the vibrations of the air in the tube *C*. On the other side of the membrane is a small gas chamber *D*. Illuminating gas flows into this chamber at *F* and burns at the jet *E*. Whenever the membrane *AB* vibrates, the gas in *D* will vibrate also; and this will cause the flame to vibrate, the tip of the flame following roughly the vibrations of the membrane. Since the vibrations of sound are too fast to be observed by the unaided eye, we have to observe the flame in a mirror which is kept in rotation. The apparatus ready for use is shown in Fig. 193. When thus observed in the rotating mirror and no sound is acting, the image of the small flame appears to be drawn out into a straight band of light; but

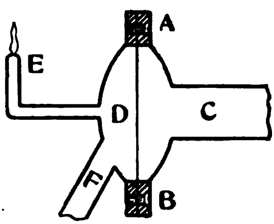


FIG. 192. THE VIBRATING FLAME

when a train of sound waves is allowed to strike against the membrane, this band is no longer straight, but its upper edge assumes a wave-like form which must correspond closely in shape to that of the waves impressed on the membrane.

Let us first send in sound waves from the tuning fork (Fig. 193), which, as we have learned, produces waves that are nearly homogeneous; the appearance of the flame in the rotating mirror will be then shown in the top band in Fig. 194. Using a second tuning

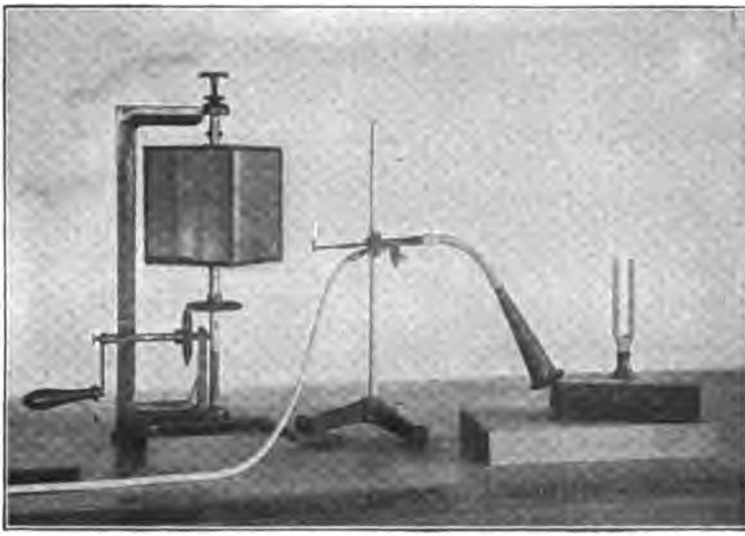


FIG. 193. APPARATUS FOR OBSERVING VIBRATING FLAMES

fork, an octave below the first, the appearance of the flame will be that shown in the second band in the figure. If now we send in the waves from both these tuning forks at the same time, the appearance of the flame in the rotating mirror will not be the same as before, for we have now added together two waves of different periods, and therefore have a complex wave. The result is shown in the third band in the figure. Referring to curve *R* in Fig. 181, page 309, we see that the shape of the top of the band of light agrees roughly with the shape of the curve there

obtained as the resultant of two waves, one of which had half the period of the other.

We thus prove that two or more simple homogeneous sound waves add themselves together just as other waves do, and produce resultant waves of complex form.

339. Are Musical Tones Complex? The question then arises, are the waves sent out by piano strings, violin strings, or the human voice simple homogeneous waves, or do they have complex forms? If they are complex, do two complex waves of the same period, but corresponding to tones of different quality, have different shapes? The answer to this question is easily obtained from observations with the little vibrating flame. For if one of us sings into the flexible tube C at the same pitch the vowels *a*, *o*, the appearance of the flame in the rotating mirror will be as shown in the fourth and fifth bands in Fig. 194. This result is a most striking confirmation of the hypothesis that tones of the same pitch but of different qualities have wave forms of different shape.

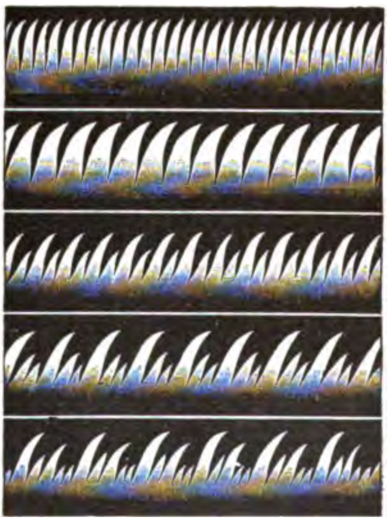


FIG. 194. APPEARANCE OF THE VIBRATING FLAME

The tones from strings, organ pipes, and other musical instruments, when analyzed in this way with the vibrating flame, show differences in wave form corresponding to their different qualities. But waves of different shape are produced by compounding simple waves in various ways. Therefore we see that a complex tone must be produced by the addition in various ways of simple tones, and that the quality of the complex tone depends on the way in which the various simple tones happen to be brought together.

340. How Musical Tones are Possible. It is easy to see how complex waves may be produced by the addition of two or more simple homogeneous waves of different lengths which originate from different sources, as from two or more tuning forks. But we have just learned that a single vibrating body, like the human vocal organ or a musical instrument, produces such complex waves. How can a single vibrating body produce several different vibrations at the same time? And if it does do so, are there any relations among the different vibrations which are thus produced at the same time? Recall the jumping rope (Art. 301). We learned that it may vibrate in one loop, in two loops, in three loops, depending on how rapidly it is turned. Similarly, a stretched string (Fig. 195) may vibrate in one loop, in two loops, in three

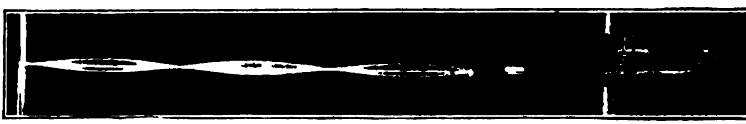


FIG. 195. THE STRING MAY VIBRATE IN THREE LOOPS

loops, etc. Since in this case the string is stretched with a constant force, and since the lengths of these loops are 1, $\frac{1}{2}$, $\frac{1}{3}$ the length of the string, etc., the vibration numbers of the notes produced are related by the simple ratios, 1: 2: 3, etc. Suppose it were to vibrate in several of these ways at once, what would be its shape? We can find out by adding together the component vibrations as in Art. 300. Thus, if we conceive the string to be vibrating in one and in two loops at the same time, and that the amplitude of the 2 loops is only half that of the 1, the result is shown at R in Fig. 181. If now, in addition, it is vibrating in three loops, and the amplitude of the 3 is but $\frac{1}{3}$ that of the 1, we add the 3 loop wave to the resultant of the other two and obtain the curve shown at R' in Fig. 181. Similarly, by adding the 4 loop vibration with $\frac{1}{4}$ the amplitude of the 1, and the 5 loop vibration with $\frac{1}{5}$ the amplitude of the 1, we get the resultant shown at P and Q, Fig. 196. If we continue this process of adding the curves that correspond to greater numbers of vibrations, each

with a correspondingly smaller amplitude, the resultant becomes more and more like the curve at *R* in the figure.

We thus see that a string may vibrate in all these ways at once if it can take the shape shown at *R*. But can strings take that shape? What would be the shape of a stretched string if a piano hammer had hit it at a point near the end? How does the violin bow act? Does it not pull the string into a shape similar to that shown in the figure until the tension of the string becomes great enough to overcome the friction of the bow? Then the string flies back. Or, if we merely pick the string with a sharp point, as in the case of the mandolin, we bring the string to the indicated shape and then let it go. So we see that a string may be made to take the indicated shape, and therefore we may infer that it can send out a compound wave similar to that composed of a number of vibrations whose periods are related to one another, as 1: 2: 3, etc., and whose amplitudes continually decrease.

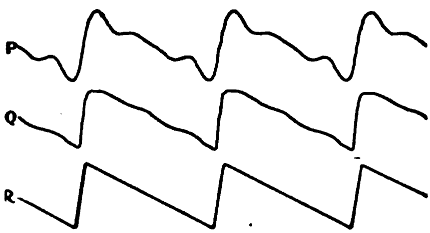


FIG. 196. COMPLEX WAVES OF A STRING

341. Fundamental and Overtones. We must now distinguish between the tones thus produced. For this purpose we call the tone that corresponds to the vibration of the string in one loop the FUNDAMENTAL. It has the smallest number of vibrations and is most intense of them all. The other tones are called OVERTONES, or harmonics. Since the number of vibrations of the string in two loops is twice that of the string in one loop, the first overtone will be the octave of the fundamental. Similarly, the second overtone is related to the first overtone by the ratio $\frac{3}{2}$, and will therefore be to the first as *g* to *c* in the musical scale. This interval is called a fifth. Similarly, the third overtone is related to the first overtone by the ratio $\frac{4}{2}$, and will therefore be an octave above the first, etc.

There are several interesting things about these overtones.

In the first place, the quality of a complex tone evidently depends on which overtones are present and how strong each is; for we have shown that notes of different quality produce complex waves of different shapes, and also that differences in shapes of waves are produced by differences in the number and strength of the simple waves of which they are composed. In the second place, we can show that a simple wave produces resonance in a body whose natural period agrees with that of the simple wave, not only when the simple wave exists alone, but also when it is a component of the complex one.

342. Overtones of Piano Strings. The simplest way of showing this is the following: Press a key, say middle *c* of the piano, gently, so that the hammer does not strike the string, but so that the muffler is lifted. The string will then be free to vibrate by resonance. Then strike the key *C*, an octave below, and let it rise again so that the muffler stops the vibrations. The tone middle *c* will be heard gently humming in the piano. But we have just learned that the lower *C* contains the upper *c* as its first overtone; so we see that, since the overtone *c* exists as part of the compound tone *C*, the string whose natural period agrees with that of this overtone is set into vibration by resonance. Similarly, if we press the note *g* above middle *c* so as to lift its muffler, and then again strike the lower *C*, the tone *g* will be heard coming from the piano, showing that *g* exists as an overtone in the vibrations of *C*. If, however, the experiment be tried with the note *f* above middle *c*, no tone will be heard from the piano after lower *C* has ceased to vibrate, because *f* is not an overtone of *C*, and therefore the vibrations corresponding to *f* do not exist in the complex tone emitted by *C*.

343. Helmholtz Resonators. The experiment is even more striking if we use as resonators, not the strings of the piano, but a series of hollow brass spheres, whose volumes are such that the natural periods of the volumes of air in them coincide with the periods of different notes of the piano. Such a series of hollow spheres was used by Helmholtz in analyzing these complex tones.

One of them is shown in Fig. 197. The little projection on one side is intended to be fitted into the ear, thus enabling it to detect very faint sounds whose periods agree with that of the resonator.

344. How the Ear Perceives a Complex Tone. Let us now expand our conception of the resonance effect of a compound tone to include all the overtones at once. What effect will be produced on a series of strings, tuned to all the notes of the scale and free to vibrate, if we sound near them a compound note whose fundamental agrees with one of the notes of the scale? Evidently those strings that correspond to the overtones will be set into vibration by resonance, while the others will remain at rest. Thus we see that the effect of a compound tone on such a series of strings, like those of a piano or harp, is similar to that produced by a skilled hand passing rapidly over the strings and touching gently those among them that correspond to the overtones and the fundamental note.

FIG. 197. RESONATOR

We have learned that the ear contains such a series of strings or fibers, and so we may imagine that when a complex note falls on the ear, not all of these fibers are excited, but only those whose natural periods agree approximately with the periods of the fundamental and the overtones of the note.

We may now reach our final conclusion concerning discords; for since it appears that all musical notes are complex, and since such a note excites in the ear not only the fibers corresponding nearly to its fundamental, but also those corresponding to the overtones, it becomes clear that to obtain harmony between two notes we must avoid beats, not only between the fundamentals, but also between the overtones. Therefore the complete answer to our question as to the reasons for discord is, *two tones are discordant when either their fundamentals or any of their overtones produce beats which are more than* 4 *or less than* 30 *per second.*

It now remains for us to show that this principle will enable us to make clear why the interval *c-g* is more pleasing than *c-f*♯. In order to do this we have merely to write out the numbers of

vibrations of the fundamentals and of the overtones and see whether we have such beats anywhere. These numbers are, for the three notes under consideration,

c	256	512	768	1024		1280	1536
g	384		768		1152		1536
f#	376		752		1128		1504

It thus appears that the discord between *c* and *f*# is due to the production of 16 beats by the second overtone of *c* and the first of *f*#.

345. Related Tones. This table brings to light another interesting fact concerning the notes *c* and *g*, viz., that some overtones are common to both. We see that both have an overtone of 768 vibrations and another of 1536. Noting this fact, Helmholtz calls such notes musical relations, i.e., he says that when two tones have two or more overtones in common, they are musically related. Such musical relationship must occur between notes whose fundamental vibration numbers are related by the simple ratios 1: 2: 3: 4: 5: 6, etc., because the vibration numbers of the overtones are related to those of the fundamentals by these same ratios.

And so, at last, we reach the answer to Pythagoras's problem. It may be stated in many ways, but perhaps the simplest is the following: *The numbers of vibrations of the overtones of strings and air columns are related to those of the fundamental by the simple ratios* 1: 2: 3, *etc. Therefore the notes of the scale must be related by the same ratios in order to avoid disagreeable beats between both fundamentals and overtones.*

346. Chimes. Do we ever use other sources of musical tone besides strings and air columns? We might answer, "Yes," and cite, as an example, chimes of bells, which are justly reputed to produce a decidedly musical effect. But did you ever hear a chime of bells played in chords, i.e., more than one note at a time? Probably not, because the overtones of the bells are not related to their fundamentals by the simple ratios 1: 2: 3, etc., and so when bells

are played in chords, the effect is musically intolerable, because disagreeable beats occur between the overtones.

Another interesting conclusion is, that fundamental tones which have no overtones would not be disagreeable, when the same fundamental tones with overtones would be so. This is easily shown to be true by sounding together two tuning forks with pitches *c* and *f*#, for instance, and comparing the effect with that produced by two organ pipes or strings of the same pitches. The combined effect of the forks is not at all disagreeable, while that of the pipes or strings is decidedly so.

There are many other interesting and perplexing questions concerning tone quality and concerning harmony and discord. For example, how can we control the tone quality, as in the organ, where we make pipes whose tones resemble flutes, violins, horns, and even the human voice? How are the wonderfully different qualities of the human voice produced? If we could magnify the records cut by a phonograph in the wax cylinder, what would their shapes be? How are the possible successions of chords in a musical composition dependent on the tone quality and the beats. These inquiries can not be pursued here, for a discussion of them would lead us far beyond the scope of this book.

SUMMARY

1. Tone quality is related to wave shape.

2. The addition of simple sound waves produces waves of complex shape.

3. A single vibrating body may send out complex waves.

4. The complex vibrations of strings and air columns are composed of simple vibrations whose numbers are related by the ratios 1: 2: 3, etc.

5. The lowest note in the complex tone is called the fundamental and the others are overtones.

6. The relations between the vibration numbers of the fundamental and the overtones of strings and air columns are expressed by the ratios 1: 2: 3.

7. Tone quality depends on the number, pitches, and relative intensities of the overtones.

8. An overtone in a compound note may produce resonance just as if it were alone.

9. Two tones are discordant when either the fundamental or any of the overtones combine to produce disagreeable beats.

10. The notes of the musical scale must be related by simple ratios because the overtones of musical instruments are so related.

11. Harmony depends on tone quality as well as on pitch.

QUESTIONS

1. Why may we assume that tone quality and wave shape are related?

2. How do we prove that this assumption is correct?

3. How can a single vibrating body send out complex waves?

4. How do we know that the numbers of vibrations of the overtones of a string are related by the simple ratios 1: 2: 3, etc.?

5. Upon what does the tone quality depend?

6. Can one component in a complex wave act to produce resonance in a body whose number of vibrations agrees with its own?

7. When the fundamentals do not produce disagreeable beats, why may two tones still be discordant?

8. What do we mean when we say two tones are musically related?

9. Why do the notes of the musical scale have to be related by simple ratios because the overtones of strings and air columns are so?

10. Could we replace the strings of a piano with bells with good musical effect? If not, why not?

PROBLEMS

1. Beginning with $c = 24$, write out the vibration numbers of the first 8 overtones of a string of that pitch. Do the same with the tone $g = 36$. How many of the eight overtones are common to both tones? Which of the overtones is the first common one? Write out the first eight overtones beginning on $f = 32$, and also on $e = 30$. How many overtones has each of these tones in common with the series beginning on 24? Which of the overtones in each series is the first common overtone? Which is the best consonance, c–g, or c–e? Which pair have the greatest number of common overtones?

2. Have the two tones beginning respectively on $c = 24$ and $d = 27$ any of their first eight overtones in common? Do any of their overtones give disagreeable beats, i.e.. more than four and less than 30 per sec? Is this interval c–d more or less consonant than the interval c–g? Can you see any connection between the consonance of a musical interval

and the number of the first overtone which is common to the two component notes?

3. In a string on a musical instrument a node cannot exist at the point where the string is either bowed, picked, or struck with a hammer. If a string is plucked at a point distant from the bridge $\frac{1}{3}$ the length of the string, what overtones will be wanting in the tone produced?

4. A piano hammer strikes at a distance of $\frac{1}{7}$ the length of the string from one of its ends. What overtones are wanting in the tone produced? Write the series of overtones for $c = 256$, and see if the seventh is apt to cause beats with its neighbors. Can the quality of the tone of a piano string be varied by changing the position of point where the hammer strikes? Why does a violinist bow near the bridge when he wishes to produce "brilliant" tones?

SUGGESTIONS TO STUDENTS

1. Construct a vibrating flame as described in Art. 338, in Hopkins's *Experimental Science*, and in Mayer's *Sound*, and see what vowel sound gives the most interesting vibrations. See if each of your voices gives the same shaped flame when singing the same vowel on the same pitch. Try other musical instruments in the same way. See if you can photograph the flame.

2. Can you find out how a phonograph or a graphophone works? What sort of curves must be cut in the cylinder or disc of the machine? Have you ever examined such a curve with a microscope?

3. Ask the organist at your church how organ pipes are made to have different qualities of tone. Examine the pipes yourself and see if you can think why the diameters of the flute and violin pipes are smaller in proportion to their lengths than those of the diapason pipes.

4. Pronounce the vowels. In which is the mouth cavity elongated so as to give resonance to the lower overtones? In which is it shortened so as to cut them out?

CHAPTER XIX

LIGHT

347. What Does Light Do for Us? A peculiar interest attaches to the study of light, because of its great usefulness to mankind. Not only is it indispensable for all human action, but also the color combinations by which it enables us to express art ideals are sources of highest pleasure and satisfaction. Can you conceive of a world devoid of light? And what a monotonous existence we should lead if light were deprived of color! Yet the very omnipresence of light often leads us to overlook its vast importance to life in the universe. In taking up the discussion of this subject, then, let us ask, first, what does light do for us?

When we ponder this question carefully, we are led to conclude that light enables us to gain information of four different kinds. First, it enables us to distinguish differences in the directions in which objects are located with reference to us and to one another. This ability to recognize differences in direction enables us to determine the shapes of objects as well as their relative positions, for the different parts of an extended object lie in different directions from our eyes.

In the second place, light makes it possible to distinguish between the colors of things. This power not only assists us in distinguishing between objects about us, but it also enables us, as we shall presently see, to observe the peculiarities of distant stars and study the mechanism of ultimate atoms.

In the third place, we are able to distinguish between intense and faint light—to recognize all the possible gradations of light and shade, whose totality produces the pictures which succeed one another with endless variety and which produce in us emotions of joy or pain, of inspiration or dejection, throughout our entire conscious lives.

And, lastly, we can not only appreciate simple color, but we can

distinguish and produce endless shades and varieties of color by mixing the simpler colors together in different ways. This last power, which we could not have without light, is fundamental in the art of painting, and is thus of far-reaching importance in our appreciation of the beautiful both in nature and in art.

Now, it may seem to many sacreligious to attempt to pry into the mechanism of light—to seek to find out how light is able to do all this for us. We must confess that we think that it would be so if it were not for the fact that this inquiry does not in any way destroy our recognition of the enormous utility of light nor diminish our appreciation of the beauties of nature and of art which it enables us to enjoy. On the contrary, the detailed study of the phenomena of light, in the way in which the physicist studies it, adds enormously to our estimation of the wonders of light by showing us the ingenious way in which it operates to serve us as it does:

348. What is the Nature of Light? After noting carefully the common experiences with light, the first thing the physicist does, is to ask what assumption or hypothesis he can adopt that will enable him to group the various phenomena together and to construct a mechanical model that will assist him in describing its action more in detail. When asked to propose such a hypothesis, what shall we say? We have just analyzed the kinds of information that light helps us in acquiring, and find them to be of four sorts, viz.: 1, As to direction; 2, as to color; 3, as to intensity; and 4, as to blending of colors. What sort of mechanism is able to bring us such information? Probably, just as in the case of sound, a wave motion would suffice; and therefore we will at the outset adopt the hypothesis that *light is a wave motion.*

But what characteristics of the phenomena of light may we identify with each of the characteristics of the wave? Clearly, the sense of direction is derived from the direction of propagation of the waves. It is also clear that the intensity of the light corresponds to the amplitude of the waves. This leaves perception of color and the composition of colors to correspond respectively

to the wave length and the wave form. Possibly simple color may correspond to wave length and complex color to wave form. Let us, then, assume that these are the relations and see to what conclusions we shall be led. To this end we must enter upon a more detailed discussion of these characteristics of light.

349. Direction. The first question that naturally arises is, how do we detect differences in direction? You answer, "With our eyes"; but how do they operate to detect the direction of propagation of waves? In order to answer this question, we must first call to mind a very familiar characteristic of light, viz., that it appears to travel in straight lines. Thus, the sunlight falling on the floor traces an outline of the window there. If we cover the window with a shutter having a small hole in it, we notice that the beam of light which passes through the hole, and whose path is revealed by the dust particles in the air, travels in a straight line, and makes a bright spot on the floor. We also notice that the path of the light is the continuation of the line joining the sun and the hole in the shutter, so that if we invert the process and draw a line from the spot on the floor to the hole, that line indicates the direction of the sun. Thus we see that we can determine the direction of the sun with reference to the shutter and the floor, because the light travels ordinarily in a straight line.

350. Image. If now we have two bright objects outside the window, like two electric lights, each will produce a bright spot on the floor or on some suitable screen held behind the hole in the shutter. When we draw straight lines from these two spots to the hole, they inclose an angle between them, and by means of this angle we are able to judge of the relative positions of the two electric lights. If now we have a large number of such bright points, for example a landscape outside the window, each point of the landscape produces on the screen a bright spot, which indicates the direction of the point with reference to the screen and the hole; these bright spots on the screen will each indicate the direction of its corresponding source, and so we obtain on the screen an image of the landscape outside (Fig. 198).

Two things are apparent concerning the image thus formed: 1. It is inverted, and 2, it is indistinct and fuzzy. A moment's

FIG. 198. THE IMAGE IS INVERTED AND FUZZY

thought will show us why it is inverted, namely, because the rays all cross at the hole, so those that were below on one side are now above on the other, and *vice versa*. Therefore this characteristic of the image is inherent in the nature of the phenomenon and can not be altered.

But why is the image fuzzy? An analysis will show us why. It is because each point of the landscape is sending out waves which spread out in all directions about that point (*cf.* Fig. 172). When these waves reach the hole in the shutter, they are divergent, and therefore make on the screen a spot of light somewhat larger than the hole, as shown in Fig. 199. Thus the image of each point of the object is a spot of light on the screeen, not a point; and therefore the entire image, which is the sum of these spots, is not clear and distinct like the object. Yet, even so, the image is enough like the object to be readily recognizable.

FIG. 199. THE SPOT IS LARGER THAN THE HOLE

351. What the Eye Does. Now, although this image is not as clear as the landscape outside, it enables us to distinguish definitely the directions of the different points of the latter. We might, therefore, conceive that the eye is able to distinguish directions by a similar device; for is not the pupil of the eye merely a small hole in a shutter, and therefore there must be formed at the back of the eye an image of the object in front? Now, an examination of the construction of the eye shows that immediately behind the pupil there is a little transparent object of hard elastic substance, called the CRYSTALLINE LENS (*L*, Fig. 200). The front and back of this lens seem to be portions of

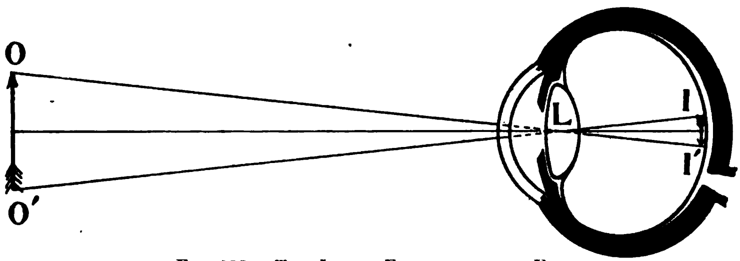


FIG. 200. THE IMAGE FORMED IN THE EYE

spherical surfaces, and it is thicker at the middle than at the edges. Behind this lens the eye is filled with water, and the rear surface, called the retina, is covered with fine nerve filaments.

352. What a Lens Does. We can see that an image would be formed at the back of the eye without the crystalline lens. What, then, is the use of this addition? On holding a piece of glass that is shaped like the lens of the eye behind the hole in the shutter, we observe that when the screen is at one particular distance from the hole the image of the landscape outside is very clear and distinct. Therefore we may conclude that the purpose of the crystalline lens in the eye is to render the image on the retina distinct, i.e., to bring the rays from a point on the object outside together on the retina in a point instead of in a spot. Referring to Fig. 204, p. 377, we see that such a lens must be able to

bend the light, so that, after passing the lens, it is convergent instead of parallel or divergent.

353. How Light is Changed in Direction. But if light travels in a straight line, how can a lens bend it? Yet, clearly, it does do so. Have you ever noticed that light bends when it passes obliquely from air into water? Place a battery jar full of water so that the sunbeam from the hole in the shutter falls obliquely on the surface, Fig. 201. Does the path of the light have the same direction in the water as it does in the air? Is the path in the water straight? Thus it becomes clear that light travels in a straight line only so long as it is moving through the same sort of matter, for when it passes from air to water it is bent. A similar effect is observed when we pass the light into glass or into any other transparent substance.

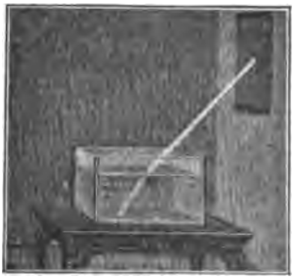


FIG. 201. THE LIGHT IS BENT WHEN IT ENTERS THE WATER

How may we conceive that this bending is effected? We have assumed that light is a wave motion. Let us then imagine that we have a beam of light of width *a b*, Fig. 202, traveling in air and approaching a surface of water *a c*. Let the direction in which the light is traveling be represented by *b c*. Then the FRONT of the wave, i.e., the line joining those points of the wave that are in the same phase, will be represented by *a b*, which is perpendicular to *b c*. When the light has entered the water, we find that it is traveling in the direction *c e*; so that the wave front, which in the water is perpendicular to *c e* the new direction of travel, has been turned from the direction *a b* to that of *c d*.

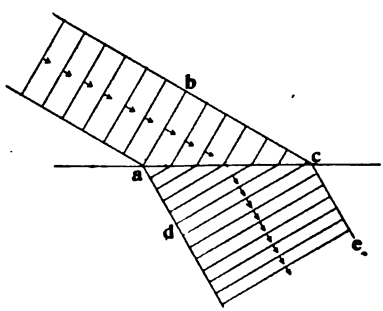


FIG. 202. REFRACTION

This result would be accomplished if that portion of the wave near b traveled the distance $b\,c$ in air in the same time that was taken by the portion of the wave near a to travel the distance $a\,d$ in water. But $a\,d$ is clearly less than $b\,c$. So we see that we can form a perfectly intelligible picture of the manner in which a ray is bent on passing from one medium to another, by assuming that the light waves travel more slowly in the water than in the air.

354. Index of Refraction. This phenomenon of the bending of a beam of light when it passes from one medium to another is called REFRACTION. How can we measure it? Clearly, the amount of bending depends on how much difference there is between the velocity of light in the two media. For if $b\,c$ remains the same, then the less $a\,d$ is the greater the bending will be. We may, therefore, measure the bending by the ratio of $b\,c$ to $a\,d$. But $a\,d$ represents the distance traveled by the light in the second medium in a certain time t, and $b\,c$ represents the distance traveled in the first medium in the same time t. Therefore $a\,d$ and $b\,c$ are proportional to the velocities of light in the two media. So we may say that the amount of bending depends on the ratio of the velocities of light in the two media. Since we can measure the amount of bending by the ratio of these two velocities, we call that ratio the INDEX OF REFRACTION. It is usually denoted by n. Therefore we define this index in the following way:

$$n = \frac{\text{velocity in first medium}}{\text{velocity in second medium}} = \frac{b\,c}{a\,d}. \qquad (18)$$

It has been proved by experiment that the velocity of light in a given medium is constant for a given color, therefore we may infer that this index remains constant for the same two media.

It is not always easy, however, to measure the velocities in the two media; therefore let us see if there is not a more convenient form of expressing this ratio. To do this, drop a perpendicular ncn' to the surface between the two media (Fig. 203). Then the angle ncb, formed between this perpendicular and the direction in which the light is traveling in air, is called

the ANGLE OF INCIDENCE and is usually denoted by i. Similarly, the angle $n'ce$ is called the ANGLE OF REFRACTION and is usually denoted by r. Now, from the figure we see that $\angle ncb = \angle bac = i$ and $\angle n'ce = \angle acd = r$. We have also learned (*cf.* Art. 304) that

$$\sin bac = \frac{bc}{ac} = \sin i, \text{ and } \sin acd = \frac{ad}{ac} = \sin r, \text{ whence}$$

$$\frac{\sin i}{\sin r} = \frac{\frac{bc}{ac}}{\frac{ad}{ac}} = \frac{bc}{ad}. \text{ Therefore [*cf.* equation (18)] } n = \frac{\sin i}{\sin r} \quad (19).$$

Thus the index of refraction is equal to the sine of the angle of incidence divided by the sine of the angle of refraction. Since these angles are easily measured, this ratio is more convenient to use than that of the velocities.

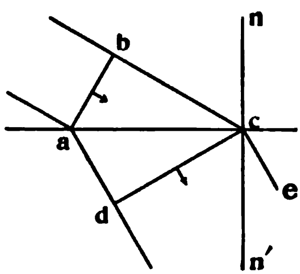


FIG. 203. REFRACTION DIAGRAM

Now, if we use light of one color and measure the angles r that correspond to various angles of incidence i, and then, with the help of a table of sines, find the corresponding values of n, we shall find that the values of n are the same for all angles of incidence. Therefore we conclude that *n is constant for any two media and for any definite color*, as we have inferred that it should be. This fact was discovered experimentally by Willibrode Snell in 1680, and is known as SNELL'S LAW, or the LAW OF REFRACTION.

355. How the Lens Forms the Image. We are now able to see why the introduction of a lens of the form of the crystalline lens of the eye improves our image; for since that lens is thicker in the middle than it is at the edges, it is evident that those portions of the wave which pass through the center of the lens travel through a greater thickness of glass. But the light travels more slowly in glass than in air, so the center of the wave is retarded more than the portions near the edges of the hole, and the wave is converted from a plane or a convex wave into a concave

wave, as shown in Fig. 204, in which the vertical lines represent the successive wave fronts.

Because after passing the lens the wave fronts are concave, they contract toward a point O, and there form a small image of the point from which they started. Thus we see that introducing a lens of the given form contracts the spot of light to a point. Therefore every luminous point of the landscape is represented by a single bright point on the screen, and so the image becomes brilliant and distinct.

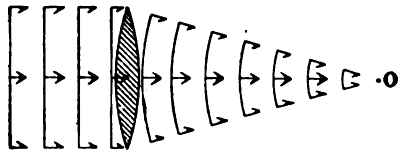


FIG. 204. THE LIGHT IS BROUGHT TO A POINT

We see, however, that the screen must be placed at the definite distance O from the lens in order to receive a distinct image. This distance from the lens to O is called the FOCAL LENGTH, and the point O is called the FOCUS. Manifestly, the focal length will depend on the curvature of the lens, its index of refraction, and the shape of the incident wave. The study of the relations between these quantities is of great importance, for the construction of all optical instruments depends on them. We can not properly understand optical instruments without a clear conception of these relations. We shall take up this study in the next chapter, as our attention is first demanded by one other important phenomenon connected with our perception of the direction of light.

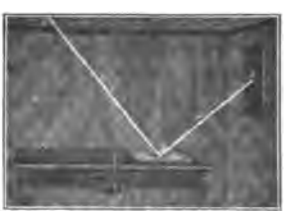


FIG. 205. REFLECTION

356. Reflection. We can best study this phenomenon by placing a mirror on the floor where our sunbeam falls. The beam is turned away from the floor and reflected to the ceiling or to some other part of the room, Fig. 205. If we turn the mirror into different positions, we note that the reflected beam is turned in different directions. But by so turning the mirror we vary

the angle at which the incident beam falls upon it. We also vary the angle at which the reflected beam leaves it. Is there any relation between these two angles?

357. Laws of Reflection. Before answering this question, we must agree as to how we shall measure the angle of incidence. We therefore define the ANGLE OF INCIDENCE to be the angle included between the perpendicular to the surface and the incident beam. This angle is measured in the plane which contains this perpendicular and the incident beam. Thus if (Fig. 206) AB represents the direction of the incident beam, BC that of the reflected beam, and NB the perpendicular to the mirror, the angle of incidence is then NBA. This angle is, as before, denoted by i. Similarly, the angle of reflection is that included between the perpendicular NB and the reflected beam BC, i.e., it is NBC.

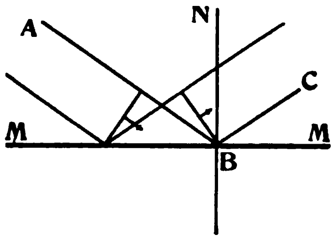


FIG. 206. REFLECTION DIAGRAM

We may now ask what relation, if any, exists between these angles. In order to answer the question, we must measure various angles of incidence and the corresponding angles of reflection. [If we do this, we find that *the angle of incidence is equal to the angle of reflection* in all cases. We further find that *the incident beam, the perpendicular, and the reflected beam, all lie in the same plane.* Therefore we conclude that these are the LAWS OF REFLECTION.

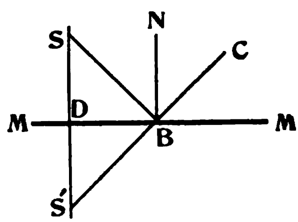


FIG. 207. THE PLANE MIRROR

358. Where the Image Appears. These principles can now be used to find out where a source of light appears to be when we observe it by reflection in a mirror. Clearly, an observer at C, Fig. 206, will receive the light as if the source were in the direction CB. But at what point in that direction? In order to find out,

we have but to alter Fig. 206 as follows: Let S (Fig. 207) be the source of light. Since it is sending beams in all directions, it will send out not only one in the direction $S\,B$, but also many in other directions. One particular ray SD will strike the mirror perpendicularly, and be reflected back along its own path, i.e., in the direction of DS.

Now, an observer at C sees the light in the direction CB and another observer behind S and in the direction DS sees it in the direction SD. In what direction does the source appear to lie? Clearly in the directions of both lines. Hence the apparent source of the light must be at the point S', where the lines CB and SD, intersect when they are extended behind the mirror.

But where is the point S' with respect to the mirror? From the law of reflection, we know that the angle NBS = angle NBC, hence we may easily prove that the two right triangles SBD and $S'BD$ are equal, and therefore $S'D = SD$. But these are the distances of the source S and its image S', measured perpendicularly from the mirror MM; so we see that this result may be stated as follows: *When light is reflected in a plane mirror the image appears to be as far behind the mirror as the source is in front of it.*

359. Diffuse Reflection. The law of reflection just stated applies clearly to all cases of reflection from metallic or other polished surfaces. If we replace the mirror by a piece of white paper, what becomes of the reflected beam? If we try the experiment with other things by placing them in the path of the sunbeam we will notice that some of them act like the mirror and reflect most of the light in a definite beam while others reflect part of it in a definite beam, and still others reflect none of it in a definite beam. Hence we see that this law of reflection is not general in its application. We may, then, ask what law applies to these other cases. On placing a white card in the path of the beam, we note that the light seems to be scattered in all directions from the bright spot on the card. In fact, the effect is the same as if the card were itself the source of the light. This phenomenon is called DIFFUSE REFLECTION in contradistinction to the other kind, which is called METALLIC REFLECTION.

The importance of diffuse reflection is seldom appreciated. We do not often realize that we see most objects because they reflect diffusely. Thus, when we look at a landscape or a picture, each part of the object affects us as if it were itself a source of light. This, then, is the law of diffuse reflection, viz.: *A body that reflects light diffusely appears as if it were self-luminous.*

We thus see that there are two kinds of reflection, metallic and diffuse, of which the latter is the more important to mankind. It is not possible, however, to classify all substances as reflecting either metallically or diffusely. At one end of the series we have the metals, which reflect in the first way only; and at the other end we have what are called PERFECTLY MATT SURFACES, like a plaster wall, which reflect entirely diffusely. Between these two extremes we have substances that reflect partly in one way, partly in the other, in all sorts of varying proportions.

SUMMARY

1. Light enables us to acquire four kinds of information: 1. As to direction of a source; 2, as to its color; 3, as to its intensity, and 4, as to its tone of color.

2. Waves also bring us four similar kinds of information, and therefore we adopt the hypothesis that light is a wave motion.

3. Light ordinarily travels in straight lines in any one medium.

4. An image of an object is formed when the light from it passes through a small hole.

5. Such an image is inverted and blurred.

6. A lens makes this image distinct and more brilliant.

7. The focal length of a lens is the distance from the lens to the point at which the image is formed.

8. The conditions necessary for the formation of a clear image are realized in the human eye.

9. The direction in which light travels is altered when it passes obliquely from one medium to another.

10. The amount of this bending is measured by the index of refraction.

11. The index of refraction is the ratio of the velocities of

light in the two media, or the ratio of the sines of the angles of incidence and refraction.

12. The index of refraction is a constant for any two given media and for a given color.

13. When light is reflected from a metallic surface, the angle of incidence is equal to the angle of reflection, and lies in the same plane.

14. The image of an object reflected in a plane mirror appears as far behind the mirror as the object is in front of it.

15. Light is diffusely reflected from unpolished surfaces.

16. A surface reflecting diffusely appears as if it were itself a source of light.

17. The reflection of light by many surfaces is partly diffuse and partly metallic.

QUESTIONS

1. What four kinds of information does light enable us to acquire?

2. What may we assume as a working hypothesis as to the nature of light?

3. Upon what property of light does our determination of the direction of light depend?

4. How is an image formed through a small hole?

5. Why does a lens improve the clearness of such an image?

6. Describe the construction of the human eye. What provision is there made for distinguishing differences in the directions of objects?

7. Why is the path of light bent when it passes obliquely from air into water?

8. How do we measure the amount of this bending?

9. What relation exists between the index of refraction and the velocities of light in the two media? between that index and the angles of incidence and refraction?

10. What is the focal length of a lens?

11. What is the difference between metallic and diffuse reflection?

12. What is the law of diffuse reflection?

13. Which kind of reflection is the more common? Which is the more useful?

14. Where does an object reflected in a plane mirror appear to be? Can you prove it by the geometrical relations?

15. What sorts of substances reflect metallically? What sorts entirely diffusely?

PROBLEMS

1. The method of finding the location of the image of a point source in a plane mirror is described in Art. 358. Replace the point source by an arrow and graphically construct the image. How far behind the mirror does the image lie?

2. If v_1 represents the velocity of light in glass and v_2 that in water, show that the index of refraction of light at a surface between glass and water is equal to $\frac{v_1}{v_2}$. If v represents the velocity in air, what is the index at the surface between air and glass? Between air and water? May the index for glass-water be obtained by dividing that for air-glass by that for air-water? Show how.

3. The index of refraction is shown to be the ratio of the sine of the angle of incidence to the sine of the angle of refraction, Art. 354. If the light falls perpendicularly on a surface of glass, so that the angle of incidence is 0, what is the value of the angle of refraction? Is the light bent when it passes perpendicularly through the dividing surface?

4. Is the period of vibration of a light wave (i.e., the color of the light) changed by passing from one medium to another, as from air to water? With the help of the equation $v = n\,l$, show that the index of refraction may also be defined as the ratio of the wave length of the light in air to its wave length in water.

5. When light passes from water into air, is the path of the light bent toward or away from the perpendicular to the surface? How do you define the index of refraction under these conditions if v_1 represent the velocity of light in water and v its velocity in air? How is it defined in terms of the sines of the angles of incidence and refraction? How does its numerical value compare with that for the converse case of light passing from air into water?

6. The index of refraction of air-water has the value 1.33, i.e., $\frac{\textit{sine } i}{\textit{sine } r} = 1.33$, or $\textit{sine } i = 1.33 \textit{ sine } r$. But the sine of an angle is the ratio in a right triangle of the side opposite the angle to the hypothenuse, and since that side cannot be greater than the hypothenuse, *sine i* cannot be greater than 1. What is the greatest value that *sine r* can have?

7. If we reverse the direction of the light and send it from water into air, is there any reason why the angle r should not be greater than the value determined as a maximum in problem 7? If we give the beam a greater inclination to the surface, so that the value of 1.33 *sine r* becomes greater than 1, what becomes of *sine i*? What does the light do? Can it escape from the water? This phenomenon is called *total reflection.*

SUGGESTIONS TO STUDENTS

1. Using the principle that the angle of incidence is equal to that of reflection, see if you can find out by graphical construction what will become of a beam of parallel light after it is reflected from a concave spherical mirror. If you have a spectacle, camera, or opera glass lens that has a concave surface, reflect the sunlight from it and see if your construction is correct. Try the same experiment with a silver spoon or a lamp reflector. Can you construct graphically in the same way the image of an arrow as formed by a concave mirror?

2. Make a pin hole camera and see if you can take a picture with it. Any light-tight box will do. To get fairly clear images, the edges of the pin hole must be smooth.

3. Reflect a sunbeam into a neighboring window with a plane mirror. When you turn the mirror through any given angle, through what angle is the reflected beam turned? See if you can devise a method of measuring it with a protractor and of making a graphical solution. The geometry of the right triangle will aid you here.

4. How are search lights constructed? What is the shape of the mirror?

5. Were you ever in a "crystal maze?" If so, explain your perplexities in a brief paper.

6. Why is a large mirror of advantage in decorating a small room? Would a living room be comfortable if its walls were nearly covered with metallically reflecting substances? Can you see a physical reason why matt surfaces are considered in better taste?

7. Make a diagram to prove that the lower half of a full length mirror is not necessary in order that a lady may see her entire figure in it. Verify the conclusion by experiment.

8. Find out why most books are printed on matt paper.

CHAPTER XX

OPTICAL INSTRUMENTS

360. Principal Focus. In the last chapter we learned that an image of a luminous point is formed by a lens at a particular distance from the lens. Is this distance always the same for a given lens, no matter where the luminous point is situated with reference to it? In order to answer this question, take a simple lens L of the shape shown in Fig. 208, and allow light from the sun to fall on it in the manner there shown. On holding a paper behind the lens, we easily find the point at which the image of the sun is distinctly formed. Now, the sun is so far away that the wave fronts of the waves that reach us are sensibly plane. Hence they may be represented by straight lines as AB, which are moving in a direction LF perpendicular to AB. Since AB is a straight line, all the perpendiculars to it, which indicate the directions of motion of the various parts of the wave, are parallel to one another and to LF. These lines are called rays. Such a series of parallel rays constitute a PARALLEL BEAM. In order to make the figure symmetrical, let us place the lens so that its central plane CD is also perpendicular to LF. The line LF, which passes through the center of the lens and is perpendicular to the plane CD, is called the OPTICAL AXIS of the lens.

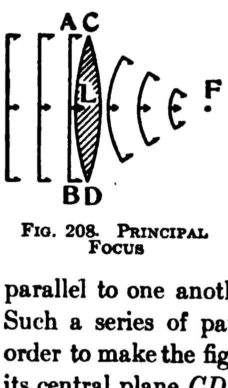


FIG. 208. PRINCIPAL FOCUS

Since the light waves in this case constitute a parallel beam, and since they are moving in the direction of the axis of the lens, it is evident from the symmetry of the figure that they will be brought together at a point on that axis such as F. Then F will be the focus. We note that the direction of motion of the central part of the wave has not been changed while the outside portions have been bent through $\angle ACF$. The point F, at

which parallel rays are brought together, is called the PRINCIPAL FOCUS. In the case under consideration, we may measure the distance LF, and this is the PRINCIPAL FOCAL LENGTH.

361. Image of a Point Source. If, with our lens, we form an image of some object that is not very distant, say of a luminous point S on the axis, Fig. 209, the image I of that point will lie farther away from the lens L than the principal focus F, because the incident waves from a near point are not plane but convex, and, since the thickness of the lens is the same as before, and the light passes through its center in the same direction as before, the same retardation will be produced in the center of the beam. But part of that retardation is now necessary to render the incident waves plane, and so less is left to make the plane waves concave; therefore, they come to a focus at some point I, farther away from the lens than the principal focus F.

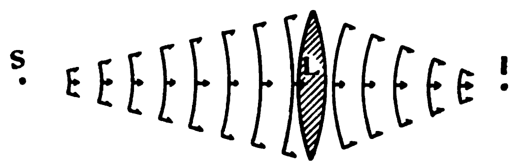


FIG. 209. IMAGE OF A POINT SOURCE

If we bring the point S still nearer to the lens, we find that its image is still farther away (Fig. 210), and when the distance of the point S from the center of the lens is equal to the principal focal length, the waves behind the lens become plane, and we have a parallel beam, Fig. 211. We see that this figure is the converse of Fig. 208. This reciprocal relation is general in optics. *If the source is placed where the image was, the image will be found where the source was.*

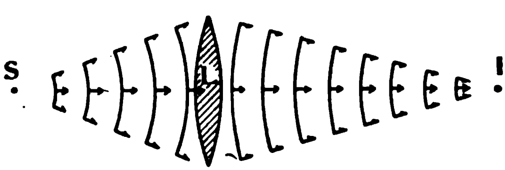


FIG. 210. AS THE SOURCE APPROACHES THE IMAGE RECEDES

362. Characteristic Rays. In discussing the formation of images of extended objects by lenses, it is simpler to consider

the rays only, and not the waves; so we shall use rays in the remainder of the explanation.

In the general discussion just given, we note that in the formation of an image two rays are particularly well defined, viz., the one that passes through the center of the lens, and the one that is parallel to the axis. Since the ray that passes through the center of the lens is not bent, its path is determined by the point source and the center of the lens. Since rays parallel to the axis, after passing the lens, go through the principal focus, this ray is determined by the distance of the ray from the axis, the direction of the axis, and the principal focus. These two rays enable us to find out many important things concerning the relations between objects and images formed by lenses.

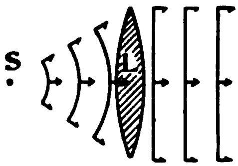


FIG. 211. SOURCE AT THE PRINCIPAL FOCUS

363. Construction of the Image. For example, suppose that we have an object OO', Fig. 212, at a distance ML in front of the lens whose principal focus is at F. Where will the image of the object be? Its position may be found as follows: From O draw a ray through the center of the lens L. This ray passes through the lens without being deflected, and the image of O must lie on this line at some point, as I. Similarly, a ray from O' through the center of the lens passes through the lens without being deflected, and the image of O' must lie on this line at some point, as I'. Thus it appears that the image must lie within the angle ILI'. We shall call this angle the LENS ANGLE of the image. The angle OLO' will be called the lens angle of the object, and we see that the two are equal. The lens angle will be found to be very useful in discussing optical instruments. It is defined as the angle subtended at the center of the lens by either the object or the image. It will

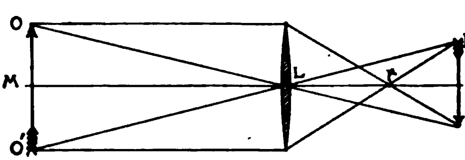


FIG. 212. CONSTRUCTION FOR THE IMAGE

be noted that *the lens angle depends only on the size of the object and its distance from the lens, and is independent of the size or shape of the lens used.*

In order to locate the points I and I' on the sides of the lens angle, from O draw a ray parallel to the axis MF. This ray must pass through the principal focus, and the image of O must lie on it. Hence the image of O must lie at the intersection I of the two rays FI and LI. When we have located the image I' of the point O' in the same way, it will be noted that the image is inverted, as was found to be the case in the last chapter (Art. 350). This simple construction is very useful and always gives us a close approximation to the position of the image; for both the center of the lens and the principal focal length can always be determined with a fair degree of accuracy, the former from the geometrical symmetry of the lens, and the latter by forming an image of the sun, as just described.

364. Size and Distance of the Image. What are the relative sizes of the objects and the image? To understand the answer to this question, we must first distinguish between ANGULAR and LINEAR size. If we mean the former, the size of the image is the same as the size of the object, i.e., *the angle subtended by the object at the center of a lens is the same as that subtended by the image at the same point.* This fact must be carefully noted, because it is fundamental in understanding the operation of optical instruments.

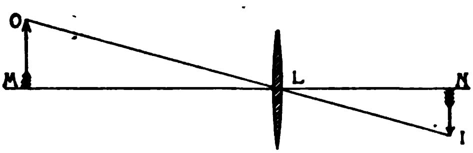


FIG. 213. RELATIVE SIZES OF OBJECT AND IMAGE

The relative linear sizes of the object and its image are different, however, for they are at different distances from the lens. In Fig. 213 the object OM and the image IN are both perpendicular to the axis MN of the lens and their lens angles are equal, i.e., $\angle OLM = \angle ILN$. Therefore the right triangles ILN and OLM are similar, and $\frac{OM}{IN} = \frac{ML}{LN}$; or, *the linear size of the object*

is to the linear size of the image as the distance of the object from the lens is to the distance of the image. These two distances are therefore of great importance in the discussion of relative linear sizes. They are called the CONJUGATE FOCAL LENGTHS. Similarly, the points M and N, so related that an object at either of them forms an image at the other, are called CONJUGATE FOCI.

We can now determine the relative linear sizes of object and image. We see that when $M\,L$ is greater than $L\,N$ the object is larger than the image; i.e., *when the object is farther from the lens than the image, the object is larger.* Conversely, *when the image is farther from the lens than the object, the image is larger, and when object and image are at equal distances from the lens they have the same size.* In this latter case the distance of the object or the image from the lens is twice the principal focal length of the lens. In this position the object and the image are as near together as they can be. Thus we see that as an object is brought nearer the lens, the image recedes from the lens and becomes bigger, as every photographer knows. This increase in the size of the image is due both to an increase in the lens angle as the object approaches, and to the increase in the distance of the image from the lens. Is there any limit to this process? Can we bring the object close to the lens and get an infinitely large image? Let us see.

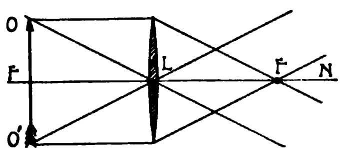


FIG. 214. THE OBJECT IS AT THE PRINCIPAL FOCUS

We have already remarked that when the distance of the object from the lens is equal to the principal focal length of the lens the emergent rays are parallel. Do such rays meet? Where, then, is the image formed? We may conceive that an infinitely large and infinitely distant image is formed when the distance of the object from the lens is equal to the principal focal length. This case is illustrated in Fig. 214.

365. A Virtual Image. Suppose we bring the object still nearer to the lens, where will the image lie? We may find out by

constructing a diagram, Fig. 215. Evidently the rays are divergent after leaving the lens. Where, then, is their point of intersection at which the image is formed? They do not intersect after leaving the lens, so that they are unable to form an image on a screen. They do, however, proceed as if they came from a point behind the lens. Yet if we place a screen at that point, we have no image formed on it, for the rays do not actually intersect there. Hence, such an image is said to be VIRTUAL. It will be noted that this virtual image is not inverted, as real images are, and that in this case, also, the lens angle of the image and that of the object are the same.

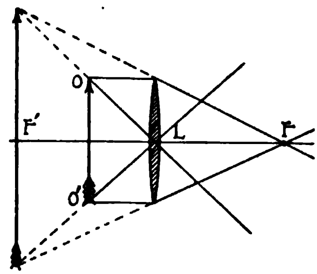


FIG. 215. THE IMAGE IS VIRTUAL

366. How the Eye is Focused. In Art. 361 we have learned that as the object is brought nearer the lens, the image is formed at a greater distance from it. In the eye, however, the distance between the lens and the retina, where the image is formed, is constant; yet we can see both distant and near objects clearly. Unless there were some means of focusing the image on the retina, it would not always appear sharply defined. The device employed is that of changing the thickness of the lens. For if (Fig. 209) the lens were made still thicker in the middle, the central portions of the beam would be retarded still more with respect to those at the rim, and the curvature of the waves would be changed more in passing through. This increased curvature would bring the waves to a focus nearer the lens.

In order to bring the images of objects that are near-by to a focus on the retina, the rim of the crystalline lens of the eye is surrounded with a small muscle which contracts, and squeezes the lens so that it bulges out and becomes thicker in the middle. Thus its focal length is shortened and the near-by objects brought to a focus on the retina instead of at a point behind it. The eye thus ACCOMMODATES itself to the different focal distances. This

accommodation of the eye is limited. In a normal eye the limit is reached when the object is about 25 cm (= 10 inches) from the eye. If we bring the object nearer than this, the image recedes behind the retina, and since there is no distinct image on the retina we can not see the object clearly. This distance—the least at which the object can be placed from the eye and still form a distinct image on the retina—is, therefore, called the LIMIT OF DISTINCT VISION.

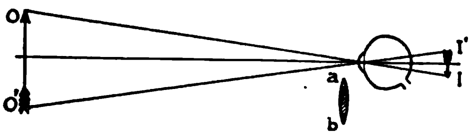


FIG. 216. FAR-SIGHTED EYE

367. Spectacles. If the crystalline lens in the eye is not normal, it does not form clear images of objects at all distances down to 25 cm. If it is too weak, i.e., too thin in the middle, the accommodating muscle must be used in forming the images even of distant objects, and will have to squeeze the lens harder in order to focus near-by objects on the retina. Therefore this muscle rarely gets any rest while its possessor is awake. This defect of the eye is called far-sightedness (Fig. 216), and is likely to cause serious results unless corrected. Far-sightedness is corrected by strengthening the crystalline lens and making it thicker in the middle. This necessitates introducing in front of the eye a lens which is thicker in the middle than at the rim. This lens is indicated by *ab* in Fig. 216. The strength of the added lens must be such that distant objects are seen clearly when the accommodating muscle is relaxed.

Conversely, an eye is near-sighted when the crystalline lens is too thick in the middle. Then the images of objects are distinct only when held very close to the eye, but those of distant objects are formed in front of the retina (Fig. 217). This defect is corrected by placing in front of the eye a lens that is thinner in the middle than at

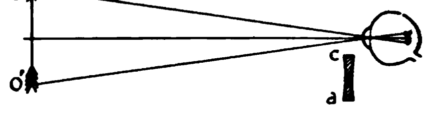


FIG. 217. NEAR-SIGHTED EYE

the rim, as indicated by *cd* in Fig. 217. In this case, also, the added lens should be of such curvature that distant objects are seen clearly when the accommodating muscle is relaxed.

368. The Simple Microscope. We are now prepared to understand the operation of the simple microscope. This consists of a single lens which is thicker in the middle than at the rim. It therefore does not differ in its action from the far-sighted spectacle lens. It merely enables us to focus clearly on an object which is much nearer the eye than the limit of distinct vision. Fig. 218 shows the object at the limit of distinct vision, and Fig. 219 shows the same object brought nearer the eye and focused on the retina with the help of the microscope lens. It is clear that in this case the lens angle *OLO′* is greater than before. Since our appreciation of the size of an object depends on the angle that it subtends at the eye, and since that angle depends only on the size of the object and its distance from the eye, it is evident that the microscope enlarges the apparent size of objects, because it enables us to see an object clearly when it is very near the eye, so that its lens angle there is large. This lens angle of the eye is generally called the VISUAL angle.

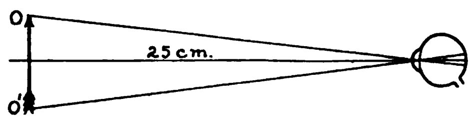


FIG. 218. LIMIT OF DISTINCT VISION

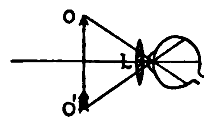


FIG. 219. SIMPLE MICROSCOPE

369. The Camera. Besides the eye and the simple microscope, probably the camera is the best known optical instrument. We have just discussed (Art. 350) the formation of an image by a small hole in a shutter, and shown how that image is made clearer by the introduction of a convex lens. Every one must recognize the image formation thus produced as identical in every respect with that of the camera. But a camera lens consists not of a single reading glass, but of several lenses mounted together in a tube. Nevertheless, in this case also the lens angles of the object and of

the image are the same, since the combination of lenses may be replaced by a single lens that would produce a similar effect. The discussion of all the reasons for thus making the camera lens of several parts, would lead us far beyond the limits of our present study. The use of the stops in such lenses, however, demands attention.

370. Stops. In the first place, every photographer knows that photographic lenses are supplied with stops which limit the amount of the lens used. The effect of a stop is twofold, viz.: 1, it reduces the amount of light admitted to the camera and so lengthens the necessary time of exposing the sensitive plate to the light that comes from the object through the lens; and 2, it makes the image on the plate sharper at the edges. The relation between the area of the opening in the stop and the time of exposure is simple; for the amount of light that enters the lens from a point on the object is proportional to that area, and therefore the intensity of the light at a point on the plate must vary as that area, other things remaining the same. Hence, with a given lens, a stop whose opening has half the diameter of the lens requires an exposure four times as long as that required for the lens without the stop, since the areas of the openings are proportional to the squares of the diameters. It will be noted that the introduction of the stop does not change the lens angle.

371. Spherical Aberration. The other effect of the stop now demands our attention, viz., that the clearness of the image around the edges of the plate is improved by reducing the size of the opening in the stop. The reason for this is rather complex and requires for its complete explanation a more exhaustive study study than can be undertaken here. Suffice it to say, the theory shows that lenses whose surfaces are portions of a sphere can not bring all the points of a plane image to a focus in a plane. Thus, if we have a plane object perpendicular to the axis at M, Fig. 220, the points of the object near the axis LN will be focused in a plane perpendicular to the axis at N. This plane is called the focal plane. But points of the object that are farther away

from the axis, will be focused either in front of or behind this focal plane.

In order to bring all the points of the object to a focus in one plane, the surfaces of the lenses would have to be portions of ellipsoids instead of spheres. But as ellipsoidal surfaces are almost impossible to grind and polish, their use is practically out of the question. However, the difference between a spherical and an elliptical surface is small if we consider only a small area of each. Therefore, when we reduce the area of the hole in the stop, we allow only the central portion of the lens to be used, and, therefore, we make its difference from the theoretically correct shape very small. So the image becomes clearer. This blurring of the image because of the spherical shape of the lens surfaces is called SPHERICAL ABERRATION. It has been found that the spherical aberration of a lens may be somewhat reduced by using several lenses instead of one, so that this is one reason why photographic lenses are made of several parts. Another reason will be discussed in the next chapter while studying color.

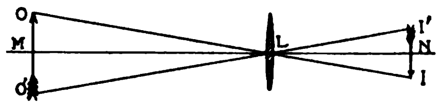


FIG. 220. THE IMAGE DOES NOT LIE IN A PLANE

372. The Astronomical Telescope. The next instrument to which we shall direct our attention is the astronomical telescope. This consists of at least two lenses of the type of the simple micro-

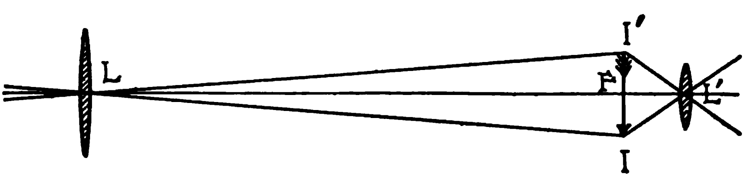


FIG. 221. THE ASTRONOMICAL TELESCOPE

scope. Inasmuch as the telescope is usually used for observing distant objects, we shall assume that the beam of light from every point of the object is a parallel beam. Thus, in Fig. 221, let the line $L\,I$ represent the path of the incident beam coming from the

tip of a distant arrow and passing through the lens L, which is called the OBJECTIVE. If F is the principal focal length of L, the image of this point will then be at I in the principal focal plane. Similarly, the image of a point at the other end of the arrow will be at I'. The lens angle of the object is then equal to that of the image. i.e., it is the angle ILI'. If we introduce a second lens behind the image in such a way that its principal focus is also at F, this second lens will render parallel the light from each point of II', so that the beam from each point on the object is a parallel beam when it leaves this second lens as well as when is strikes the objective.

The image II' is now the object for the combination consisting of the second lens and the eye, and its lens angle with respect to this combination is $IL'I'$. Since the image II' is nearer to the eye combination than to the objective, its lens angle at that combination is larger than its lens angle at the objective, i.e., $IL'I' > ILI'$; and, therefore, the object appears enlarged. It will be noted that the image is inverted. Since the second lens is near the eye, it is called the EYEPIECE. The combination of an eyepiece or a simple microscope with the eye will be called the eye combination.

The reason why the telescope makes things appear larger is now apparent. The visual angle of the object, when viewed without the telescope, is small, because the object is usually at a great distance. This angle is very nearly the same as the lens angle of the object at the objective of the telescope. But the objective forms an image close to the eye combination. Since this image is close to the eye combination and at a much greater distance from the objective, its lens angle in that combination is larger than the visual angle of the object. The magnification may be defined as $\dfrac{\text{lens angle of the eye combination}}{\text{lens angle of the objective}} = \dfrac{IL'I'}{ILI'} = \dfrac{IL'F}{ILF}$.
In Art. 6 we learned that an angle may be measured by its tangent, and that the tangent of $IL'F = \dfrac{IF}{L'F}$. Similarly, tangent $ILF = \dfrac{IF}{LF}$.
Therefore the magnification is $\dfrac{\text{tangent } IL'F}{\text{tangent } ILF} = \dfrac{LF}{L'F}$.

PLATE VIII. THE 40-INCH YERKES TELESCOPE

But LF is the principal focal length of the objective, and $L'F$ is that of the eyepiece; so we see that *the magnification of the telescope is determined by the ratio of the focal length of the objective to that of the eyepiece.* Therefore, if we wish to make telescopes that shall have large magnifying powers, we must so construct the objective that it will have a great principal focal length, while the eyepiece must be made to have a small focal length.

Plate VIII is a photograph of the Yerkes telescope, which is the largest in the world. It has a focal length of about 25 m; therefore, with an eyepiece which had a focal length of 0.5 cm, its magnification would be 5000. Such a high magnification can seldom be used, on account of the unsteadiness of the atmosphere. Since the more the image is enlarged the fainter becomes the light in each cm^2 of it, it follows that when large magnifications are used large lenses are necessary, in order to gather as much light as possible into the image. The objective of this telescope is 100 cm in diameter.

373. The Concave Lens. If we examine a common opera glass, we find that the eyepiece is thinner in the middle than at the rim. Such a lens is called CONCAVE; those that are thicker in the middle being CONVEX. We have noted (Art. 367) the action of the concave lens in aiding near-sighted persons to see more clearly. To gain a better understanding of the action of concave lenses, consider Fig. 222. Light waves from the point S spread out and fall on the concave lens L. Since the lens is thicker at the rim than in the center, the waves that pass through the rim are more retarded than those passing through the center. The result is that the divergent beams from S are rendered more divergent, so that the waves behind the lens will appear to come from some point as I. Thus, I is the image of S, and it is virtual. When the point S is far away, so that the incident beam is parallel, the principal focus will be near I and will also be virtual. Con-

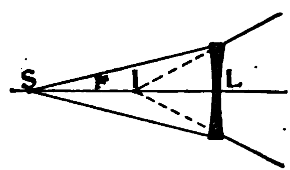


FIG. 222. THE CONCAVE LENS SCATTERS THE LIGHT

versely, if we have a convergent beam that would otherwise come to a focus at the principal focus F of a concave lens, the interposition of this lens will make the beam parallel (Fig. 223). If we place an arrow at S and construct the image as described in Art. 363, we find that the lens angle of the object is equal to that of the image, i.e., $OLO' = ILI'$, Fig. 213.

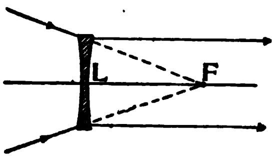


Fig. 223. Parallel Beam Formed by Concave Lens

374. The Opera Glass. Let us now construct a diagram to represent the visual angles as they occur in the opera glass. The objective alone would form an inverted image II' of the distant object at its principal focus F. The lens angle of the object is, then, equal to $I'LI$. The concave lens is introduced in front of the image, with its principal focus also at F. Then, since the rays which fall on L' from any point of the object are converging toward a point in the plane of its principal focus, they will be rendered parallel by the eyepiece (*cf.* Art. 373). The direction of each such parallel beam will be that of the corresponding point of the image II' from the center of the lens L'. Hence, the lens angle

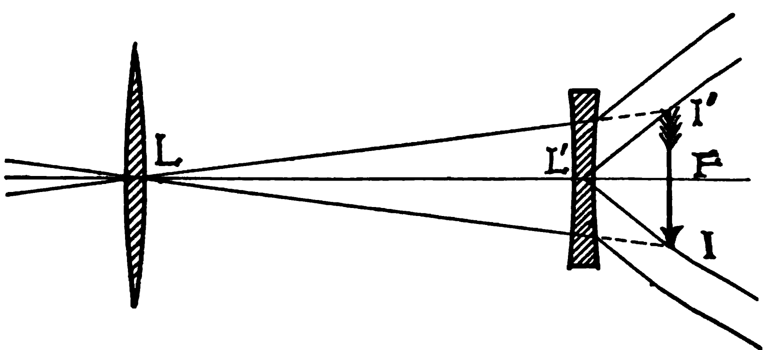


Fig. 224. The Opera Glass

of the image after passing the lens L' will be $IL'I$; and therefore, as in the case of the astronomical telescope, the magnification is the ratio of the focal lengths of the objective and the eyepiece. For viewing ordinary objects this instrument has an advantage

over the telescope in that the image formed by it is upright. The opera glass is often called the Galilean telescope, since it is the kind invented by Galileo and with which he discovered the satellites of Jupiter.

375. The Compound Microscope. This instrument differs from the telescope only in the fact that the object is placed in front of the objective near its principal focus, so that its lens angle may be made as large as possible. A real image is formed by the objective near the eye combination, so that the lens angle of this real image with respect to that combination is also large. Fig. 225 shows the arrangement, ILI' being the lens angle of the

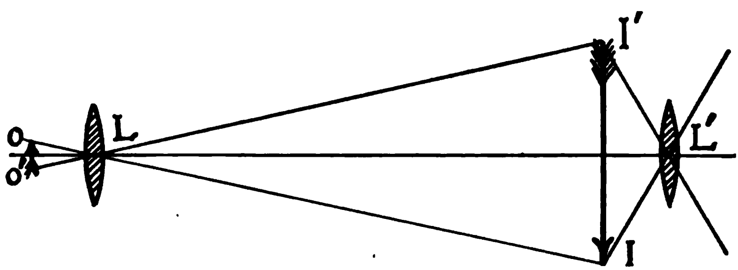


FIG. 225. THE COMPOUND MICROSCOPE

object, and $IL'I'$ the lens angle of the image with respect to the eye combination. By using an objective of very short focal length, real images may be obtained when the object is brought very close to the objective. But, provided it remains outside of the principal focus, the nearer the object is brought to the objective, the larger is the lens angle of the object, and also the larger is the linear size of the image. Consequently, by this arrangement very large magnifications may be obtained, the only limit being the intensity of the light; for as the image gets larger it becomes fainter. When we reach a magnification of about 2500 we have reached the practical limit for eye work. Photographs can, of course, be made with still larger magnifications, but the exposures must be very long.

One interesting point may yet be mentioned. Although we can magnify an object without limit, a point is eventually reached

beyond which greater magnification fails to reveal further detail in the object. What is that limit? Though we can not here give the reasons for the conclusion, we may, nevertheless, state it. When the distance between two points of an object is less than 1-100,000 of an inch, we are not able, by any known optical device, to distinguish whether there are two points or only one. Thus, when an object has been magnified until points one-hundred-thousandth of an inch apart are separated, further magnification will not reveal any further details of the construction of the object. This point is of interest, because the ultimate particles of matter—atoms and molecules—are much closer together than this in solids, and therefore we know that we can never see them with our eyes, though we may be able to know them in other ways.

376. The Photometer. In the discussion of the astronomical telescope and of the microscope, we have found that the intensity of illumination of the image is a matter of importance. In practical life it is a matter of even greater importance, since all artificial lighting by gas, or electricity, is measured and rated according to its intensity. The unit in which intensities are measured is the CANDLE-POWER. This is the rate at which light is radiated from a candle of specified construction burning a specified amount of sperm per minute. The ordinary electric glow lamps are generally equivalent to 16 standard candles, and are therefore called 16 C.P. (candle-power) lamps.

Intensities are compared by means of a photometer. The two lights to be compared, e.g., a lamp and a standard candle, are set about 2 or 3 m apart, and a piece of paper with a grease spot on it is supported between them. This paper is moved backward or forward until the spot can no longer be seen. When this is the case, the *illumination* of the two sides of the paper is the same. The *intensities* of the two lights must be directly proportional to the squares of their distances from the paper. If the distance of the standard candle from the paper is 20 cm and of the lamp 80 cm, or 4 times that of the candle, then the intensity of the lamp is 4^2 times that of the candle, or 16 C. P. This may be proved by considering that the light is spreading out in all directions

from each source, so that the energy that spreads over 1 cm^2 on a surface 1 m from the light is spread over 4 cm^2 on a surface twice as far from the light. This form of photometer is called the Bunsen photometer, after its inventor, and as we have just seen, it is based on the principle that *the amount of light which falls from a given source on each* cm^2 *of a surface is inversely proportional to the square of the distance of the source from the surface.* This is a geometrical consequence of the straight paths of the rays; but it is strictly true only when the distance is large compared with the size of the source, so that the light may be regarded as diverging from a point.

SUMMARY

1. When parallel rays are brought to a focus by a convex lens, the distance from the lens to the focus is called the principal focal length.

2. As the object is brought nearer the lens, the image recedes from it. When the image is real, it is inverted.

3. When an image is formed by a convex lens, the ratio of the linear size of the object to that of the image is equal to the ratio of the distance of the object, from the lens, to that of the image, from the lens.

4. When the distance of the object from a convex lens is less than the principal focal length, the image is erect and virtual.

5. When an object and its real image have the same size, the distance of each from the lens is equal to twice the principal focal length.

6. No real image can be formed by a convex lens when the object and the screen are nearer together than four times the principal focal length of the lens.

7. The eye is focused by changing the thickness of the crystalline lens.

8. A normal eye can not form on the retina clear images of objects that are nearer to the eye than 25 cm. This distance is, therefore, called the limit of distinct vision.

9. An eye is far-sighted when the accommodating muscle must be used to see distant objects clearly. It is then unable to

bring the images of near objects to a focus on the retina without straining the accommodating muscle.

10. An eye is near-sighted when its crystalline lens is too thick in the middle. It cannot bring the images of distant objects to a focus on the retina.

11. When an image is formed by a lens, the lens angle of the object is the same as that of the image.

12. The simple microscope enables the eye to focus clearly on the retina the images of objects that are less distant from the the limit of distinct vision. The object appears enlarged, because it then subtends a larger visual angle.

13. In the telescope and the opera glass, the light from each point of the object is parallel when it enters the objective, and also parallel when it leaves the eyepiece. The magnification is due to the fact that the real image formed by the objective is nearer to the eye combination than it is to the objective, so that the lens angle of this image with respect to that combination is larger. The telescope and the opera glass therefore give the same effect, enlarging the visual angle of the object.

14. A concave lens causes the light rays to diverge, and forms only virtual images.

15. The practical unit of light is that furnished by a standard candle.

16. The intensity of the light that falls on 1 cm^2 of a surface is inversely proportional to the square of the distance of the surface from the source of light.

PROBLEMS

1. If you wish to copy a photograph full size with a lens of 15 cm principal focal length, how great must be the distance between the picture and the photographic plate? What will be the relation between the distances from the lens to the object and image respectively? If you wish to enlarge the picture to twice its size, what will be the relation between those distances? If you wish to get the number of cm in this case, you must know the relation between the two conjugate focal lengths and the principal focal length. If u represent the distance of the object from the lens, v that of the image, and f the

principal focal length, the relation among the three is found to be $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. Verify this equation in the case of object and image the same size ($u = v = 2f$).

2. Draw a diagram of a convex lens with a principal focal length of 4 cm. Draw an object 6 cm from the lens and construct the image by the method of Art. 363. What is the relative size of the image? Measure the distance from the lens to the image and see if the construction verifies approximately the equation in problem 1.

3. A 16 c. p. incandescent lamp and an arc lamp give the same illumination on a screen when the distance from the screen of the incandescent lamp is 10 cm and that of the arc lamp is 100 cm; what is the candle-power of the arc lamp?

4. If the arc lamp in problem 3 takes 9 amperes at 55 volts, while the incandescent one takes 0.5 amperes at 110 volts, at what rate in watts is energy supplied to each? Which lamp is the more efficient? What is the ratio of their efficiencies?

5. The amount of light that passes through a camera lens is proportional to the area of the opening of the lens, i.e., to r^2, if r is the radius of the opening. If we have two lenses with different sized openings of radii, r_1 and r_2, and if i_1 and i_2 represent the intensities of illumination of the light on a cm_2 of the plate at a fixed focal distance f, write the proportion which expresses the relation between the intensities and the radii of the openings

6. If we have two camera lenses of the same area of opening but of different focal lengths f_1 and f_2, the same amount of light will pass through each under like conditions, but since the focal length of one is greater than that of the other, the intensity of illumination of the light on one cm^2 of the plate will be inversely as the squares of the focal lengths, i.e., $i_1 : i_2 = f_2^2 : f_1^2$. Does this explain why a long focus lens is "slower" for taking pictures than a shorter focus one of the same aperture?

7. Multiply together the two equations of problems 5 and 6 and extract the square root. The result is $i : i_2 = \frac{r_1}{f_1} : \frac{r_2}{f_2}$. What angle is measured by $\frac{r_1}{f_1}$? Twice this angle, or that subtended by the rim of the lens at a point on the plate, is called the ANGLE OF APERTURE. Since the "speed" of a photographic lens depends on the intensity of light on the plate, may we use this ratio, or its double, the ratio of the diameter of the lens to the focal length, as a measure of speed? What is the meaning of the marks $\frac{f}{8}$, etc., on the stops in some camera lenses?

SUGGESTIONS TO STUDENTS

1. If you have a camera, measure the diameter of the stop marked $\frac{f}{8}$ and then measure the focal length of the lens—preferably by forming an image the same size as the object—and dividing the distance between object and image by 4. Do you find any relation between the things measured and the numbering on the stop?

2. Take your opera glass lenses out, and measure the focal lengths of the objective and eyepiece. This may be done by letting sunlight pass through each and measuring the distance at which the sun's image is formed by the objective. The eyepiece spreads the light, but its focal length may be found by measuring the diameter of the lens, and the diameter of the spot of light formed by it on a screen at a measured distance. What is the magnification? Determine it by looking with one eye at a brick wall and with the other at the same brick wall through one tube of the opera glass; you then see two images of the bricks, one without the glass the other with it; the magnification is the number of bricks in the first image which cover one in the second. You can also determine the magnification of telescopes in this same way. Would a plainly marked scale of equal parts be better than the brick wall?

3. Examine the projecting lantern in the schoolroom and see if you understand the operation of its "condenser." Is its projecting lens different from a camera lens? Measure the distances from slide to lens and lens to screen, and see if the diameter of the slide is to that of the image on the screen as the respective distances are to each other.

4. There are many useful books on photography. You will also find Wright, *Optical Projection* (Longmans, New York), valuable. There are some interesting experiments in optics in Hopkins's *Experimental Science*. The best book on light for young students is Professor S. P. Thompson's *Light, Visible and Invisible* (Macmillan, New York). This is a series of the Christmas Lectures at the Royal Institution in London. It gives a very clear and fascinating account of the latest experiments and theories. See also Tyndall's *Six Lectures on Light*, which gives an account of the work of Newton and Young. Tyndall's style is a model in clearness and precision, and the charm of the man is reflected from every page.

CHAPTER XXI

COLOR

377. Newton's Experiment. Having studied in the last two chapters the way in which light serves us by enabling us to distinguish differences in direction, we shall now pass on to the discussion of color phenomena. Although color has been observed and used extensively from time immemorial, little was known concerning the reasons why different substances appear to have different colors, until the time of Newton. In 1675, Newton discovered that a beam of sunlight when admitted through a small hole H in a shutter into a dark room and then sent through an ordinary glass prism P, was not only bent from its path, but was also spread out into a band of various colors, extending from red to violet (Fig. 226). He further found that if a second prism, in all

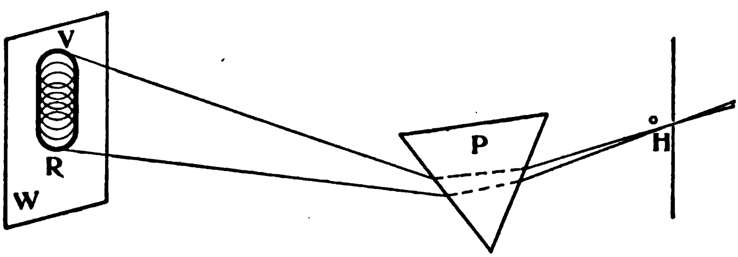


FIG. 226. NEWTON'S EXPERIMENT

respects like the first, was introduced behind the first in such a way that the two together made a thick plate of glass with parallel sides, the colored band was reduced to a colorless spot. From these experiments Newton concluded that white sunlight is composed of all the colors of the rainbow, and this conclusion has been verified by further investigation.

Now, this experiment shows that white light is a mixture of all different colors in certain proportions, but it does not tell us

wherein the colors differ from one another. What is the physical difference between lights of different colors; for example, between red light and blue light? To this question Newton gave no satisfactory answer, because he did not conceive light to be a wave motion. But when we adopt the theory that light consists of waves, we are able to form a clear conception of the physical nature of differences in color, viz., that the different colored lights correspond to waves of different lengths, just as sounds of different pitch correspond to waves of different lengths. How can we prove that this is so; i.e., how can we measure the lengths of the waves of light, in order to find out if they are different for different colors?

378. Interference Fringes. This may be done in a number of ways, but probably the simplest is the following: Carefully clean two pieces of the best plate glass, and clamp them together so that they touch along one edge and are held apart by a fine hair or fiber at the other (Fig. 227). We thus have formed between the two plates a wedge of air. If now we allow light of one color, like that from a flame colored with salt, to fall perpendicularly on these plates, we do not see the familiar image of the flame, but, instead, there appears a series of bright bands of equal width, separated by dark spaces, which are also of equal width. These bands are called interference fringes, and they plainly show that there is something periodic about the light, just as the phenomenon of beats makes evident the periodic nature of sound.

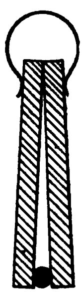


FIG. 227. AIR WEDGE

No very satisfactory reason can be given for the appearance of the black bands unless we conceive that the light consists of waves. We have learned in the chapters on wave motion and sound that two waves may add themselves together so as to produce no motion when their phases are opposite. Similarly, in the case of the two plates of glass, we may suppose that waves of equal length, but in opposite phases, are thus adding themselves together so as to destroy each other's effects, therefore darkness

results. Whence do we get the two waves? By reflection from the inner surfaces of the two plates. Thus, let *PQ* and *PR*, Fig. 228, represent these two surfaces. When the light falls on the surface *PQ*, part of it is reflected and part passes through. When that which has passed through falls on the second surface *PR*, part of it is reflected and thus we have two reflected beams, one *a d* from the first surface, and the other *b e* from the second. Now, it is clear that the light reflected at *b* will, when it reaches *c*, be somewhat different in phase from that reflected at *a*, because the light at *c* will be some wave lengths behind that reflected at *a*, since it has traveled a distance *abc* more than the light at *a*. Therefore, if this extra distance is half a wave, the light at *c* will be half a wave behind that at *a*, and so the two beams will come together in opposite phases, cancel each other's effect, and together produce darkness (*cf.* Art. 299). Similarly, when the extra distance *abc* is a whole wave, the two rays *ad* and *ce* will be in the same phase, and so when they are added together, they produce light. Therefore, at that place we see a bright band crossing the glass. When the distance *abc* is three half waves, darkness again results, and so on. Therefore the successive bright bands occur at places where the successive extra distances *abc* traveled by the second beam differ by a whole wave, and that the dark bands occur when these distances differ by half a wave.

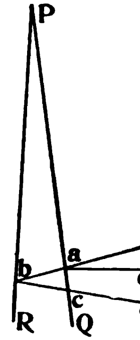


FIG. 228. AIR WEDGE DIAGRAM

Thus, in Fig. 229 the bright bands are found at the points marked *b*, and the dark ones at the points marked *d*. It is clear from the figure, that between two bright bands the distance from plate *A B* to plate *C D* increases by one-half of a wave. This distance can be measured by determining the diameter of the fiber, its distance from the edges of

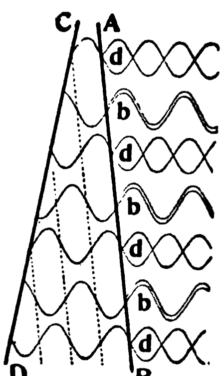


FIG. 229. INTERFERENCE OF WAVES IN THE AIR WEDGE

the plate, and the distance between the dark and bright bands. For example, if the diameter of the fiber is 0.01 mm, and if it is 60 mm from the place where the plates touch, and if the distance between the bands in red light is 2 mm, we may see 30 bands on the plate. Since in this case 30 bands correspond to a change in distance between the plates of 0.01 mm, one band corresponds to a change of $\frac{0.01}{30} = .00033$ mm. But this change in distance corresponds to half a wave length of red light; therefore $2 \times .00033 = .00066$ mm = one wave length of red light. There are several other ways of determining the lengths of light waves, all of which are vastly more accurate than this. They are all based on the principle of interference, and the values of the wave lengths obtained by the different methods agree closely with one another.

379. Lengths of the Waves of the Colored Lights. If we now illuminate this same pair of plates with green light, the dark bands will appear narrower and nearer together—there will be about 40 on the plate. Therefore the distance between the plates increases $\frac{.01}{40}$ for each band, i.e., .00025 mm. Therefore one wave length in green $= .00025 \times 2 = .00050$ mm. Proceeding in a similar manner for the other colors, we find the lengths of the corresponding waves to be approximately:

red,	.00066 mm
orange,	.00060 mm
yellow,	.00055 mm
green,	.00050 mm
blue,	.00045 mm
violet,	.00040 mm

We thus find that different colors really do have different wave lengths. It is interesting to note how extremely minute the waves are. Thus, there are about 2500 blue waves or 2000 green ones, or 1500 red ones in one millimeter or 38,000 red waves in an inch.

380. Interference Fringes in White Light. What will happen if we allow white light, instead of light of one color, to fall on the

two glass plates? We find a most beautiful array of many-colored bands. The colors are not so marked, however, as those in the spectrum. These bands are easily produced with an ordinary soap solution, such as is used for blowing bubbles. A drinking glass is dipped in the soap suds and then set on its side, so that the soap film over its end is vertical. As the water drains out from between the two sides of the film, a thin wedge of soap solution is formed, and the colored bands will be seen stretching horizontally across the film. The hypothesis that light consists of waves enables us to give a simple explanation of the formation of these colored bands. For if all the colors are present in white light, and if the different colors correspond to waves of different lengths, then each color will have a set of bands corresponding to it, and these bands will be of different widths. When these different sets of colored bands are all present, they overlap, so as to give us color mixtures. These mixtures of the different colors produce the various tints or tones observed in the two glass plates or the soap film.

Thus one of the mysteries of our early childhood is solved; for the colors often seen in a crystal or a piece of ice which has a crack in it, are formed in this way. So, also, are the colors in a soap film, or those seen on the surface of oily water. It is by reflection from the two surfaces of the bubble, or the crack, that these wonderfully colored interference bands are produced. The iridescence of polished shells, and of certain kinds of glass, may be accounted for in a somewhat similar manner by the interference of waves reflected under suitable conditions from their surfaces.

381. Dispersion. Having thus learned that different colors correspond to different wave lengths and that white light is a composition of all the colors, let us return for a moment to Newton's experiment with the prism, and ask how the prism is able to separate the colors and spread them out into a band. Clearly, the prism must be acting differently on the different colored lights. If we repeat Newton's experiment, and interpose a red glass in the beam of sunlight, we find that the path of the red light is bent a certain amount by the prism, and we get on the screen a

red spot only. On changing the red glass for a blue one, we observe a spot on the screen, but the blue spot does not fall on the screen at the same place as was occupied by the red one. On examination we find that the prism changes the direction of the blue light more than it does that of the red. Thus the prism separates the white light into a series of spots of color and the entire band of color is thus seen to consist of a series of spots of color, each overlapping the adjacent ones.

The bending of the light is due to the refraction of the glass of the prism, and the amount of that refraction is measured by the index of refraction, which is the ratio of the velocity of light in the glass and in the air (*cf.* Art. 354). Hence, since the index of refraction of blue is found to be different from that of red, we conclude that the velocity of blue light in the glass differs from that of the red. Since the blue is bent more than the red its index is greater, and therefore its velocity in glass must be less than that of the red. Thus, we conclude that the separation of white light into colors by the glass prism is due to the fact that the waves of the different colored lights travel with different velocities in glass. This phenomenon of the separation of light into colors by a prism is called DISPERSION. It is of interest to find out whether all transparent substances separate light into colors to the same extent that glass does, i.e., how this separation depends on the substance of the prism.

382. The Spectrum. It would not lead to accurate results if we tried to measure dispersion with the apparatus as used by Newton, because each color in the white light forms a spot of color at a definite place, and these spots overlap to form the band of colored light. Since each spot is larger than the hole (*cf.* Art. 350), the band is fuzzy and not clearly defined. Hence, we must so improve the apparatus that we may measure the position of a given spot with some degree of accuracy. As you may have surmised, this is done by introducing a convex lens L, Fig. 230, in such a way that it forms on the screeen images of the hole in the shutter. Another improvement is to replace the round hole in the shutter by a narrow slit parallel to the edge of the prism, because then the

successive colored images of the hole do not overlap as much as do the round images, as shown in the figure. When we make these changes we find that the appearance of the band of color,

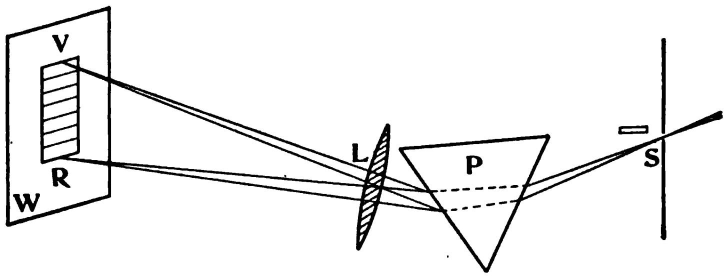


FIG. 230. THE SPECTRUM IS CLEAR AND BRILLIANT

the SPECTRUM, as it is called, is much improved. The edges are now clearly defined and the colors in the center are both brighter and more distinct.

383. Bright Line Spectra. However, one other question needs settlement before we can compare dispersions, viz.: What colors shall we compare; for the spectrum appears to contain all sorts of reds, greens, etc.? We must, then, select some particular ones for comparison. Which shall they be? If, with the new arrangement of the apparatus, we interpose a red glass, we find that even now the spectrum of the transmitted light is not sufficiently well defined to admit of definite measurement; for the red spot, though rectangular, is broad and contains many different shades of red, i.e., is not monochromatic. Nature has furnished us with a means of producing suitable monochromatic colors, for we find that if we burn a metal, sodium, for example, the light is nearly monochromatic. Thus, if we send the sodium light through our apparatus—a SPECTROSCOPE, let us call it—we note that the spectrum consists mainly of a clearly defined bright yellow image of the slit through which the light passed before entering the prism. Since this image is a clearly defined yellow line, we may measure its position with great accuracy. In a similar way, if we burn mercury, we find that the corresponding spectrum consists of a

bright yellow line, a bright green line, and a violet line. Zinc, in the same way, gives a spectrum consisting of a red, a green, and two blue lines. Other metals give still other lines.

Now, the great advantage of these lines is that they are very clearly defined, and, so far as we have been able to detect, they always have the same colors; i.e., the wave length corresponding to a given line is always the same in air. Therefore they furnish convenient points of reference by which to measure dispersion.

384. Measurement of Dispersion. We are now in a position to compare the powers of two different prisms. Let us take, for example, one of water and one of glass, both of the same size. We first place the water prism in the spectroscope, and using mercury light, for example, mark on the screen the positions of the yellow, the green, and the violet lines. We then do the same with the glass prism, and find that it not only bends each of the rays more than the water does, but also that the distance between the successive lines is greater. Thus, the glass not only gives the rays a greater deviation, but also the dispersion of the rays is greater.

Another complication now arises, for if we turn the prism about a vertical axis, we find that both deviation and dispersion vary; i.e., both depend on the angle of incidence of the light on the prism. It will be found, however, that there is one position of the prism for which the deviation is smaller than for any other. Hence we may compare prisms when in this position, which is called that of MINIMUM DEVIATION. On setting the prism in this position, it will be found that the light enters and leaves it at the same angle with the front and rear faces respectively. We may, then, compare the powers of prisms when they are in this particular position.

385. Achromatic Lenses. The importance of this discussion of deviation and dispersion becomes manifest when we apply the principles we have learned to lenses. For it must be clear that if the different colors travel with different velocities in glass, then when white light passes through a lens, some of the colored beams

will be refracted more than others by the lens, so that the different colors will not all come to the same focus. Thus, if the object is a star on the axis of a lens, the blue waves, being much more retarded by the lens than the red, will come to a focus nearer the lens than do the red waves. Hence, when we wish to observe the image of a star, we find that there is, strictly speaking, no image, for at one point there is a red image surrounded by fuzzy blue light and at another a blue image surrounded by fuzzy red light, and so on for other colors. This phenomenon is called CHROMATIC ABERRATION. It thus becomes clear that unless we can find means of correcting this aberration, lenses for fine work are useless.

Since chromatic aberration is a result of dispersion, it is clear that the solution of the difficulty must be sought in a study of that phenomenon. We may then ask, Do prisms of the same size, but made of different substances, always differ in the deviations which they produce, and also in their dispersion? Or may we have two prisms that have the same index of refraction for some one color, and yet have different dispersions? The answer to this question is, of course, only obtained from careful investigation of numerous cases. The result of such investigation is that substances have been found which produce the same deviation, and yet have different dispersions.

In order to understand how this fact may be used to produce colorless images, we must first remember that deviation alone is what we need for this purpose. Therefore we must find two prisms of such nature that they produce deviation without color. This is done in a manner similar to Newton's method of recombining white light by putting two prisms together with their angles in opposite directions (*cf.* Art. 377). But in Newton's experiment the two prisms were of the same size and of the same substance. Therefore, by introducing the second prism, he canceled not only the dispersion of the first, but also its deviation. But suppose the second prism is of some other substance, and of such an angle that it possesses the same dispersive power as the first but less deviating power. Then the dispersions of the two will cancel each other and leave a colorless beam, while some of the deviation produced by the first prism will remain.

This method is the one actually used in constructing lenses. If you take out a telescope or opera glass objective, you will find that it consists of two lenses, one convex, made of crown glass, and the other concave, made of flint glass, as shown in Fig. 231. These two lenses are so shaped that their powers of dispersion are equal and that they act in opposite directions, while the deviating power of the convex one is greater. Therefore, in the combination the chromatic aberration is corrected, but the lens is still able to deviate the rays, and so it forms an image that is clear and free from the fuzzy halo of color.

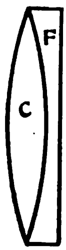


FIG. 231. ACHROMATIC LENS

We owe the solution of this problem to John Dollond, an English optician, who produecd the first achromatic objectives in the year 1757. Newton had been unable to find suitable glasses for the constructon of such a lens, and had therefore thought that it was impossible. By the discovery of new sorts of glass modern investigation has shown that it is possible to produce other types of achromatic lenses than the one just described.

386. Spectrum Analysis. Having thus seen how the phenomenon of dispersion nearly prevented us from constructing lenses that were useful for fine work, let us ask if dispersion is truly useful in any way. To this we must answer, yes, indeed; and bring forward the following explanation in justification of the answer. We have noted, in what has just preceded, that the light from burning sodium, mercury, or zinc, when analyzed in the spectroscope, is found to consist of certain well defined colors. Each of these colors is pretty nearly pure, i.e., it consists of waves not differing much from one another in length. Further, the colors emitted by each substance are found to be different, so that each element, when burned, produces in the spectroscope a series of bright lines that are characteristic of it. Therefore the spectrum furnishes us with a means of determining the nature of a substance. For if we burn a substance and analyze with a spectroscope the light emitted, we can recognize the lines and so tell what the substance burned is. Thus the dispersion of prisms

furnishes us with a powerful and accurate method of chemical analysis.

In this use of the spectrum we are not confined to the study of substances of the earth only, for we receive light from the sun, the stars, and other heavenly bodies. Because their light can be analyzed in this way, we are able to discover the composition of these bodies. So the dispersion of prisms enables us to extend our chemical analysis to the farthest visible regions of the universe. The results of such study are most interesting, for we find that many heavenly bodies produce spectra consisting of bright lines, while others produce spectra that are of the same nature as that of our sun. So we are able to find out that some of the heavenly bodies are composed of burning metals, while others are more like our sun.

387. Continuous Spectra. What is the difference between a spectrum consisting of bright lines, and one similar to that of the sun, which seems to contain all possible colors? We can find out by experiment; for if we heat a metal, say a piece of zinc, in a flame, it grows gradually hotter. As we have learned in Art. 153, it sends out long heat waves at all temperatures, and as it grows hotter, shorter waves are added to the complex mass of waves, until, when the temperature reaches about 520° C., it begins to send out red light waves. If its light is then passed through the spectroscope, the spectrum will be found to contain mainly red waves. But as the temperature increases, the shorter waves appear, until the entire spectrum is produced on the screen. The zinc is then said to be white hot. But it has not yet melted. On examining the spectrum thus formed, it will be found to be CONTINUOUS, i.e., to contain all the wave lengths that correspond to the different visible colors.

But if the zinc is heated still further until it catches fire and burns, i.e., forms a luminous vapor, the spectrum will suddenly change to one consisting of the bright lines characteristic of the zinc. Thus it appears that incandescent solids produce continuous spectra, while incandescent vapors are the source of the bright line spectra. Applying this fact to the spectra of the

heavenly bodies, we see that if those spectra consist of bright lines, we are justified in concluding that the heavenly bodies that produced them consist of incandescent vapors; while if the spectrum appears continuous, the body must be more like an incandescent solid.

388. Dark Lines in the Spectrum. Is our sun a vapor or a solid? On examining the solar spectrum carefully, it will be found to be neither continuous nor yet to resemble a bright line spectrum; for though it appears continuous when the dispersion used is small, a higher dispersion reveals the fact that it is crossed by a large number of black lines. What do these black lines mean? Evidently, that certain wave lengths are not present in the solar light when it reaches us. A comparison of the position of these dark lines with those emitted by burning metals shows that the position of the two sets in the spectrum are the same. Thus, for example, there are found in the solar spectrum black lines at the same positions that are occupied by the bright lines of burning sodium. Similarly, for the other metals. What, then, can be the nature of the sun that it sends us all other vibrations excepting those that the substances we know on earth emit? Is the sun made of substances that are totally different from those of the earth?

The explanation of the presence of the dark lines in the solar spectrum was first given by Kirchhoff in 1859, and rests on the principle of resonance explained in Art. 314. We there learned that bodies that emit powerfully waves of a certain period are set into violent vibration when waves of the same period are impressed on them. We have found a marked example of this in sound (*cf.* Art. 342), for there we discovered that a body may be set into violent vibration by resonance, when its natural period of vibration agrees with the impressed period. Further, when a body is thus set into vibration by resonance, it absorbs the radiant energy that falls on it, begins vibrating, and thus becomes itself a new source of waves which spread out in all directions about it. Thus, the energy of the waves which are traveling in the direction of the resounding body is scattered by that body in all directions, and therefore less of it is passed on in the original direction.

Now, Kirchhoff reasoned in a similar manner concerning light waves. For he said, "Since the sodium particles have natural periods of vibration which correspond to the bright lines produced by their radiations in the spectrum, they must be able to vibrate by resonance whenever they are acted on by waves whose period agrees with their own. Further, when they are thus set into vibration by resonance they must absorb the energy of the incident waves, and scatter it in wave motion in all directions. Therefore the black lines in the solar spectrum, which indicate the absence of the waves that agree in period with those emitted by sodium, show that there are, between us and the source of the solar light, particles of sodium which absorb the sodium vibrations from the sunlight, and scatter them in all directions."

389. The Sun's Atmosphere. Where can such particles be? They do not exist in the earth's atmosphere. They do not exist free in the cold regions of space, since they must be in the form of a hot vapor in order to execute their free vibrations. They must, therefore, be in the solar atmosphere. So we reach the conclusion that sodium vapor is a constituent of the sun's atmosphere. Reasoning in like manner, we conclude that the other elements, whose bright lines coincide with dark lines in the solar spectrum, must exist as vapors in the solar atmosphere. Careful study of the bright lines of elements and of the dark lines of the solar spectrum shows that almost every known bright line has a corresponding dark line there; and, therefore, we conclude that the atmosphere of the sun is composed, not of substances different from those in the earth, but of the same materials. The lines in the spectra of the stars also agree in position with those of substances known here. So we conclude that all bodies in the visible universe are composed largely of the same substances as are found on the earth.

390. Complex Colors. In Art. 379 we found that colors correspond to the lengths of the light waves, and that simple colors are those that are produced by waves of a definite length. We have learned that incandescent solids send out white light

consisting of all imaginable visible colors, while an incandescent vapor sends out certain simple colors characteristic of it. We have seen how particles of definite natural periods of vibration absorb and scatter the energy of waves whose periods of vibration agree with their own. We are now ready to enter on the investigation of the reasons for the appearance of color in the ordinary objects about us. Nothing is more familiar to us than that different objects, illuminated by the same sunlight, appear to have different colors. Why do the leaves appear green, the violets blue, the goldenrod yellow, etc.?

The complete answer to this question is at present unknown. However, in the light of the principles just learned, and with the aid of the spectroscope, we can give a partial answer. If we pass sunlight through the spectroscope, and then interpose various pieces of colored glass in the path of the light, we note that the resulting spectra are all different. Thus, when red glass is introduced, the yellow, green, and blue of the spectrum are absorbed, while the red and some orange come through. When green glass is introduced, the red and the blue disappear, while the green and some yellow come through. It will, however, be noted that this sort of absorption differs from that which produces the dark lines of the solar spectrum. For in the case of absorption by metallic vapors, very definite colors are absorbed—what we have termed simple colors, which correspond to one particular wave length. But the colored glass absorbs a vast number of wave lengths and lets another vast number pass through. So we see that though the color of the glass, red, for instance, is due to the absorption of part of the waves in the sunlight, yet a large number of different waves are transmitted, so that the light which is thus sifted by the glass is still very far from simple.

391. Colors of Ordinary Objects. We thus see that the colors of ordinary objects are complex in the same way that sounds are, in that both consist of a large number of different waves. There is, however, this distinction, that the various notes in a complex musical sound are related by simple numerical relations, while among the waves that produce ordinary colors, no such

relations exist. Consequently we have no color scale, i.e., we have no standard universally accepted as to harmony and discord of colors. Many attempts have been made by metaphysicians to construct a color scale which should correspond to the musical scale; and Newton's statement that the spectrum consists of the seven colors, violet, indigo, blue, green, yellow, orange, red, has frequently been used for this purpose. We are now able to see that such attempts are entirely artificial, since there is no simple numerical relation between the component parts of a complex color, and so our judgment of harmony and discord of colors has no such physical basis as has our recognition of tonal relationship in music.

392. The Eye. We can, perhaps, make this clearer if we compare the mechanism by which the eye detects colors with that by which the ear detects sounds. Experiments in the reproduction of colored pictures have shown that most of the colors recognizable by the eye can be produced by combinations of red, green, and blue light. This fact has led to the theory of Young and Helmholtz, that there are on the retina of the eye three sets of nerves, one sensitive to red, one to green, and one to blue. These three nerves must be of such a nature that exact coincidence between their natural period and the impressed period is not necessary in order to make them respond to the action of the waves. Our perception of the colors, then, probably depends on the relative intensity of the excitation of these three sets of nerves. The ear has a large number of nerve fibers, each tuned to a different note, therefore it resolves complex tones into their components and hears the components separately. The eye, on the other hand, has three different sets of fibers, of which a large number are tuned to red, another large number to green, and still another to violet blue. Therefore red, green and violet lights, acting together stimulate all three sets of nerves and give the sensation of white. Blue and yellow together do the same, while green and violet lights together give peacock blue, a tint between green and violet, red and violet give purple, red and green give yellow, and so on.

393. Paints and Dyes. The action of paints and dyes is similar to that of colored glass in one way, but different in another. It is different in that the light we receive from painted or dyed surfaces is reflected, not transmitted. It is similar, in that the light that comes to us after reflection is what remains of the incident light after some of its colors have been absorbed. Thus, red glass absorbs most of the blue, green, and yellow from white light, and transmits only the red and some of the orange. In like manner, red paint absorbs most of the other colored lights, and returns only the red and orange by reflection.

That this is the action of paints and dyes is shown by the fact that the colors of substances appear so different when viewed by gas light and in the daylight. For daylight contains all the different colors in large amounts, while gas light lacks much of the blue and violet. Since the pigments can send back only certain waves from among those which they receive, the blues and violets appear dull in gas light; and in a red light they appear nearly or entirely black, because they absorb all the reds, and there are in the red light no waves that they are able to reflect.

394. Mixing Lights and Mixing Pigments. Why is it that if we mix two colored lights, say blue and yellow, we see a white mixture on a white screen; but if we mix blue and yellow paints or dyes, and color the screen with the mixture, the result is green? The answer is easily obtained by considering carefully the action of the painted and the unpainted screen on the light. The screen is white in daylight, because it receives and reflects a mixture of all colored lights. If we send two beams of colored light, one yellow and the other blue, to the same point on the screen, both beams are reflected and make their impressions at the same place on the retina of the eye. The result is that all three sets of nerves at that place in the eye, the red, the green, the blue, are sufficiently excited to give us the impression of white.

The sensation arising from the light reflected by a mixture of blue and yellow paint is green, because the blue pigment absorbs all the incident light except the green and the blues, which it reflects; while the yellow paint absorbs all but the orange,

yellow, and green, which it sends back. Hence, when the two pigments are mixed, the mixture returns only those colors that are not absorbed by either the yellow or the blue, i.e., it reflects the greens only.

395. Complementary Colors. If we look intently at a brightly colored blue spot for several minutes, and then turn our gaze to a white wall, there appears to be on the wall a yellow spot of the same size and shape as the blue one. In like manner, if the spot is pink, that which appears to be on the wall will be pale green. But we have just seen that a mixture of yellow and blue light produces white. Hence these colors are said to be complementary. This phenomenon of complementary colors may be accounted for with the help of the Young-Helmholtz theory of color vision. For when we look at a brightly colored spot, the nerves sensitive to that color become tired, so that when we look at a white surface, which is sending out all colors, the "blue" nerves do not respond, and the sensation corresponds to white with the blue left out, i.e., it is that of the complementary color, yellow

SUMMARY

1. White light is a mixture of a vast number of waves of different lengths.
2. Difference in color corresponds to difference in wave length, the red waves being longer than the blue.
3. The wave lengths may be measured by means of the interference fringes.
4. The velocities of different colored lights in transparent substances, like water and glass, are different.
5. The separation of composite light into its colors is called dispersion.
6. Dispersion makes the formation of clear images by a single lens impossible on account of chromatic aberration.
7. Chromatic aberration may be corrected by properly combining two lenses of equal and opposite dispersive powers, but of unequal deviating powers.

8. Every incandescent vapor sends out waves of definite lengths which correspond to a few simple colors that are characteristic of it.

9. Substances may be analyzed by the spectroscope. This analysis may be extended to include the sun and stars.

10. Dark lines in the spectrum are due to the absorption of definite waves by metallic vapors.

11. The atmospheres of the sun and other celestial bodies consist of the simple substances known on the earth.

12. The colors of ordinary objects are very complex. They absorb large numbers of waves of certain lengths and reflect waves of certain other lengths, which give them their characteristic tints.

13. There is no simple numerical relation between the vibration numbers of the components of a complex color, and so we have no color scale and no physical basis for judging concerning harmony and discord of color.

14. The eye seems to have three sets of nerve fibers, sensitive respectively to red, green, and violet blue.

15. The ear analyzes a complex tone and hears the elements separately. The eye receives one sensation as the resultant of the action of a complex color.

16. Two colors that produce the sensation of white when they are mixed together are said to be complementary.

17. The results of mixing colored lights are quite different from those of mixing colored pigments.

QUESTIONS

1. How may we prove that white light consists of a large number of colors?

2. How may we prove that different colors correspond to waves of different lengths?

3. What is meant by dispersion? How does a prism deviate a beam of light? How does it separate it into colors?

4. What is chromatic aberration? How is it corrected in lenses?

5. How is dispersion used for chemical analysis? What improvements do we have to make in Newton's apparatus in order to produce a clear spectrum?

6. Are the colors emitted by incandescent vapors simple?

7. What is the difference between the spectrum of an incandescent solid and that of a vapor? How does this difference depend on the temperature?

8. How does resonance enable us to explain the dark lines in the solar spectrum?

9. How is the sun's atmosphere analyzed?

10. How is the complexity of ordinary colors different from that of a musical tone?

11. Why do we have no color scale to correspond to the musical scale?

12. How does the eye detect differences in colors? Has it, like the ear, a separate nerve for each separate number of vibrations?

PROBLEMS

1. When salt is burned in alcohol, the light emitted by the flame is the characteristic yellow light of sodium. Why do people appear ghastly in this light? If a yellow gas flame, which has a similar effect, is surrounded with a red glass globe, does it make the people appear more natural? Why?

2. If a piece of blue paper is illuminated with red light, what color does the paper appear to have? What color does it appear to have when illuminated with yellow light obtained by passing daylight through yellow glass or a solution of bichromate of potash? If the yellow light were that of sodium, what would be the result?

3. When you look through a prism at a window or a broad band of white paper, you see a broad patch of white with red, orange and yellow on one edge, green, blue and violet on the other; but if you look at a narrow slit through which white light is coming or at a narrow band of white paper, you see a continuous spectrum. Explain the cause of the difference.

4. Why is it that the complementary of any one of the spectrum colors is a complex tint and not a pure color?

5. Why do interference fringes in white light give complex tints instead of pure spectrum colors?

SUGGESTIONS TO STUDENTS

1. Make a soap solution such as you use for blowing soap bubbles, and form a soap suds film over the open end of an ordinary drinking glass with straight sides, not tapering. Set the glass on its side so that the film stands vertically. See if interference bands appear as in the air wedge mentioned in Art. 378. If the bands do not appear, the solution is too strong, if the film breaks, the solution is too weak. Does a black spot appear at the top of the film? Can you

find out how thick the film is at the bottom? Remember that the thickness of the film increases half a wave length for every black band. If you view the film in red light, and the wave length of the red light in air is 0.000065 cm, how long is it in water, i.e., in the soap solution if the index of refraction is 1.33?

2. Can you find out how the rainbow is formed? How does the light pass into and out of each drop of water so as to be separated into its colors? Why is the bow always curved? Has the rainbow an end? Why do we sometimes see two bows?

3. Consult a book on botany and see what you can find out about chlorophyll and its relation to the green color of leaves. What rays does it absorb, and which does it reflect?

4. You can buy a small prism from an optician for about 30 cents, and with it make a number of interesting and instructive experiments. See Twiss' *Laboratory Exercises in Physics* (Macmillan, N. Y.) Exercise 43. This will suggest others; e.g., examine with the prism, as there directed, narrow strips of the Milton Bradley colored papers, which may be obtained at your bookstore, and the lights from red and green fire, which you can buy at your druggist's.

5. The Milton Bradley color top, which can be bought postage prepaid for 6 cents, is an endless source of amusement and instruction. By all means get one and use it as directed in the Bradley color book, Hopkins' *Experimental Science*, or Mayer and Barnard's *Light*.

You will find the following books of interest in connection with the subjects discussed in this chapter: O. N. Rood, *Modern Chromatics* (Appleton, N. Y.); N. Lockyer, *Spectrum Analysis* (Appleton, N.Y.); Vanderpoel, *Color Problems* (Longmans, N. Y.); *Elementary Color* (Milton Bradley Co., Springfield, Mass.); Mayer and Barnard, *Light* (Appleton, N. Y.). This is full of beautiful home experiments. Thompson's *Light, Visible and Invisible*, Lommel's *Nature of Light* (Appleton, N. Y.), and Professor D. P. Todd's *New Astronomy* (Am. Book Co., Cincinnati), also contain much that is interesting and not difficult to read.

CHAPTER XXII

VELOCITY OF LIGHT

396. The Medium. In the preceding chapters we have assumed that light is a wave motion. We have learned how, by ordinarily traveling in straight lines, it enables us to judge the directions of the objects. We have then discovered some of the consequences of considering that differences in color correspond to differences in wave length. We now pass to the problem of determining more exactly the nature of the light waves. If light is a wave motion, in what medium does it travel, for we have learned that a medium is necessary for the propagation of waves? It is not air, because light comes from the filament of an incandescent lamp even though the air is entirely pumped out of the bulb, as was noted in Chapter VIII. Also, light comes from the sun and stars to us, although we are certain that our atmosphere does not extend that far. But how shall we find out more about the medium in which light waves are propagated? How determine its properties? One possible way is by determining the velocity of light, as we determined the properties of air from a study of the velocity of sound in it (*cf.* Art. 313). But how shall we measure the velocity of light? It is well known that its velocity is very great; for, as was noted when discussing the velocity of sound, light appears to travel short distances instantaneously.

397. Galileo's Method of Measuring the Velocity of Light. The first to propose a method of measuring the velocity of light was Galileo. He suggested stationing two observers on distant hills and supplying each with a lantern fitted with a shutter that could be closed and opened quickly. Then observer 1 opens his lantern; when observer 2 perceives the light, he opens his. Then observer 1 has but to note the time that elapses between opening his lantern and seeing the flash of the other lantern,

and this time should be that taken by the light to pass from observer 1 to observer 2 and back. So reasoned Galileo. The experiment was tried, but without satisfactory results, for it was found that the time was too short to measure accurately. Further, the method is inaccurate, because there is introduced the time taken by observer 2 in becoming conscious of the appearance of the light from observer 1 and opening his lantern. This time is greater than that taken by the light in traveling the entire distance. The experiment is correct in principle, but the application of the principle can be improved.

398. Fizeau's Method. The first improvement consists in replacing observer 2 by a mirror. Since the action of the mirror is automatic, this change eliminates the inaccuracy due to the operations of observer 2. But even then it is found that observer 1 sees the light before he can get his lantern fully open, i.e., he is too slow for the light. Therefore we have to replace the shutter on his lantern by a mechanical device that will open and shut the lantern more quickly. This was done by Fizeau in 1817. The principle of his experiment is illustrated in Fig 232. Light from the source S passes through the hole h in the box and thence to the distant mirror M. It then retraces its path and is partly reflected at the glass plate G to an observer at E. On one end of the box is a toothed wheel W, which is so placed that when it revolves the light is cut off whenever a tooth t of the wheel is in the path of the light, and allowed to pass whenever a notch n is in that position. As the wheel rotates, the light is alternately cut off and let through. By rotating the wheel rapidly we are able to make these openings and closings of the lantern follow one another as rapidly as desired.

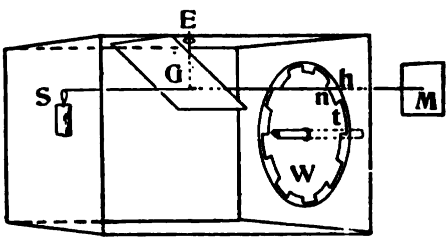


Fig. 232. Principle of Fizeau's Experiment

But it is still necessary to be able to measure the time accurately. This may be done by the notched wheel itself, for if we arrange matters so that the observer sees through the notches of the wheel the light reflected from the distant mirror, the returning light will be cut off if the time taken by it in traveling from the wheel to the distant mirror and back is the same as that taken by the wheel in turning, so that a tooth takes the place previously occupied by a notch. Now, it is easy to measure the angular velocity of the wheel, and also to count the number of notches in its rim; we may, therefore, determine accurately the time taken for a tooth to replace a notch. Twice the distance of the mirror divided by this time will then be the velocity of light.

The experiment has been made many times in this and in other ways, and the result is that light has been found to travel at the rate of $186{,}000 \ \frac{\text{miles}}{\text{sec}} = 3 \times 10^{10} \ \frac{\text{cm}}{\text{sec}}$. Since the circumference of the earth is about 25,000 miles, it appears that light is able to traverse in one second a distance equal to about 7 times the circumference of the earth.

Though this seems a surprisingly great velocity, it is not so tremendous when we consider the distances from the earth to the sun and the stars. Thus, it has been found that it takes about 8 minutes for light, traveling at this rate, to come from the sun to the earth; and the distance of the nearest star is so great that it takes light over three years to travel from it to the earth. Further, the most distant stars that we can see are so far away that it takes light some 5,000 years to come to us from them, and probably there are stars still farther away. These figures show us that the ratio of the circumference of the earth to the distance of a faint star is that of $\frac{1}{7}$ sec to 5,000 years, or $\frac{1}{1104000000000}$. Thus the knowledge of the velocity of light helps us to form some idea of the immensity of space and of the relative size and importance of the earth in comparison with the universe about it.

399. The Ether and Its Properties. But what of a medium that can propagate waves at the rate of $3 \times 10^{10} \ \frac{\text{cm}}{\text{sec}}$? Can we apply equation (15) Art. 296, which gives the relation between velocity, elasticity, and density? If we can, it is evident that the

elasticity must be enormously great and the density exceedingly small in order that the square root of the quotient may be so large a number. But how can we measure the elasticity of this medium? In the case of air the elasticity is easily measured by compressing the air and measuring its change in volume. Can we apply pressure to the medium in which light travels? Can we measure its density? These quantities are evidently so small that we can not measure them by mechanical means; for when we have pumped the air out of an incandescent lamp bulb as far as is possible, our gauges indicate no measurable pressure, and we have not been able to weigh or measure the medium in any mechanical way. But though we are not able to determine the numerical value of these factors for the medium, we can give it a name. So we call it the ETHER. We shall have occasion to learn more of the properties of the ether as we proceed. At present we can only conclude that whatever properties it may have, it does not react in any measurable way to the ordinary mechanical forces, such as pressure, torsion, and the like.

Since we thus learn that we can not measure the properties of ether as we can those of air by mechanical means, we are compelled to seek elsewhere for some method of finding out what its nature is. Are there not other phenomena which involve the ether, and by means of which we may study its character? We have already noted, in the study of electricity, that electricity and magnetism do not act by means of air, but through some other medium (Art. 286). This fact has led scientists to investigate carefully whether those properties of the ether which may be discovered by means of experiments in electricity and magnetism can in any way assist us in framing a theory concerning the properties of light waves.

400. Electric Waves. When a Leyden jar is discharged, the electric discharge vibrates very rapidly back and forth a number of times between the terminals (*cf.* Art. 197). Therefore we have in the electric spark a vibrating something which may send out waves. Can we detect at a distant point the discharge of a Leyden jar in any other way than by the sound

and light of the spark? If we place a second jar, in every way like the first, in close proximity to the first, we notice that a spark passes between its terminals at S, Fig. 233, whenever one passes between the terminals of the first jar. We must then conclude that the second jar has electric oscillations generated in it by resonance. But resonance implies waves; and so we conclude that the spark, probably, is a source of electric waves. These waves, as is now well known, can also be detected in other ways; in fact, they are the waves with which wireless telegraphy operates, for wireless messages are sent by discharging electric sparks. So we learn that electric sparks generate electric waves which travel to distant points and may there be detected.

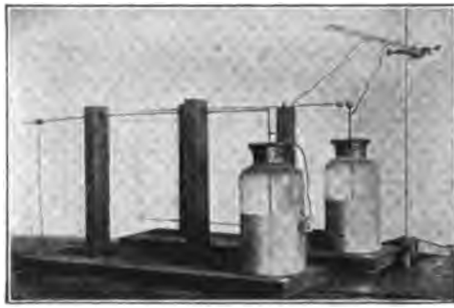


FIG. 233.. ELECTRICAL RESONANCE

If electricity acts by means of the ether, these electric waves must be ether waves. Therefore it is of interest to find out how fast they travel, because this knowledge will enable us to compare them with light waves. We can measure their velocity by means of the equation $v = n\, l$ (Art. 296), since the number of vibrations can be calculated from the dimensions of the jar used in sending the sparks; and the wave length can be measured by causing the impulses to form stationary waves on long wires, and measuring the distance between the nodes (Art. 301). The first experiments of this nature were performed by Hertz in 1888, and his result ushered in a new epoch in physical science, for he found that these electric waves travel with the same velocity as light does.

The conclusion which we draw from this remarkable result is that the ethers of light and of electricity are the same; and, further, that light is an electric vibration, not an elastic one. Since the announcement of this admirable theory, many other

facts have been discovered that all add weight to the argument, and make us more content with the simplicity and the elegance of the conclusion.

401. Wireless Telegraphy. When these electric waves are used in transmitting messages without wires, they are started by the spark from an ordinary induction coil. The receiving apparatus (Fig. 234) is very interesting. It consists of a small glass tube *C*, to which are fitted two metal plugs *p p*. The ends of these plugs are about 1 mm apart, and the space between them

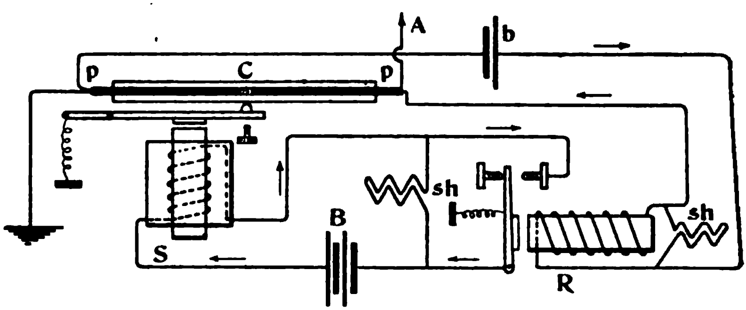


FIG. 234. DIAGRAM OF RECEIVING STATION FOR WIRELESS MESSAGES

is filled with loose nickel or silver filings. A current from a small battery *b* passes through this tube and the relay *R*, and back to the battery. When the circuit is thus completed, very little current flows, because the resistance of the loose filings is large, and therefore the relay armature is not moved. But when electric waves fall on the filings, their resistance is in some way diminished. The current through the filings and the relay is thus increased, and the relay armature is pulled toward the magnet. This action of the filings is called COHERENCE, and the tube *C* is called a COHERER. When coherence takes place, the relay closes a local circuit in which a sounder *S* is operated by a battery *B*, as in the ordinary telegraph system.

In order to break the circuit through the coherer, it must be tapped so that the filings are shaken up. It is, therefore, usually placed so that the armature of the sounder strikes the coherer

when it flies back. The sensitiveness of the apparatus may be much increased by connecting one end of the coherer to earth and the other to a long wire stretched vertically on a high pole. Such a vertical wire is called an ANTENNA. In like manner, the power of the sending coil may be increased by attaching one of its terminals to an antenna and the other to earth. A Leyden jar across the terminals of the coil often helps. The points at which the coherer and the sounder circuits are broken must be shunted with coils of fine wire of high resistance, for if a spark occurs when a circuit is broken, it sends out waves that affect the sounder and confuse the signals.

402. The Complete Spectrum. We are now in a position to enlarge our ideas of the spectrum. Since the electric waves travel with the velocity of light, and since their numbers of vibrations vary from about 1000 to 6×10^{10} per second (*cf.* Art. 197), their wave lengths vary from 3×10^7 to 0.5 cm. In Art. 397 we learned that a long red wave has a wave length of 0.000066 cm and a short blue one a length of 0.00004 cm; we now find that the electrical vibrations send out waves of greater length than those corresponding to red light. We have also noticed that as a body is heated, it sends out first longer heat waves, then shorter heat waves, and finally, at a temperature of about 520° C., it begins to emit red light waves. Putting these three facts together, we may conclude that the entire spectrum is much longer than appears to the eye, for it must contain heat and electrical waves beyond the red. This portion of the spectrum is therefore called the ULTRA-RED, or heat spectrum. Investigation shows that this spectrum really exists; for if we place in the solar spectrum beyond the red an instrument capable of detecting heat—for example, a thermometer —we shall find that it indicates heat action there.

Further, if we take a photograph of the visible spectrum, we find that the photograph indicates the presence of waves beyond the violet end; i.e., we can photograph more than we can see. So we conclude that there are electric waves shorter than those corresponding to violet light, and that these waves can act on a photographic plate. This extension of the spectrum is called

the ULTRA-VIOLET, or photo-chemical spectrum. The shortest of these photo-chemical waves that has been measured, has a wave length of .00001 cm.

We thus see that the entire spectrum contains a large number of waves in addition to those that produce the sensation of sight, for it contains waves varying in length from 3×10^7 cm to those of length .00001 cm. Of these waves, those lying between the limits 3×10^7 and .5 cm are called electric waves, and are able to act on electrical apparatus; those between the limits .008 and .00008 manifest heat action, while those between the limits .00008 and .00004 affect the eye and are called light. Those that are shorter than .00004 cm may be detected by their action on a photographic plate. It is possible that waves exist beyond these limits, but they have not been detected by any of the devices at present known to science. It is the present belief of scientists that all these waves are of the same nature, i.e., that they are all electric waves.

403. What Spectra Tell Us. The detailed study of spectra is of great importance in physics, not only because we can thereby analyze chemical compounds, but also because it opens a possible way of discovering the nature of the vibrating particles which are the sources of light waves. The principles on which this study is based have been discussed in what has preceded, but it may be well to bring them together here for the sake of showing their relations. Thus, we have seen (Art. 340) how a vibrating string sends out waves whose lengths are related by the simple ratios, 1, $\frac{1}{2}$, $\frac{1}{3}$, etc. Conversely, when we receive such a complex sound wave, and on analysis find that its component vibrations are related by these simple ratios, we are justified in concluding that the source of the wave is either a vibrating string, a rod, or some other body which is capable of sending out that particular series of overtones. Now, all bodies that send out such a series of overtones are long and thin, i.e., they resemble a straight line in geometrical form. So when we have analyzed a complex wave and found it to consist of a series of simple waves, whose lengths are related by the ratios, 1, $\frac{1}{2}$, $\frac{1}{3}$, etc., we conclude

that the geometrical form of the vibrating source of the waves is a straight line.

If, however, the complex wave is found, on analysis, to contain the overtones characteristic of a bell, we are justified in concluding that the vibrating body is shaped like a bell; and, similarly, if the component vibrations are those characteristic of a solid ring, or of a sphere, or of a body of any other particular geometrical form, we infer that the source of the waves has that particular form. A similar conclusion may be drawn concerning the geometrical form of a vibrating source of light; for we have learned how each incandescent vapor sends out a certain series of simple vibrations characteristic of it. We can sort out these simple vibrations by the spectroscope, determine their vibration numbers, and so find out whether any numerical relation exists between the vibration numbers that correspond to the bright lines in the spectrum of any element. Careful experiments have proved that such numerical relations do exist among the vibration numbers that correspond to the bright lines of many of the chemical elements. What, then, is the geometrical form of the body that can vibrate in such a way as to send out the corresponding vibrations? What is the geometrical form of the vibrating body, i.e., of the atom?

To this question science has not yet found an answer; for though we know the relations among the vibration numbers of the series of waves sent out by the atoms of many substances, that relation is very complex, so that we have not yet been able to show what geometrical form is capable of vibrating in such a way as to send out that series of vibrations. The problem, however, is not hopeless; and during the last few years considerable progress has been made towards its solution. All that can be said at present is, that this study of spectra offers a possible way of enabling us to form some conception of the shape and construction of atoms.

The importance and wonders of spectra thus become clear; for we see that from them we can not only discover the make-up of stars at almost infinite distances from us, but we can also study the mechanism of the tiniest thing that the human mind has been

able to conceive. We see that we are surrounded by phenomena of marvelous complexity, yet governed by simple principles—how these phenomena and the principles that govern them extend to both the infinitely great and the infinitely small, without any conceivable limit. For who can set an outside boundary to the universe, or who can measure the size of the smallest particle which plays its part and has its own use in the economy of Nature? Yet throughout this vast complexity of relations and interrelations, among infinitely small atoms and the infinitely great universe, we can discern the operation of comparatively simple principles, which are manifest in the least, as well as in the greatest, of the operations of the universe.

SUMMARY

1. The velocity of light is $3 \times 10^{10} \frac{\text{cm}}{\text{sec}}$.
2. The elastic constants of the ether can not be measured by mechanical means.
3. The electric spark generates electric waves varying in length from 3×10^7 to 0.5 cm.
4. These long electric waves travel with the velocity of light.
5. Heat and light waves are of the same nature as electric waves, and the ether is the medium which transmits them all.
6. The complete spectrum contains electric, heat, light and photo-chemical waves.
7. The spectrum enables us to study the geometrical form of atoms as well as the constitution of the heavenly bodies.

QUESTIONS

1. How is it possible to measure the velocity of light?
2. How long does it take light to travel a distance equal to the circumference of the earth? How long to come from the sun? From a star?
3. What medium transmits light? Can we measure its elasticity by mechanical means?
4. What other phenomena besides light depend on the ether for their manifestations?
5. How are electric waves generated? How detected? Describe a wireless telegraph system and explain its operation.

6. What is the velocity of propagation of electric waves? How is their velocity measured?

7. Why do we conclude that the waves of heat and light are electric in their nature?

8. What different kinds of waves make up the complete spectrum?

9. Can we study the geometry of an atom by the spectrum? How?

10. What are the limits of the domain within which the study of the spectrum is valuable?

PROBLEMS

1. A certain Leyden jar is discharged and the oscillations of the spark are found to have a period of $\frac{1}{100000}$ sec; what is the length of the electric wave started by the spark? If the wave length of sodium light is 0.000059 cm, how many oscillations does a sodium particle execute per second?

2. Since incandescent bodies are radiating long heat waves as well as light waves, much of the energy supplied to them is wasted, as far as light making is concerned. For example, the unit of light is the candle, and it has been found that a unit candle radiates light energy at the rate of about $19 \times 10^5 \frac{\text{ergs}}{\text{sec}}$. At what rate does a 16 candle incandescent lamp radiate light energy? Such a lamp requires for its maintenance to be supplied with energy at the rate of about 55 watts; what is its efficiency as a light producer?

3. In Art. 170 the heat of combustion of illuminating gas was given as 18×10^4 gm cal per cubic foot. An ordinary gas flame consumes $5 \frac{\text{ft}}{\text{hour}}$ and radiates with an intensity of about 16 candles. What is the efficiency of the flame as a light producer?

4. An arc lamp radiates with an intensity equal to that of 2000 candles and requires an expenditure of about 500 watts, what is its efficiency?

5. Is it more economical to burn illuminating gas in a gas engine, and use the engine to run a dynamo, and let the dynamo feed an arc lamp, than it is to burn the gas directly for its light?

6. A cm^2 of the earth's surface, perpendicular to the path of the sun's rays, receives light energy from the sun at the rate of $7 \times 10^5 \frac{\text{ergs}}{\text{sec}}$. A cm^2 distant 1 m from a standard candle, and perpendicular to the rays, receives light energy from the candle at the rate of $15 \frac{\text{ergs}}{\text{sec}}$. How many candles at the distance of 1 m would be necessary to give the same intensity of illumination per cm^2 as is given by the sun? If the distance to the sun is 15×10^{10} m, how many candles at the distance of the sun would be required to illuminate the earth with the same intensity as the sun does?

SUGGESTIONS TO STUDENTS

1. If there is a wireless telegraph station near you, visit it and find out how it works. Ask the operator if they are able to tune the receiving instrument so that it will respond only to messages intended for it.

2. What can you find out about the efficiency of a fire-fly? How do they produce so much light with so little expenditure of energy? May the phosphorescence of decaying wood be a similar phenomenon?

3. You will find interesting reading on the topics of this chapter in Oliver J. Lodge, *Signalling Through Space Without Wires* (Van Nostrand, N. Y.); R. T. Glazebrook, *J. Clerke Maxwell and Modern Physics;* Lodge, *Modern Views of Electricity* (Macmillan, N. Y.); and Thompson's *Light, Visible and Invisible.*

4. It is not a very difficult task, with the help of Professor Lodge's book, to make a coherer and connect it as in Fig. 234, so as to receive wireless signals from the sparks of a small induction coil or electrostatic machine. Many boys have made their own outfits, including the induction coil, and operated them successfully over considerable distances.

CHAPTER XXIII

ELECTRONS

404. Origin of Light Waves. In this final chapter we shall endeavor to give an outline of the argument on which our present theory concerning the nature of the ultimate particles of matter is based. What has preceded has made us familiar with the main facts on which this argument is founded.

The first point in the theory is the hypothesis that light consists of ether waves, and from this we must infer that the source of those waves is a vibrating something. Further, since these waves are so minute, the vibrating something must be very small, so as to be able to vibrate very rapidly—some 6×10^{14} times a second. We are thus led to conceive that the sources of light waves are minute vibrating particles.

405. Electrons. Since the waves of light are waves in ether, the vibrating something must be of such a nature that it is able, by its vibrations, to disturb the ether, and so become a source of waves. Therefore, in order to conceive how these minute vibrating particles can disturb the ether, we are led to the further hypothesis, that each of them carries an electric charge; for we have learned that a vibrating electric charge, such as that with which we have become familiar in the sparks from a Leyden jar, can start electric waves similar to the light waves (Art. 402). The hypothesis that there are in Nature tiny particles which carry electric charges is further justified by the phenomena of electrolysis; for we have learned (Art. 286) that the actions of the ions in an electrolyte is described most simply by assuming that they carry electric charges. So we have come to believe that light waves have their origin in the vibration of minute electrically charged particles. These particles have been named ELECTRONS.

Do they manifest themselves in any other way than as sources of light waves? Are they the same as the ions of electrolysis? How large are they? How are they set into vibration?

406. Crookes' Vacuum Tubes. Taking up these questions in order, we may answer to the first that we have good reason to believe that the phenomena observed in connection with electric discharge in vacuum tubes are due to these electrons. What are some of these phenomena? Let us analyze the action in a vacuum tube as the air pressure is gradually diminished. Thus, if we send the spark from an induction coil through one of these tubes, the discharge inside the tube, if none of the air has been pumped out of it, will, of course, be exactly like that in air. But if we connect the tube with an air pump, and begin to pump the air out, the appearance of the discharge changes. It first becomes less like the familiar, sharply defined, zigzag flash, Fig. 111, and spreads out in a broad band, giving a diffused purple glow. Presently, as more air is pumped out, this light appears to grow whiter and fill the tube. As the exhaustion is continued, the whiteness disappears, and we notice a pale blue streamer extending from the negative electrode and perpendicular to its surface. Since this streamer proceeds from the negative pole or cathode, it is called the cathode beam and is said to consist of CATHODE RAYS.

407. Cathode Rays. These rays have been carefully studied and found to possess many peculiar properties. We find, in the first place, that where they fall on the glass walls of the tube or on certain substances placed in their path, they cause these substances to emit light. They are then said to produce FLUORESCENCE. In the next place, the cathode rays travel in straight lines perpendicular to the surface of the cathode, for such fluorescent light always appears in the direction perpendicular to the surface of the cathode, and the shadow cast by a metal screen is perfectly well defined, as shown in Fig. 235. Finally, they are sensitive to a magnetic field; for if a magnet is brought near the tube, the

positions of the fluorescent spots change. In the light of these facts, what assumption may we make as to the nature of these rays? All the phenomena exhibited by them may be simply described if we assume that they consist of minute particles, carrying electric charges, and shot out from the cathode by the electric force there acting. Since the cathode is the negative pole, it is clear that the charges carried away from it by the particles must be negative.

FIG. 235. SHADOW IN CATHODE RAYS

It is easy to see how such negatively charged particles, when shot from the cathode, would be electrostatically repelled along straight lines perpendicular to the surface of the cathode. It is also easy to comprehend how they can produce fluorescence when they strike, by jarring the smallest particles in the substance on which they strike, and so causing them to shiver and send out light. But why should the cathode rays be bent from their straight path by a magnet? To understand this, we must recall the fact that a charged particle in motion produces a magnetic field (Art. 227). Therefore, if the cathode rays consist of charged particles traveling in straight lines, they should produce a magnetic field; and if they do so, of course, the cathode beam itself would be magnetic and would be repelled or attracted by an outside magnetic field.

We can thus understand how the phenomena exhibited by the cathode rays are simply described by assuming that those rays consist of negatively charged particles shot out from the cathode and traveling in straight lines. So we have found that in the phenomena of the vacuum tube we may be dealing with electrically charged particles or electrons. We may then ask whether these cathode particles may not be similar to those whose vibrations we have conceived to be the source of light waves. We can best arrive at a conclusion on this matter if we first find out something

concerning the size of the electrons in the cathode rays. But can we determine their size?

408. Size of Electrons. Science has been unable to answer this question until very recently, for it is no easy matter to determine the mass of particles so small that the weight of millions of them together could not be detected by our most sensitive balance. However, the bending of the cathode rays by a magnetic field enables us to approach a solution in the following way: If the cathode rays consist of particles each carrying an electric charge e and traveling with a velocity v, it is clear that the strength of the magnetic field generated by them will be proportional to both e and v (*cf.* Art. 227). Therefore, the force acting between the field of each particle and that of the external magnet, H, will be proportional to $e\,v\,H$ (Art. 205). But this force gives the particle a sideways acceleration, and this is clearly proportional to the magnetic force $e\,v\,H$, and also inversely proportional to the mass m of the particle. Therefore the sideways acceleration is proportional to $\frac{e\,v\,H}{m}$. Now, it is easy to measure H, and it has been possible to measure v; and since we can also measure the sideways acceleration, we can determine the value of $\frac{e}{m}$; i.e., of the ratio of the charge of a particle to its mass. This ratio is found to have the numerical value of 1.87×10^7. The value of this ratio is the same no matter of what substance the cathode is made.

Now, we have conceived (Art. 227) that the particles acting in electrolysis are charged particles, and that they move under the action of the electric forces in the solution. Evidently the velocity of their motion will be proportional to the charge e which each carries, to the strength E of the electrostatic field, and inversely proportional to the mass m of the particle; i.e., their acceleration will be proportional to $\frac{e\,E}{m}$. It is easy to measure their acceleration, and also to determine E, and so we reach another value of this ratio $\frac{e}{m}$; but this time it is for hydrogen 10^4.

What is the reason for this difference in the two results? Are the charges e the same, and the mass of the cathode particles less, or are the masses the same, and the charges of the cathode particles greater? The answer to this question we owe to the skill and ingenuity of J. J. Thomson, of Cambridge, England; for he has succeeded in showing by experiments, whose description would lead us too far from our present argument, that the charges in the two cases are the same, and therefore that the masses of the cathode particles are much smaller than those of the ions with which we deal in electrolysis. But in electrolysis we have every reason to believe that the smallest masses of the elements involved, i.e., the ions, are the atoms. Therefore we conclude that the cathode particles or electrons are much smaller than atoms.

The relative sizes of the atom and the electron may be obtained by dividing one value of the ratio $\frac{e}{m}$ by the other. Thomson thus arrived at the conclusion that *electrons are so small that it takes* 1870 *of them to make a hydrogen atom.* Experiments with other elements than hydrogen tend to show that the ratio $\frac{e}{m}$ in electrolysis is inversely proportional to the masses of the atoms, and therefore we must conclude that the number of electrons necessary to produce a mass equal to that of an atom of any element is 1870 times the mass of an atom of that element. This conclusion has recently been confirmed by other experiments along wholly different lines; so we now believe that an atom is composite and contains many electrons in its make-up.

Thus, at last, science seems to have found in the electron a particle which is smaller than the atom and which may be found to be the ultimate particle of matter.

409. Radio-Activity. But are electrons found free in nature, or do we know them only in vacuum tubes? The study of the recently discovered phenomena of radio-activity leads us to believe that *the radium rays consist of the emission, by radium and other similar substances, of electrons carrying negative charges;* for the

emanations of these substances act in many ways like the cathode rays. Probably many of you have seen one of Professor Crookes' spinthariscopes, in which a small particle of radium is mounted over a screen of some substance, like zinc sulphate, which becomes fluorescent when cathode rays fall on it. On observing the screen in a dark room, it is seen to be scintillating all over with tiny sparks. These are believed to be produced by the bombardment of electrically charged particles that are shot out by the radium so as to strike the fluorescent screen. This phenomenon and many others lead us to believe that electrons exist free in nature, since they are emitted by the radio-active substances.

410. How Electrons Start Ether Waves. We may now return to the phenomena of light, and ask how the conception that atoms are made up of large numbers of electrons assists us in conceiving a mechanism to describe the origin of light waves. We have already learned that light waves are brought into existence by sufficiently heating any substance. We must then try to form a picture of the way in which the heat energy may be converted into energy of vibration in solids and in gases. The first point that must be noticed in this connection is that heat expands the body; and therefore, if we conceive the body to consist of small particles, these must be separated from one another by the action of the heat. Further, we have noticed how all bodies are sending out long heat waves at all temperatures (Art. 149), and from this fact we must conclude that the small particles of which the body consists are vibrating even at the low temperatures. Therefore we may imagine that heating a body both increases the amplitude of the vibrations of those small particles, and also separates them further from one another.

When these particles form a solid, they are very close together, and therefore, when their vibrations are rendered more intense by heating, they must collide with one another more frequently and with greater energy than at a lower temperature. When two particles collide, each receives a shock, which must cause its component parts to shiver and send out for a brief instant a large number of waves. These waves are not those corresponding

to the natural periods of vibration of the particles. Such vibrations are called forced vibrations (Art. 333), and they die out very rapidly. But *when the particles make up a solid substance, the impacts between them take place so frequently that almost all the vibrations sent out by a solid are forced vibrations. Therefore the spectrum of a solid sending out vibrations under these conditions, contains all possible wave lengths, and so it is a continuous spectrum.* We may thus draw a mental picture of the mechanism by which continuous spectra are formed.

But *when the particles are separated by considerable distances, as they are when the substance becomes a vapor, the collisions between the particles are less frequent, and the particles travel a considerable distance between those impacts.* Now, although at the instant of impact they send out forced vibrations of all sorts of periods, these quickly die away, and thus leave the particle free between impacts to send out its own natural vibrations; *therefore the spectrum of incandescent vapors consists mainly of vibrations that are characteristic of the particles themselves,* for each has its own particular natural period and has plenty of room in which to vibrate.

411. X-Rays. Another interesting form of radiation is that found in the X-rays. The apparatus used for generating these rays is shown in Fig. 236. So much has been written in the magazines about these rays, and their applications to surgery have brought them so prominently before the public, that we need only mention some of their most marked characteristics. These are: 1. They appear to originate wherever cathode rays fall on matter of any kind. 2. They travel in straight lines, but, unlike light, they are not reflected or refracted. Thus they can not be deviated by a prism or brought to a focus by a lens. 3. Unlike the cathode rays, they are not deflected by a magnet. 4. The depth to which they penetrate material substances is nearly proportional to the densities of the substances. Thus they pass readily through wood and animal tissues, which are opaque to light. 5. They produce fluorescence in some substances, like platinum-barium cyanide and calcium tungstate.

These substances are therefore used in making the screens with which shadows of the bones may be seen. 6. They act on

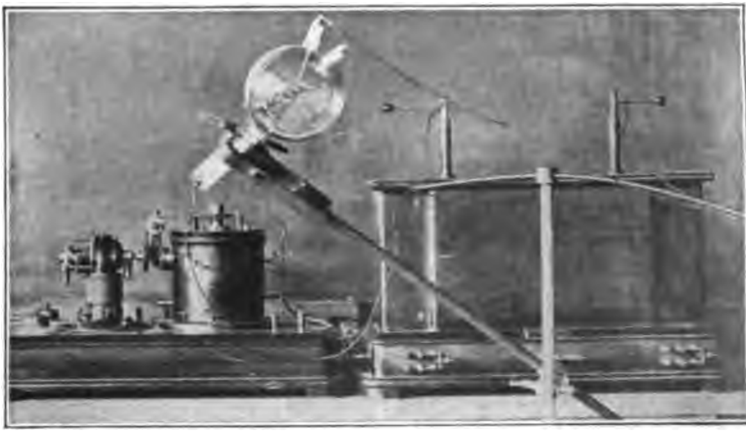


FIG. 236. A MODERN X-RAY COIL AND TUBE

a photographic plate, so that shadow pictures of the bones may be made. Fig. 237 is such a picture of a hip joint. 7. When they fall on an electrically charged body, the charge quickly escapes.

What are these rays? Since they are not refracted or reflected as light is, they probably do not consist of waves. Further, they are not deflected by a magnetic field, and therefore they probably are not projected charged particles. In order to gain some conception as to what they may be, consider the way in which they are generated. A negatively charged particle, traveling with a great velocity, strikes on some sort of matter. Such a traveling electron produces, while in motion, an electric current; but when its motion is suddenly stopped the current it is producing stops also, and this sudden stopping of the current must give the ether a quick jar or impulse. This impulse in the ether might be likened to that produced in air when a hammer strikes a block of wood. There is a sudden, sharp impulse of sound and we hear a click. Such an impulse is not a wave, though it is propagated like a wave in that it travels with the velocity of sound in air,

and also in that it can be heard. Similarly, we are led to the conception that X-rays may be a series of such sudden impulses in ether—an ether phenomenon analogous to the sound phenomena produced by a Fourth of July celebration. Such a series of

FIG. 237. X-RAY PHOTOGRAPH

impulses would travel with the velocity of light, would not be reflected or refracted as light is, and would not be deflected by a magnet; i.e., it would possess the properties exhibited by a beam of X-rays.

412. White Light. This conception of a series of impulses in ether may help us to conceive of the nature of ordinary white light. For we have seen how such light consists of a vast complex

of waves, each originating at one electron in a complex atom. We have also noted how the nature of the complex vibration changes with every impact of the atom. Therefore we may conceive that white light consists of a vast complex of comparatively short wave trains, each containing a few hundred thousand waves. Hence a wave front can not be said to exist in white light, since there is no plane or line in which all the particles are in the same phase at the same time. This same conclusion holds even for monochromatic light. Thus we find that the wave fronts of which we have talked are only convenient mathematical fictions, which make it possible for us to discuss in a rough sort of way the marvelously complex phenomena of light.

413. Properties of Electrons. Before closing this discussion it will be well to recall the facts and theories which we have been studying, and to see if we can arrange them into a satisfactory whole. To do this, let us review the main outlines of the argument, and then try to show how the results obtained by it may lead to clear conceptions as to the mechanism of these phenomena of Nature. 1. We have learned that it has been possible to recognize the existence of particles smaller than atoms—about $\frac{1}{1800}$ of the size of a hydrogen atom. 2. These particles have been found to carry negative electric charges, and to travel with high velocity. 3. It has also been possible to conceive how their oscillations within atoms may produce the waves which we call heat and light. 4. We have seen how these charged particles, or electrons, seem to be of the same size and nature, no matter from what substance they come. 5. We have learned that X-rays are generated when these particles strike on matter. 6. We have found that electrons are continually being shot out by radioactive substances.

414. Positively Charged Particles. We may now ask, Are positively electrified particles known? Electrons are always charged negatively. The answer to this question may be obtained from a further study of radio-activity. It is found that radio-active substances emit three kinds of particles. These

three kinds have been named, from the Greek letters, the alpha, the beta, and the gamma particles, and they have different properties. The beta particles are found to act in many ways like the electrons; i.e., they have a mass about $\frac{1}{1800}$ of that of the hydrogen atom; they travel with a velocity nearly equal to that of light, and they carry negative charges. Further, their ability to penetrate into substances depends only on the density of the substance, being inversely proportional to it. The alpha particles, on the other hand, are much larger, travel with a velocity of only $\frac{1}{10}$ that of light, and carry positive charges. These particles can penetrate into substances much less easily than the beta particles. However, on account of their greater mass, they possess greater kinetic energy than the beta particles. Their mass is found to be about the same as that of an atom of helium; i.e., about that of two hydrogen atoms. The nature of the gamma particles has not yet been determined.

415. The Theory of Atomic Structure. The phenomena of radio-activity are believed to consist in the spontaneous breaking up of the atoms of the radium or of the other substances. And since this disintegration produces both positive and negative particles, we have to conceive that the atoms consist of both. In fact, we should have to conceive that both exist in atoms in order that they remain stable, for a large number of negatively charged particles would repel one another and not stay long together in one group, unless they were held there by some positive attracting force. So we are led to believe that *the atom consists of a positively charged particle about which a number of electrons are rotating, like the planets about the sun;* i.e., we imagine that the atom of matter is constructed on the same general plan as the solar system, which may thus be considered as an atom of the universe.

Professor J. J. Thomson, to whom we are particularly indebted for the experimental work on which this new theory of matter is based, has shown how such complex particles might be formed, and what arrangements of electrons about the central, positively charged particle are mechanically possible. He has

compared his results with the order of the chemical elements according to their chemical properties as the chemists have arranged them, and finds almost entire agreement. While this fact is of great importance and of wonderful interest, it must not be taken to be a proof that atoms are really so formed. All that we can prove is that it is one possible way.

Those who are interested in the remarkable and rapid progress that has been made in the last ten years by science in thus prying into the nature of atoms will find a very good account of the theory and its consequences in Whethan, *Recent Development of Physical Science* (London, Murray, 1904). The limitations of this, our work, make it possible to give only the barest outlines of the subject.

416. Conclusion. We must not, however, be led to think that science has now solved the riddle as to the nature of matter. For, even if we have discovered the mechanism of atoms, we have only pushed the bounds of ignorance one step further back. Though we may be able to say that the atom is not indivisible, but is constructed in such and such a way, *we have still to show what electrons are, what the ether is, what an electric charge is, and whence they all come.* It must, nevertheless, be clear to every one who has read this book carefully, that nature is not a vast chaos of chance happenings, but a well ordered and governed whole. When we study thoughtfully the phenomena about us, we must realize that there are some simple and universal principles which are manifest in them all. Therefore, let us leave our study with this idea: that the universe in which we live is a marvelously organized and governed unit. And when we try to imagine how such a unit could have been developed, we are compelled to recognize that it could not have come to its present perfection if it originated in an unthinkable chaos, and organized itself solely by the interaction of blind matter and undirected motion.

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